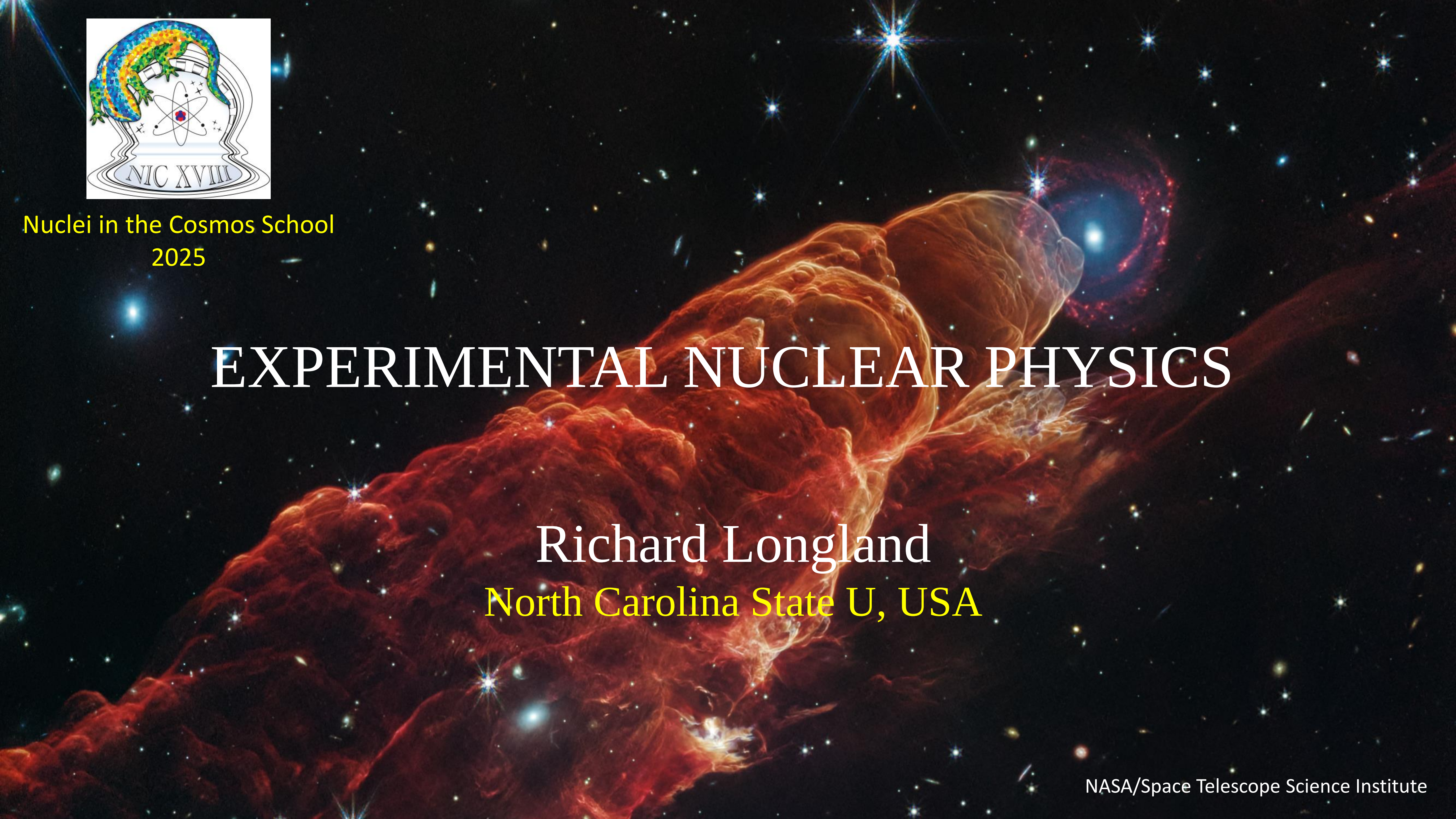

Nuclei in the Cosmos School
2025

EXPERIMENTAL NUCLEAR PHYSICS

Richard Longland
North Carolina State U, USA

NASA/Space Telescope Science Institute

Outline

1. Introduction
   - What's happening in the star?
   - Nuclear reaction cross sections

2. Determining cross sections experimentally
   - Experimental yields
   - A brief primer on stopping powers
   - Extract cross sections from yields
   - Experimental yield
   - Absolute cross sections
   - Count rates

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Example: the $^{16}O(p,\gamma)^{17}F$ reaction

Happening in our sun right now

- Mass of burning region M = 0.1 $M_\odot$
- Temperature T_9 = 15 MK
- Density ρ = 150 g/cm^3
- Assume
  - mass fraction of hydrogen $\sim$ 99%
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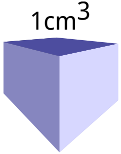
Total reaction rate

$$R = 1 \times 10^{35} \text{ s}^{-1}$$

Reproduce this in the lab

- What if we make a small plasma (of the same temperature and chemical make-up) in the lab?



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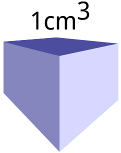

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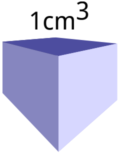

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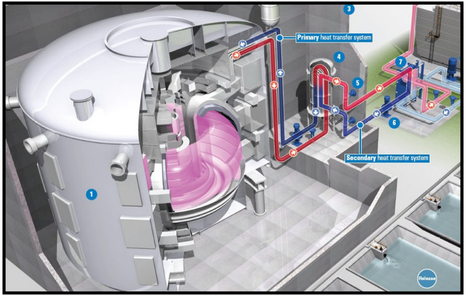

- Let's make a big plasma!
- Model reactor as a sphere with r = 6 m: V = 9 × 10^8 cm^3

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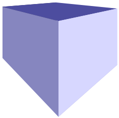
$1\,\mathrm{cm}^3$



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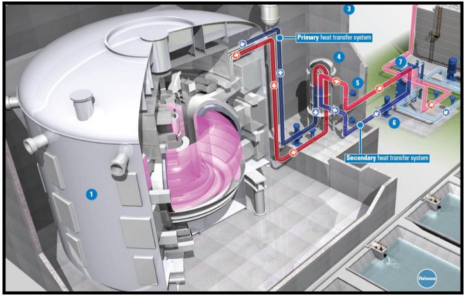
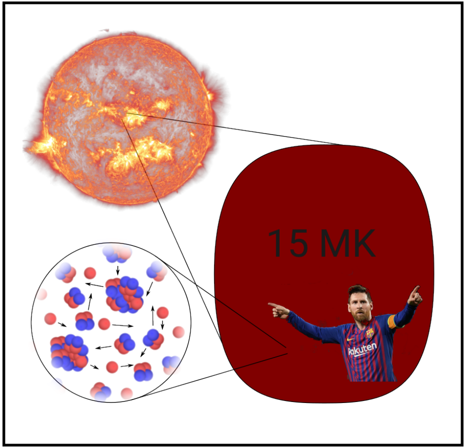


- Let's make a big plasma!
- Model reactor as a sphere with r = 6 m: V = 9 × 10^8 cm^3
- 150 reactions per year

Time to get smart



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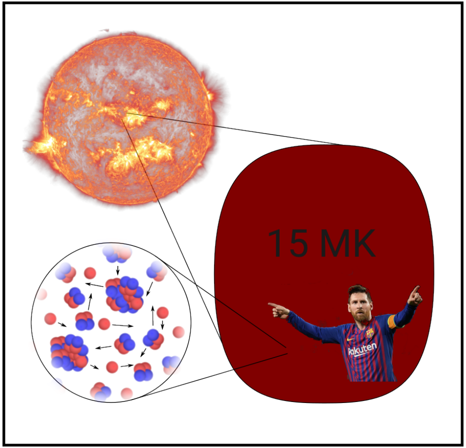

- Particle velocities distributed according to a Maxwell-Boltzmann distribution

$$N_A\langle\sigma v\rangle_{01} = \left(\frac{8}{\pi\mu_{01}}\right)\frac{N_A}{(kT)^{3/2}}\times \int_0^\infty E\,\sigma(E)\,e^{-E/kT}\,dE$$

($cm^3\ mol^{-1}\ s^{-1}$)

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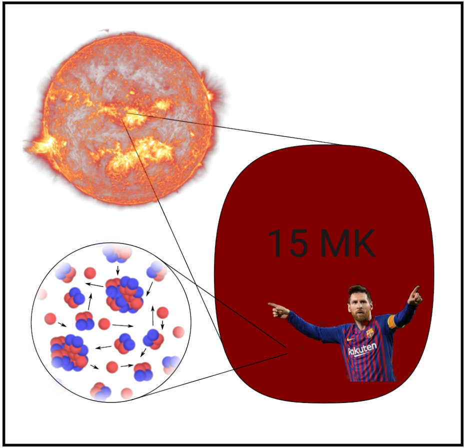

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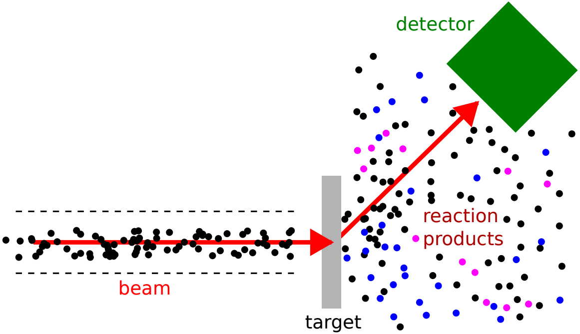
(cm^3 mol^{-1} s^{-1})

- For a known temperature, we only need to know $\sigma(E)$ to calculate reaction rate
- For the rest of my lectures, we will concern ourselves with *how*, exactly, to measure σ(E)

What, exactly, is σ(E)

Cross section

$$\sigma(E) \equiv \frac{\text{number of reactions per time}}{\text{(number of incident particles per area per time)(number of target nuclei in the beam)}}$$



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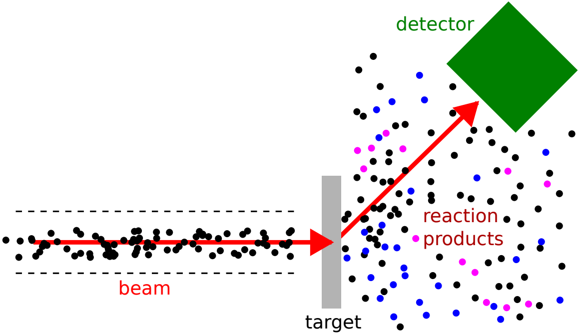
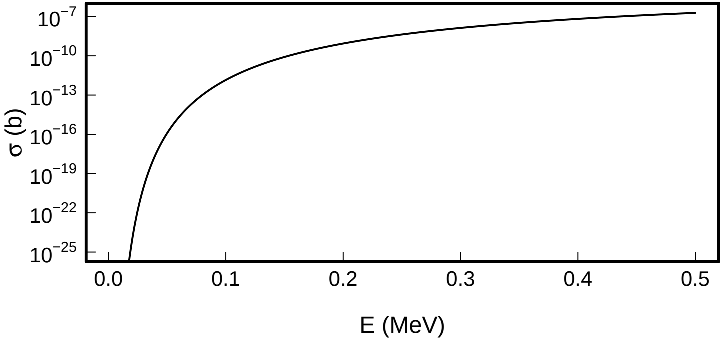
- Number of reactions
  - We need a detector that can count outgoing radiation from our experiment (Faïrouz Hammache will cover details)
- Number of incident particles
  - Need a beam of particles (Faïrouz again!)
- Number of target nuclei
  - Need to create a target (yes, Faïrouz will cover this too!)



Planning

- Stay with $^{16}O(p,\gamma)^{17}F$ reaction
- First question: *where* (i.e. what energy) should I measure cross section?

$$\langle \sigma v \rangle = \sqrt{\frac{8}{\pi\mu}} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E e^{-E/kT} dE$$



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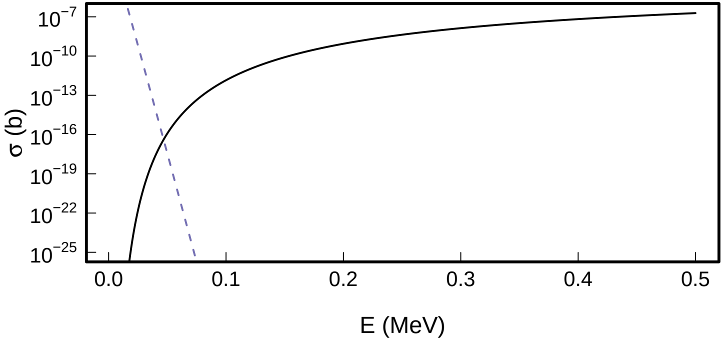

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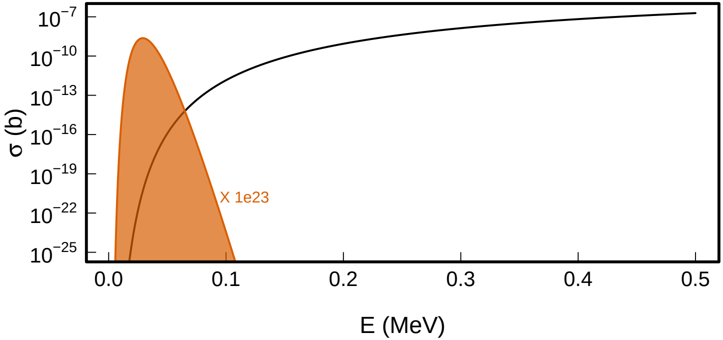
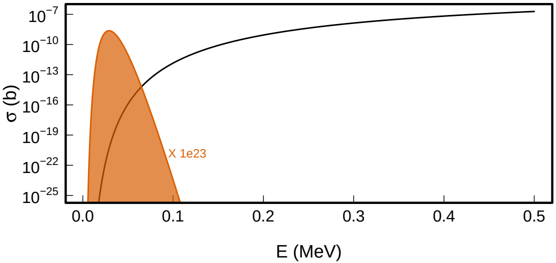


Gamow Peak

- Useful in planning *where* to measure a cross section, but comes with some caveats
- Assumption 1: cross section dominated by s-wave Coulomb barrier penetration
- Assumption 2: it can be approximated by a Gaussian



Gamow Peak

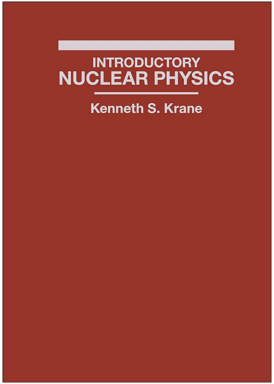
$$E_0 = \left[\left(\frac{\pi}{\hbar}\right)^2 \left(Z_0 Z_1 e^2\right)^2 \left(\frac{\mu}{2}\right) (kT)^2\right]^{1/3}$$

$$\Delta = \frac{4}{\sqrt{3}\sqrt{E_0 kT}}$$

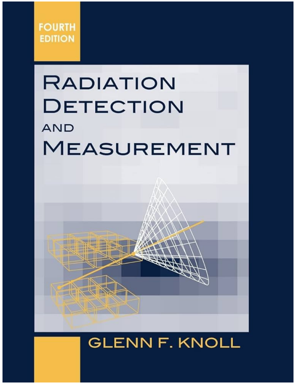
For ^{16}O(p,γ)^{17}F at 15 MK, E_0 = 30 keV and ΔE = 15 keV

Outline

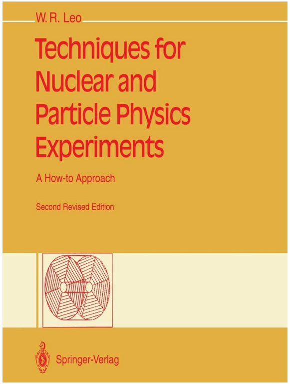
Get these books



Krane: Introductory Nuclear Physics



Knoll: Radiation Detection and Measurement



Leo: Techniques for Nuclear and Particle Physics Experiments

Preparing for a cross section measurement

Cross section

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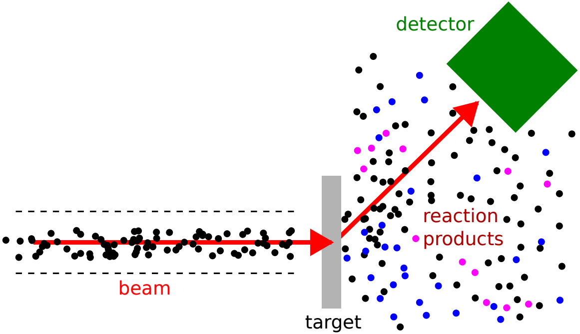
Preparing for a cross section measurement

Cross section

$$\sigma(E) = \frac{N_R/t}{(N_b/tA)(N_t)}$$

- Complication
  - Assumes that beam particles interact with target nuclei at one energy, E
  - (they don't)
  - Particles slow down as they traverse the target - we need to account for this
- In the laboratory we measure Yield: $Y \equiv N_R/N_b$
- Conversion of yield to cross section depends on nature of experiment



Experimental Yield

- Experimental yield vs. beam energy is a yield curve
- Yield includes effects of the beam slowing down in the target

- No way of knowing (for now) where the interaction took place
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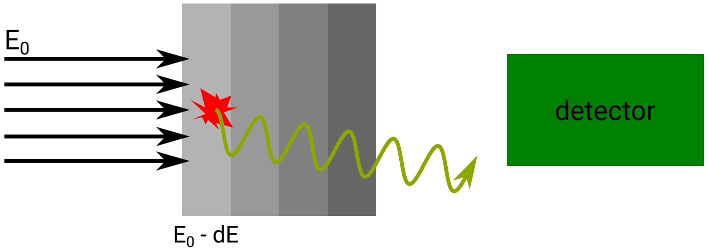

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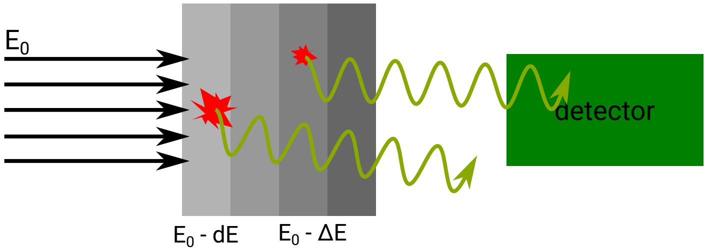

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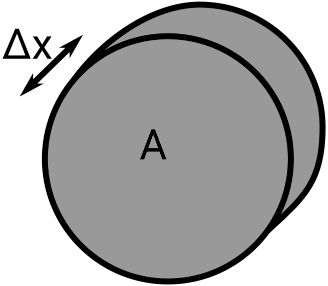
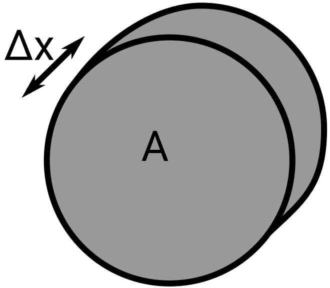


- In slice i

$$Y_i = \frac{N_{R,i}}{N_b} = \sigma_i \frac{N_{t,i}}{A} = \sigma_i \frac{N_i}{V_i} \Delta x_i$$

Experimental Yield



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- Turn this into an integral and change variables

$$Y(E_0) = \int \sigma(x) \frac{N(x)}{V} dx = \int \sigma(x) \frac{N(x)}{V} dx \frac{dE}{dx} \frac{dx}{dE}$$

- Use a new definition: Stopping power (ev cm^2/atom)

$$\epsilon(E) = -\frac{V}{N} \frac{dE}{dx}$$

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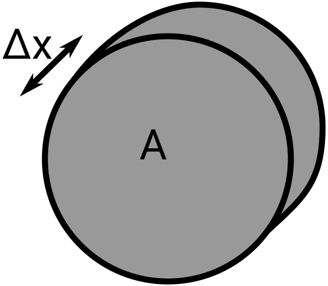

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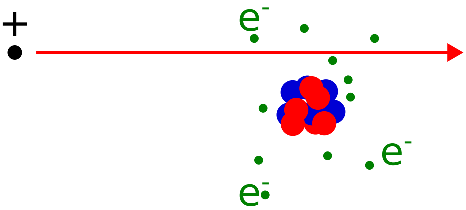
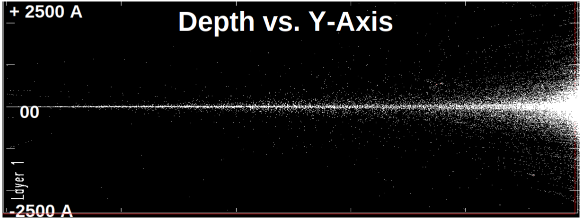
Interactions of radiation with matter

- Short story: as radiation interacts with matter, it deposits energy into the material.
- Charged Particles
  - Ion (usually positive) interacts simultaneously with many electrons
  - Energy deposit, ion slows down, electrons gain energy
  - Electrons undergo excitation or ionization





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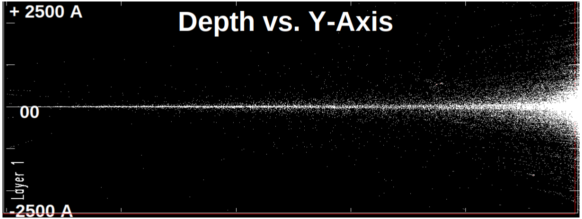
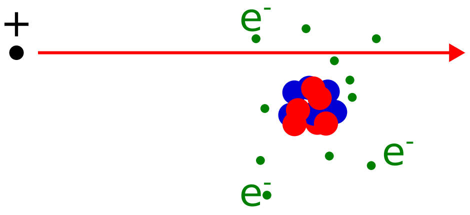




- Ionization maximum energy: $4Em_e/m$
- proton: 0.2% of total energy
- Expect many interactions
- Expect ~straight lines

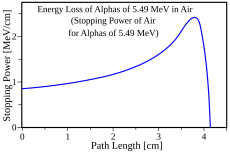
Charged particle interactions with matter

- Stopping power, S

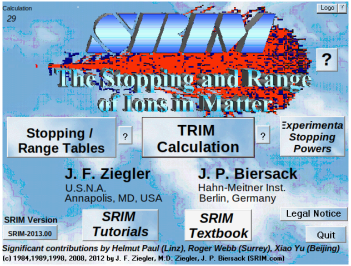
$$S = -\frac{dE}{dx}$$

- Integrate this to get range
- For charged particles, a useful approximation:

$$S \propto -1/E_k$$



- Conversions (assume Δx in cm and ρ in g/cm^3)
  - Thickness in g/cm^2:
    $\rho\Delta x$
  - Thickness in keV:
    $\Delta x \, dE/dx$
  - Angstroms from μg/cm^2:
    100 t / ρ
    e.g. 1.5 μg/cm^2 carbon foil (ρ=2.26 g/cm^3) = 66 Å
- Most people use SRIM for this: srim.org



Cases to consider

We need to determine σ(E) from Y(E_0)

$$Y(E_0) = \int_{E_0-\Delta E}^{E_0} \frac{\sigma(E)}{\epsilon(E)}\, dE$$

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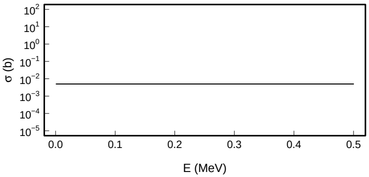

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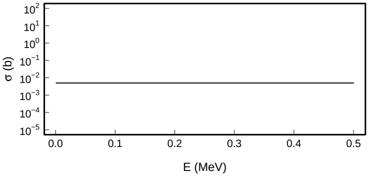
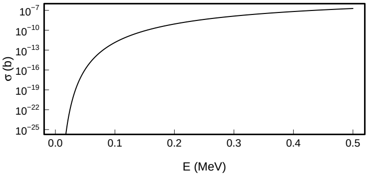


- Moderately varying σ and constant ϵ over target thickness

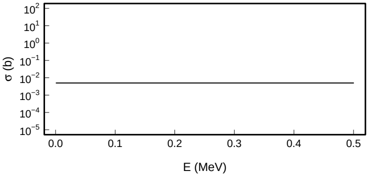


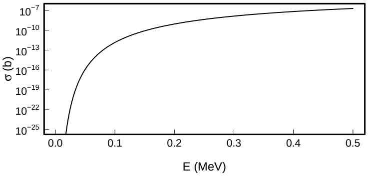
Cases to consider

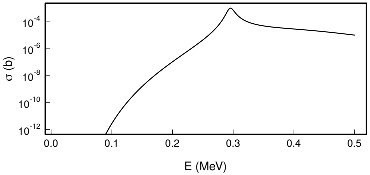
We need to determine σ(E) from Y(E_0)

$$Y(E_0) = \int_{E_0-\Delta E}^{E_0} \frac{\sigma(E)}{\epsilon(E)}\, dE$$

- Constant σ and ϵ over target thickness
- Moderately varying σ and constant ϵ over target thickness
- Strongly varying σ and constant ϵ over target thickness







Constant σ and ϵ (very thin targets)

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- Simplify to:

$$Y(E_0) = \frac{\sigma(E_0)}{\epsilon(E_0)}$$

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- Nope! Let's be more careful
- σ and ϵ are constant

$$Y(E_0) = \frac{\sigma}{\epsilon(E_0)} \int_{E_0 - \Delta E}^{E_0} dE = \frac{\sigma}{\epsilon(E_0)} \Delta E$$

- What energy was this cross section measured at?

Constant σ and ϵ (very thin targets)

$$Y(E_0) = \int_{E_0-\Delta E}^{E_0} \frac{\sigma(E)}{\epsilon(E)}\, dE$$

- Simplify to:

$$Y(E_0) = \frac{\sigma(E_0)}{\epsilon(E_0)}$$

- Nope! Let's be more careful
- σ and ϵ are constant

$$Y(E_0) = \frac{\sigma}{\epsilon(E_0)} \int_{E_0-\Delta E}^{E_0} dE = \frac{\sigma(E_{\mathrm{eff}})}{\epsilon(E_0)} \Delta E$$

- What energy was this cross section measured at?
- The "effective energy" i.e. the average energy in this case

Moderately varying σ and constant ϵ (thin targets)

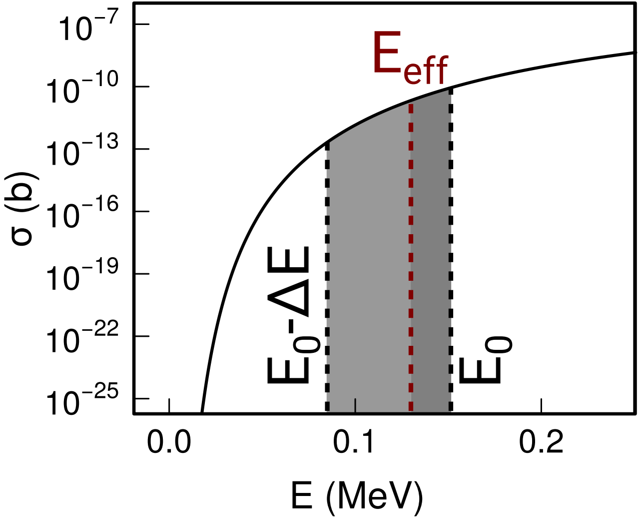
$^{16}O(p,\gamma)^{17}F$

$$Y(E_0) = \int_{E_0-\Delta E}^{E_0} \frac{\sigma(E)}{\epsilon(E)}\, dE$$

- Result from before:

$$Y(E_0) = \frac{\Delta E(E_0)\,\sigma(E_{\text{eff}})}{\epsilon(E_0)}$$

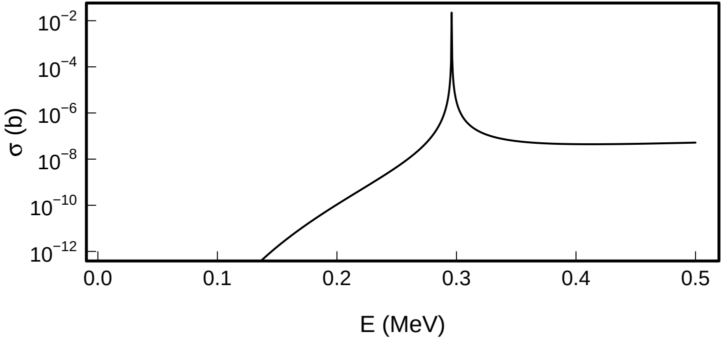
- But now the effective energy has to be carefully calculated - energy where 50% of the total yield is obtained
- Calculate numerically (maybe by approximating linear cross section)



Strongly varying σ and constant ϵ (thick targets)

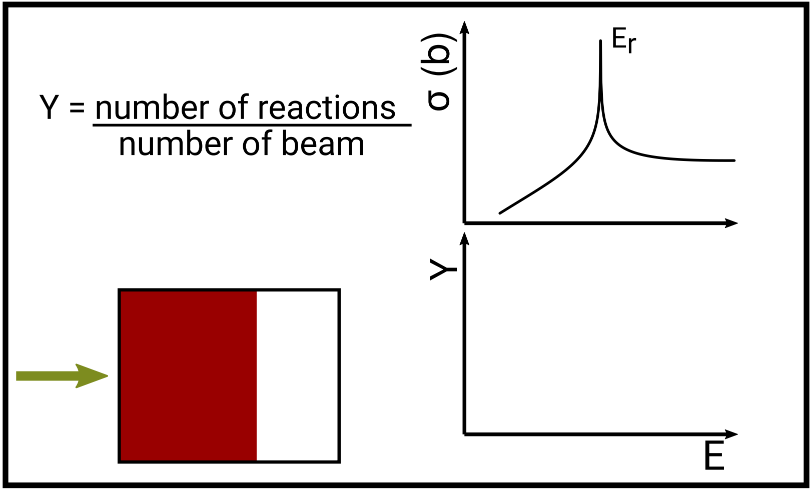
$$Y(E_0) = \int_{E_0 - \Delta E}^{E_0} \frac{\sigma(E)}{\epsilon(E)}\, dE$$

- Resonant cross section



- Cannot resolve the full shape of the resonance
- E_{eff} is the resonance energy

Strongly varying σ and constant ϵ (thick targets)



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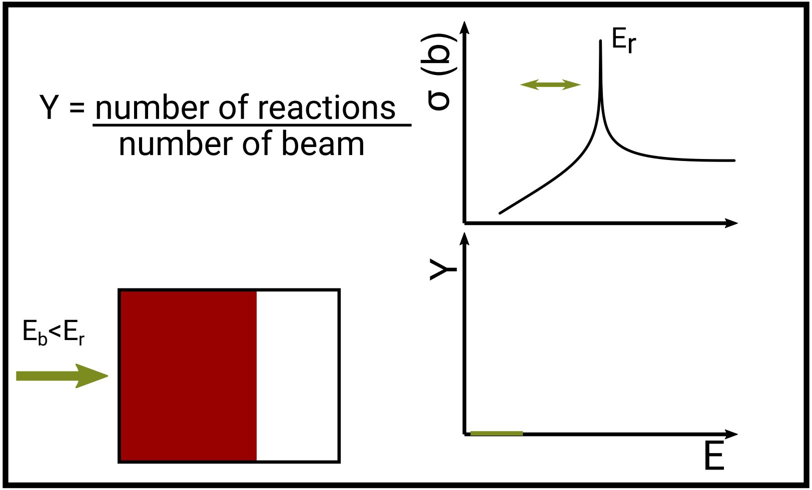

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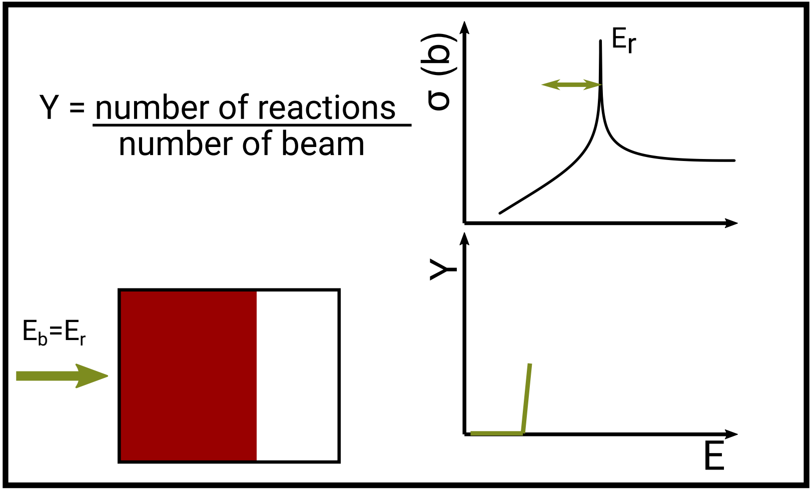

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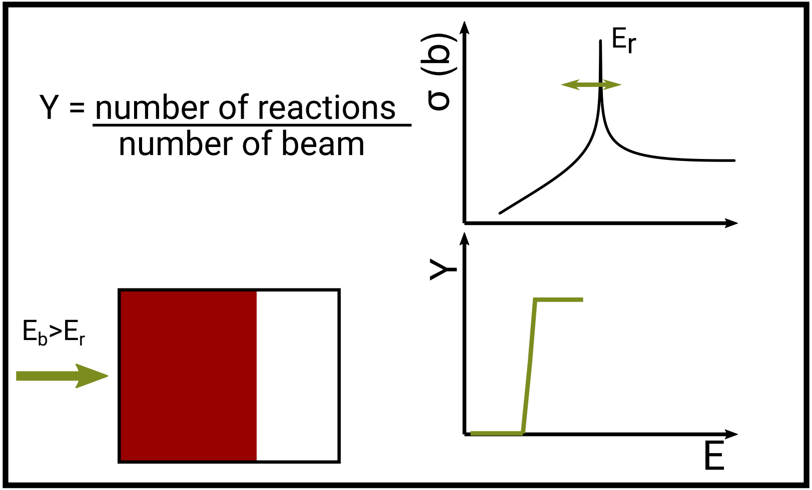

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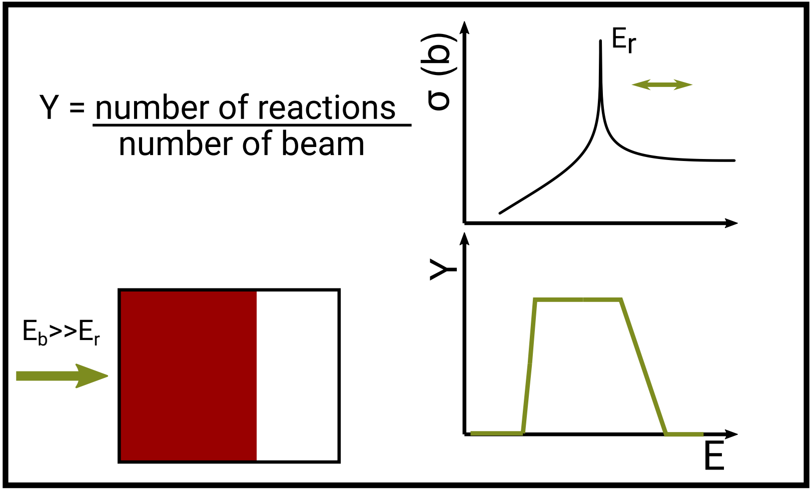


Strongly varying σ and constant ϵ (thick targets)

$$Y(E_0) = \int_{E_0 - \Delta E}^{E_0} \frac{\sigma(E)}{\epsilon(E)}\, dE$$

- Narrow resonance

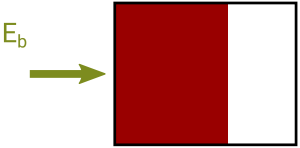
$$\sigma(E) = \omega \frac{\Gamma_a \Gamma_b}{(E - E_r)^2 + (\Gamma_a + \Gamma_b)^2/4}$$

- Integrate

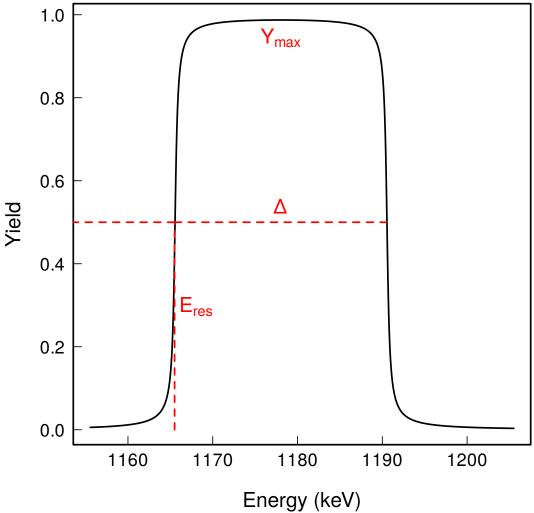
$$Y_{\rm max} = \frac{\lambda_r^2}{2} \frac{\omega\gamma}{\epsilon_r}$$

- $\omega\gamma$ is the resonance strength

Thick target yield curve details



- Thick target yield curve has some shape coming from width of resonance



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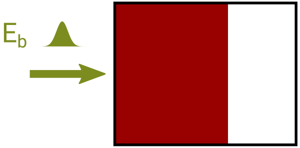
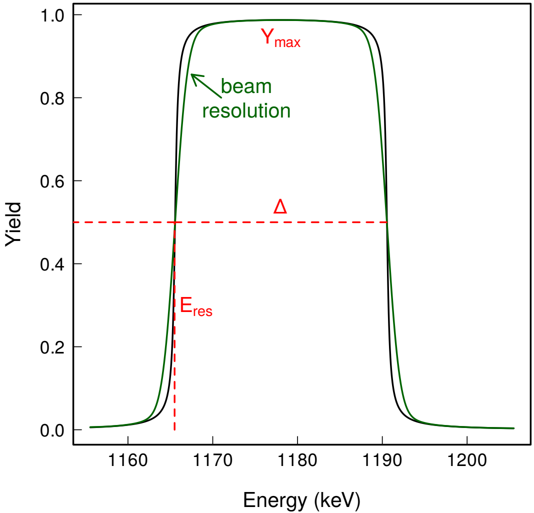


- Thick target yield curve has some shape coming from width of resonance
- Careful! Beam resolution also affects shape of resonance
- Beam and resonance width are roughly degenerate



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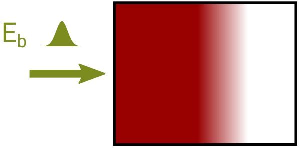
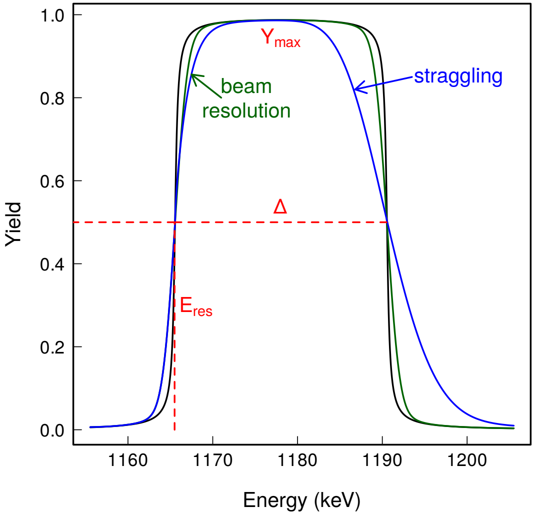
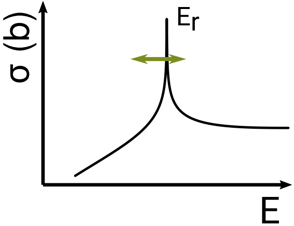


- Thick target yield curve has some shape coming from width of resonance
- Careful! Beam resolution also affects shape of resonance
- Beam and resonance width are roughly degenerate
- Beam and target straggling make yield curve asymmetric



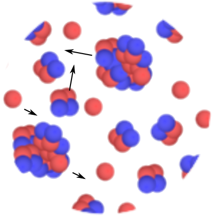
Strongly varying σ and constant ϵ (thick targets)

In the lab



$$Y(E_0) = \int_{E_0-\Delta E}^{E_0} \frac{\sigma(E)}{\epsilon(E)}\, dE$$

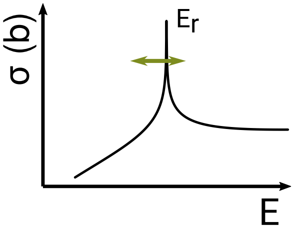
In the star



$$N_A\langle\sigma v\rangle_{01} \propto \int_0^\infty E\,\sigma(E)\,e^{-E/kT}\,dE$$

Strongly varying σ and constant ϵ (thick targets)

In the lab

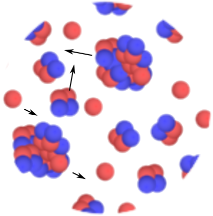


$$Y(E_0) = \int_{E_0 - \Delta E}^{E_0} \frac{\sigma(E)}{\epsilon(E)}\, dE$$

- If beam energy is just above resonance $\rightarrow$ integrate resonance

$$Y_{\max} = \frac{\lambda_r^2}{2} \frac{\omega\gamma}{\epsilon_r}$$

In the star



$$N_A \langle \sigma v \rangle_{01} \propto \int_0^\infty E\, \sigma(E)\, e^{-E/kT}\, dE$$

- If resonance is in the Gamow window $\rightarrow$ integrate resonance

$$N_A \langle \sigma v \rangle_{01} = N_A \left(\frac{2\pi}{\mu_{01} kT} \right)^{3/2} \hbar^2 \omega\gamma e^{-E_r/kT}$$

Impact of using narrow resonance assumption on reaction rates

- On the previous slide I used a key word: *if*
- When is the resonance strength no longer accurate?

$$N_A\langle\sigma v\rangle_{01} \propto \int_0^{\infty} E\,\sigma(E)\,e^{-E/kT}\,dE \qquad \propto T_9^{-3/2} e^{cE_r/T_9}$$

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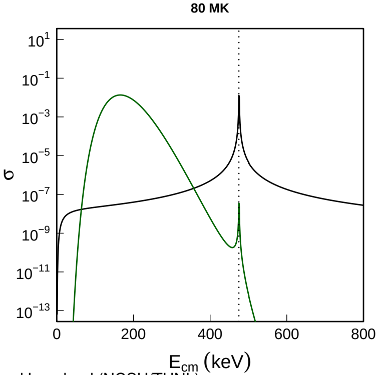
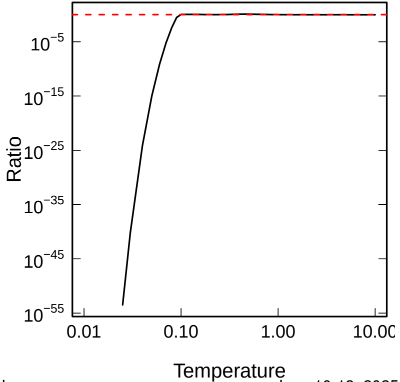
- Taking into account the tails of the resonance





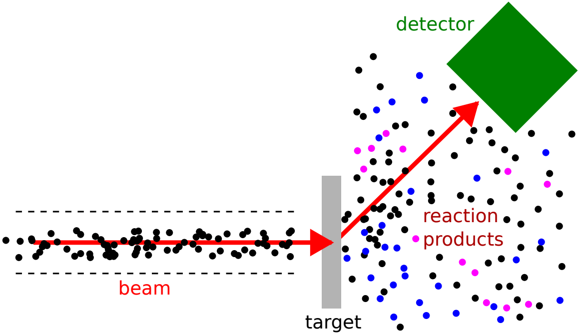
Experimental yield

Experimental yield

$$Y = \frac{N_R}{N_b}$$

- Detector doesn't measure all reaction products
- Reaction products aren't always emitted isotropically

$$Y = \frac{I}{N_b B \eta W}$$



- I: intensity of measured reaction product
- B: branching ratio if reaction product only carries part of strength
- η: detector efficiency
- W: Angular distribution correction

Resonance strength measurement

$$\omega\gamma = \frac{2}{\lambda_r^2}\epsilon_r \frac{I_{\max}}{N_b B \eta W}$$

There are several quantities in this equation that are hard to determine

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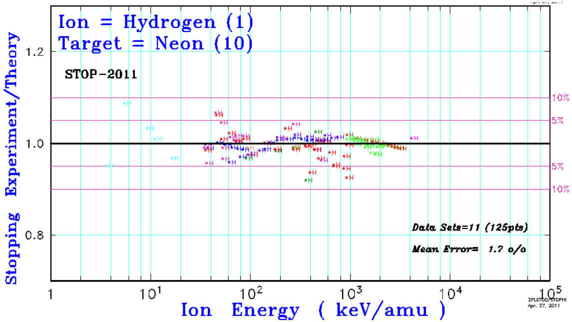
- ϵ_r: depends on
  - stopping powers
  - assumptions about target composition
  - target stability
- Stopping powers
  - SRIM.org
  - Be careful to check where measurements are available



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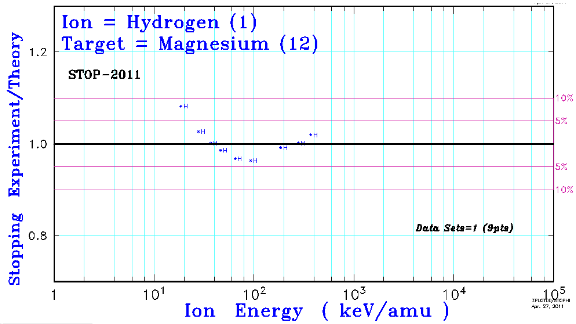
  - e.g. 1 MeV protons in magnesium



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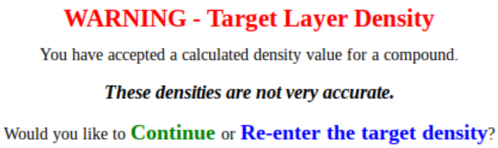
  - e.g. 1 MeV protons in magnesium
- e.g. what is the molecular model you're using?



Beam current

$$\omega\gamma = \frac{2}{\lambda_r^2}\epsilon_r \frac{I_{\rm max}}{N_b B \eta W}$$

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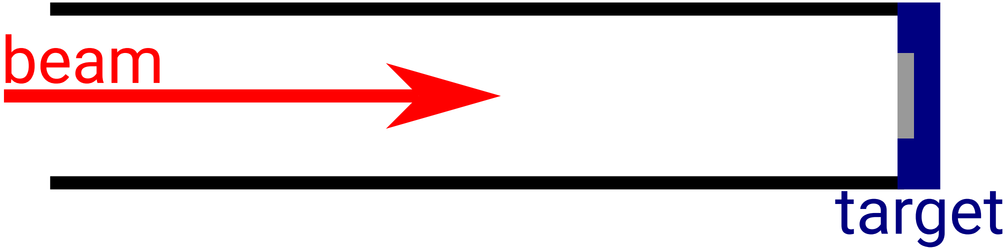

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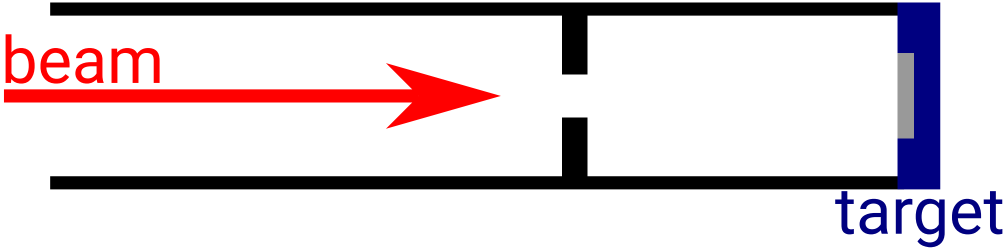

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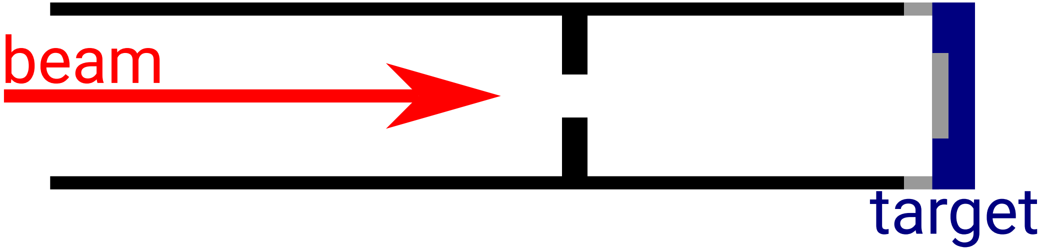

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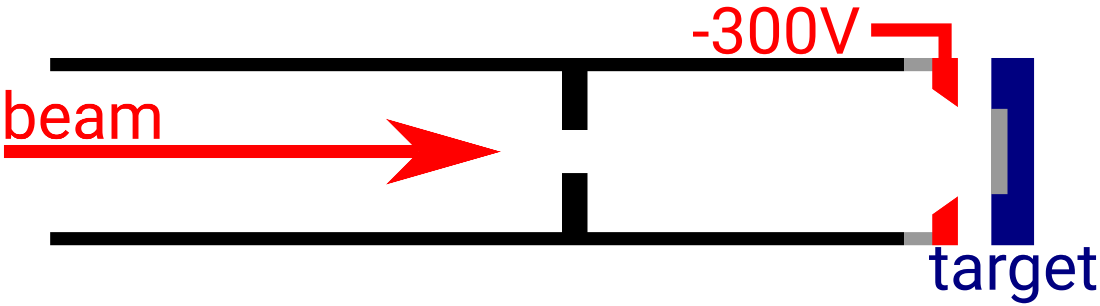
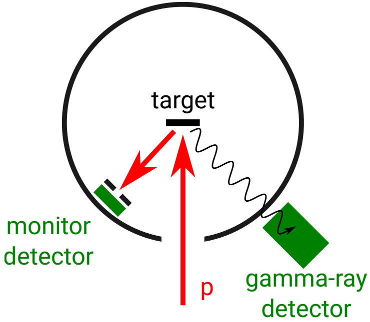


Performing measurements relative to Rutherford scattering

- Low-energy: beam scatters according to Rutherford



- Resonance strength from integrated yield

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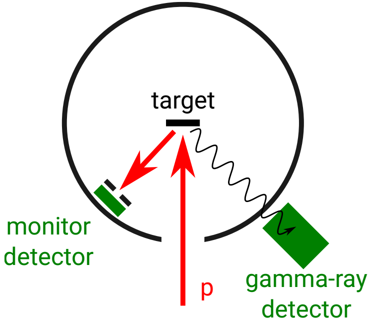

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- Yield from Rutherford scattering

$$Y_{\text{Ruth}} = \frac{N_{p'}}{N_p \Omega_{\text{mon}}} = \int_{E_0-\Delta E}^{E_0} \frac{\sigma(E)}{\epsilon(E)} dE \approx \sigma n_t$$

- Yield from reaction

$$A = \frac{1}{B_\gamma \eta_\gamma W_{\gamma(\theta)}} \int \frac{N_\gamma(E)}{N_p(E)} dE$$

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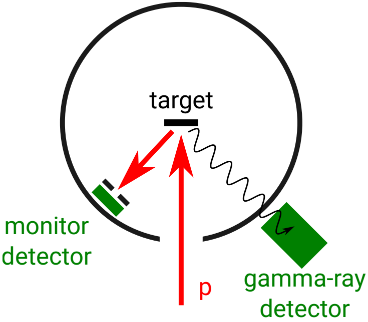

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- Yield from reaction

$$A = \frac{1}{B_\gamma \eta_\gamma W_{\gamma(\theta)}} \int \frac{N_\gamma(E)}{N_p(E)} dE$$

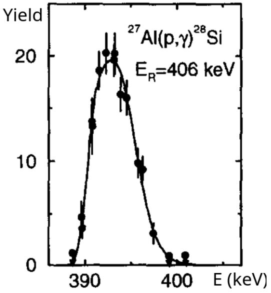
- Combining everything above

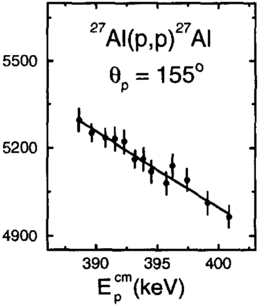
$$\omega\gamma = \frac{2}{\lambda_r^2}\frac{1}{B_\gamma \eta_\gamma W_{\gamma(\theta)}} \Omega_{\text{mon}} \int \frac{N_\gamma(E)}{N_{p'}} \sigma_{\text{Ruth}} dE$$

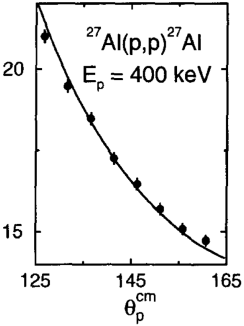
Performing measurements relative to Rutherford scattering

$$\omega\gamma = \frac{2}{\lambda_r^2} \frac{1}{B_\gamma \eta_\gamma W_{\gamma(\theta)}} \Omega_{\text{mon}} \int \frac{N_\gamma(E)}{N_{p'}} \sigma_{\text{Ruth}} dE$$

Powell et al., Nucl. Phys. A 644 (1998) 263







Absolute resonance strengths

Reaction	E_r^{lab}	$\omega\gamma$ (eV)	Uncertainty
$^{23}Na(p,\gamma)^{24}Mg$ (a)	512	8.75 (120) × 10^{-2}	14%
$^{23}Na(p,\alpha)^{20}Ne$ (d)	338	7.16 (29) × 10^{-2}	4.1%
$^{30}Si(p,\gamma)^{31}S$ (a)	620	1.89 (10)	5.3%
$^{18}O(p,\gamma)^{19}F$ (b)	151	9.77 (35) × 10^{-4}	3.6%
$^{27}Al(p,\gamma)^{28}Si$ (c)	406	8.63 (52) × 10^{-3}	6.0%

a) Paine and Sargood Nucl. Phys. A 331 (1979) 389

b) Panteleo et al., Phys. Rev. C 104 (2021) 025802

c) Powell et al., Nucl. Phys. A 644 (1998) 263

d) Rowland et al., Phys. Rev. C 65 (2002) 064609

And many more

Count rates (for narrow resonance)

- Recall our two expressions for yield: experimental and theoretical

$$Y = \frac{I}{N_b B \eta W} \qquad \text{and} \qquad Y_{\text{max}} = \frac{\lambda_r^2}{2}\frac{\omega\gamma}{\epsilon_r}$$

- Combine and find an expression for intensity of reaction products at top of yield curve

$$I = \frac{\lambda_r^2}{2}\frac{\omega\gamma}{\epsilon_r} N_b B \eta W$$

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- For convenience, convert N_b to $I_b/q_b t$
  - I_b is beam current
  - q_b is beam charge in Coulombs
  - Δ t is length of experiment

$$\frac{I}{t} = \frac{\lambda_r^2}{2}\frac{\omega\gamma}{\epsilon_r}\frac{I_b}{q_b} B \eta W$$

Example: ^{23}Na(p,γ)^{24}Mg reaction

$$\frac{I}{t} = \frac{\lambda_r^2}{2}\frac{\omega\gamma}{\epsilon_r}\frac{I_b}{q_b}B\eta W$$

Narrow resonance at 138 keV

- $\omega\gamma < 5 \times 10^{-9}$ eV
- Beam current: 200 μA
- Target effective stopping power: $\epsilon_r = 1.9 \times 10^{-14}$ eV cm^2/atom
- Detector efficiency: η = 1%
- Isotropic γ-ray emission: W=1

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Going forward

- By digging into the theory of cross sections a little more, we can use other techniques to learn the same information... next time!