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Search for high frequency gravitational waves in electromagnetic cavities

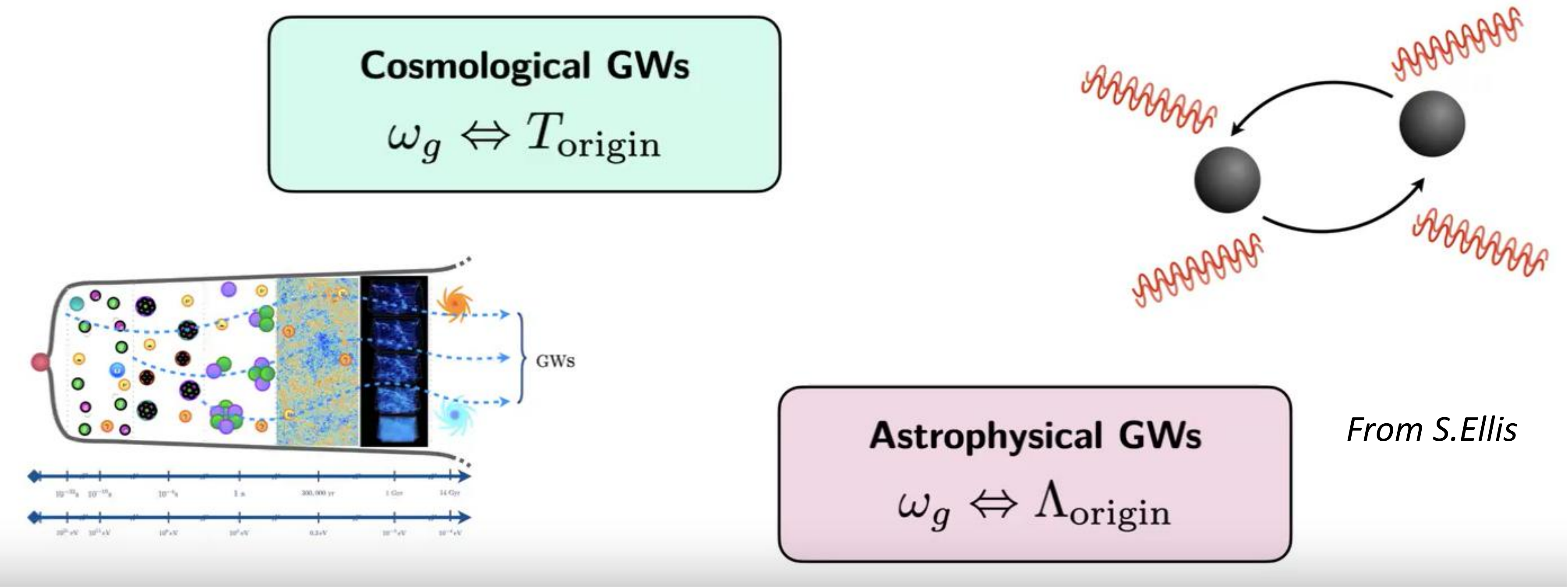
Jordan Gué

IFAE, Universitat Autonoma de Barcelona

In collaboration with T. Krokotsch (Universität Hamburg)

7th Barcelona Initiative for Gravitation

HFGW astro/cosmo signals



From S.Ellis

→ Higher GW frequency ⇔ Higher energy scale/Lower length scale we can probe

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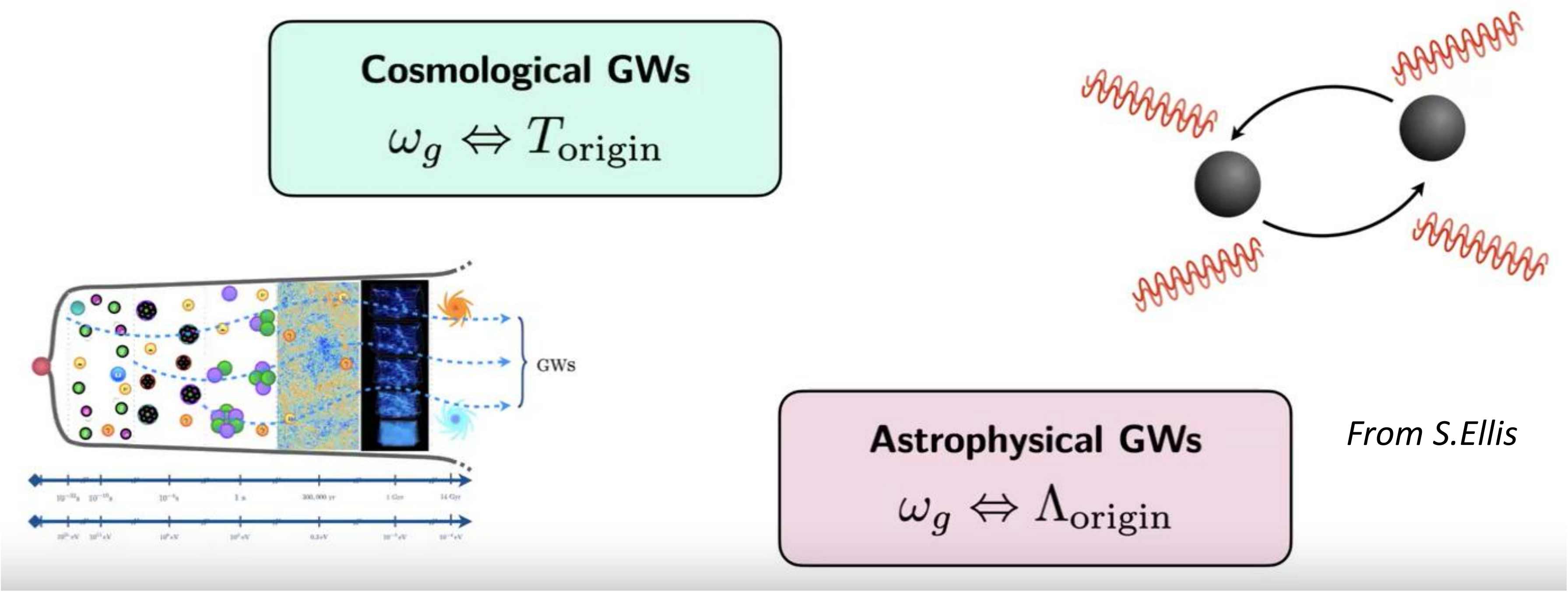

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Some astrophysical sources :

- Mergers of PBH
- Mergers of ECO (boson stars,…)

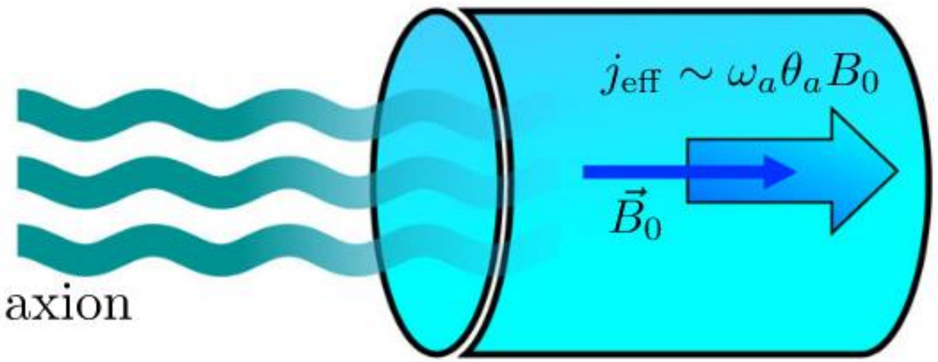
…

- First order phase transition in neutron stars
- Superradiant boson clouds orbiting SMBH (**monochromatic**)

See N. Aggarwal et al, arXiv 2501.11723

→ Many of those sources are BSM

Analogies with axion dark matter

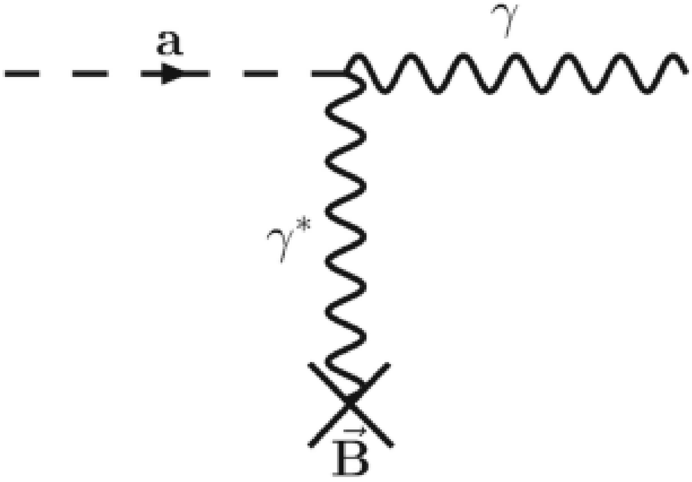


$$S = \int d^4x \left(-\frac{1}{4} g_{a\gamma} a \, \eta^{\mu\alpha} \eta^{\nu\beta} F_{\mu\nu} \tilde{F}_{\alpha\beta} \right)$$

A. Berlin et al, PRD ***105*** *(2022)*

- Simple way of looking for axions coupled to EM is through inverse Primakoff effect

→Use of microwave cavities to search for GHz axions



JK Vogel et al (2023)

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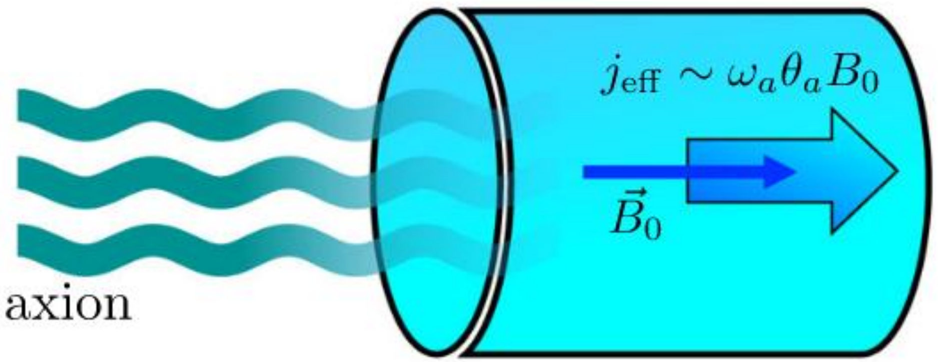

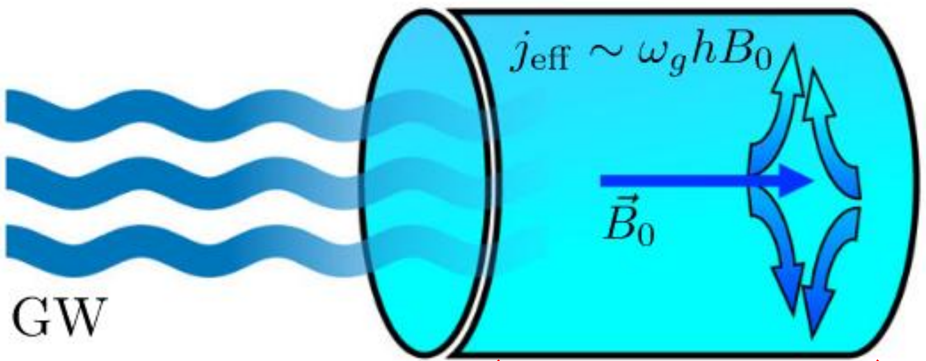

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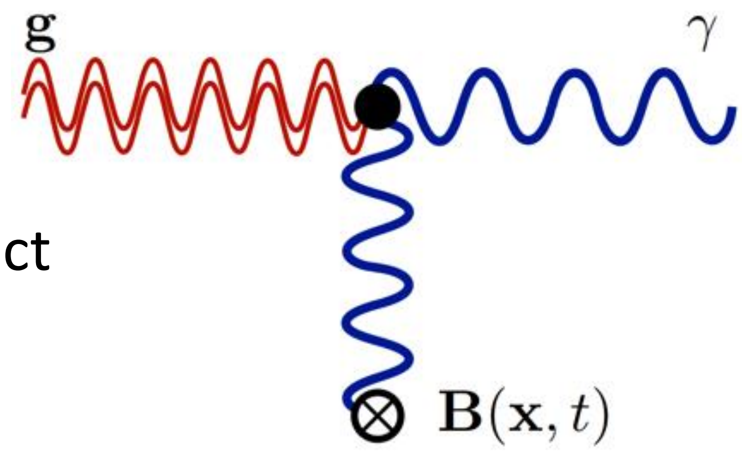
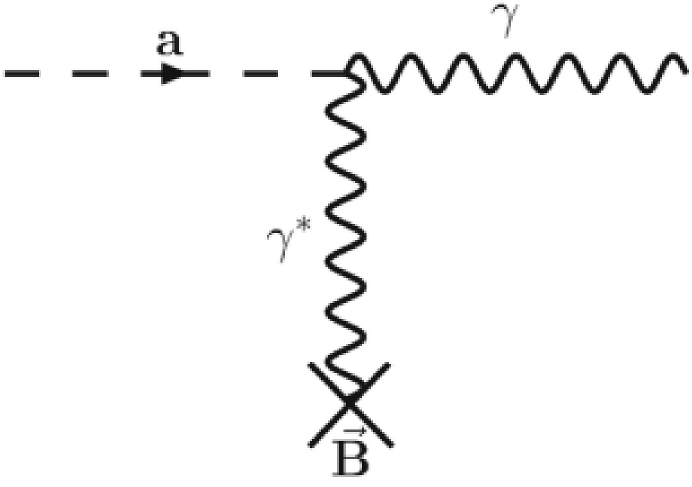
- GW analog : inverse Gertsenshtein effect

→ Same apparatus is sensitive to HFGW



Credit : S. Ellis



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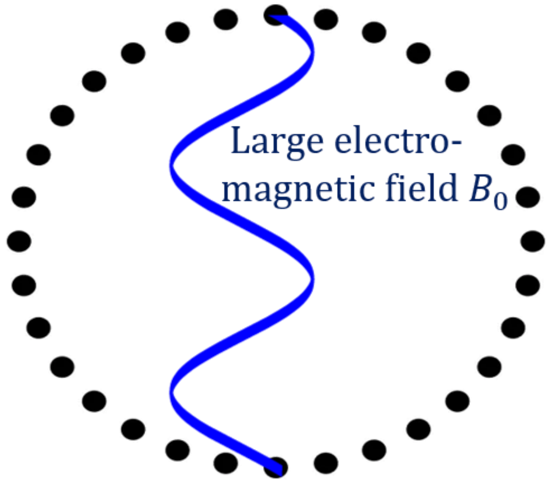
Expected GW signals



Expected GW signals

- $S = \int d^4x\sqrt{-g}\left(-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}\right) \rightarrow \partial_\nu \delta F^{\mu\nu} \equiv j^\mu_{\mathrm{eff}} \propto \omega_g h B_0$

→ GW couples to EM energy

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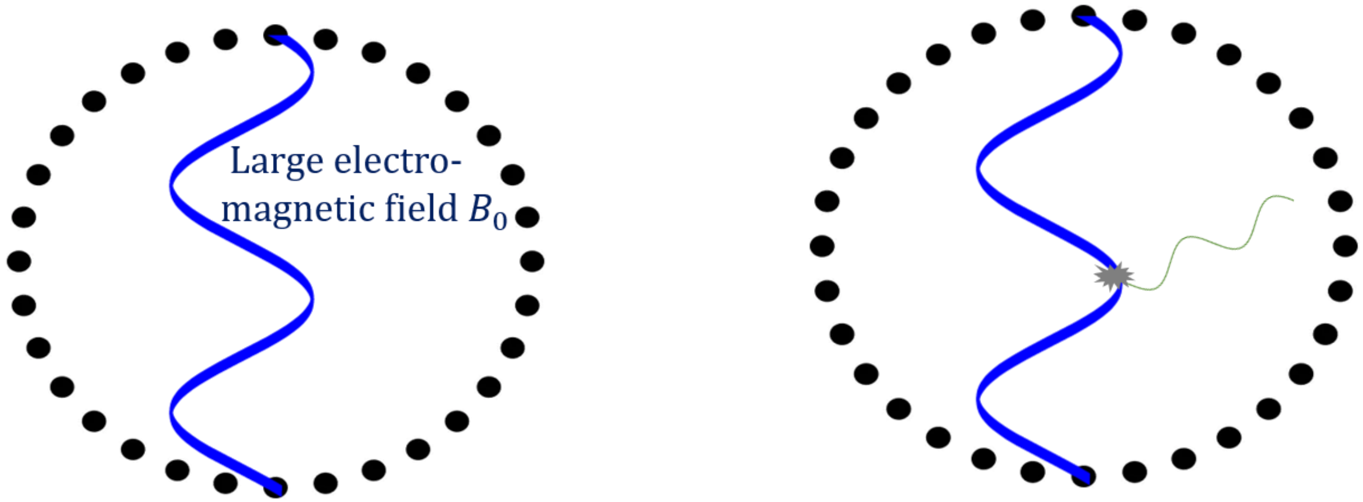
V. Domcke et al, PRL ***129*** *(2022)*



From T. Krokotsch

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*M. Hudelist et al, CQG **40** (2023)*

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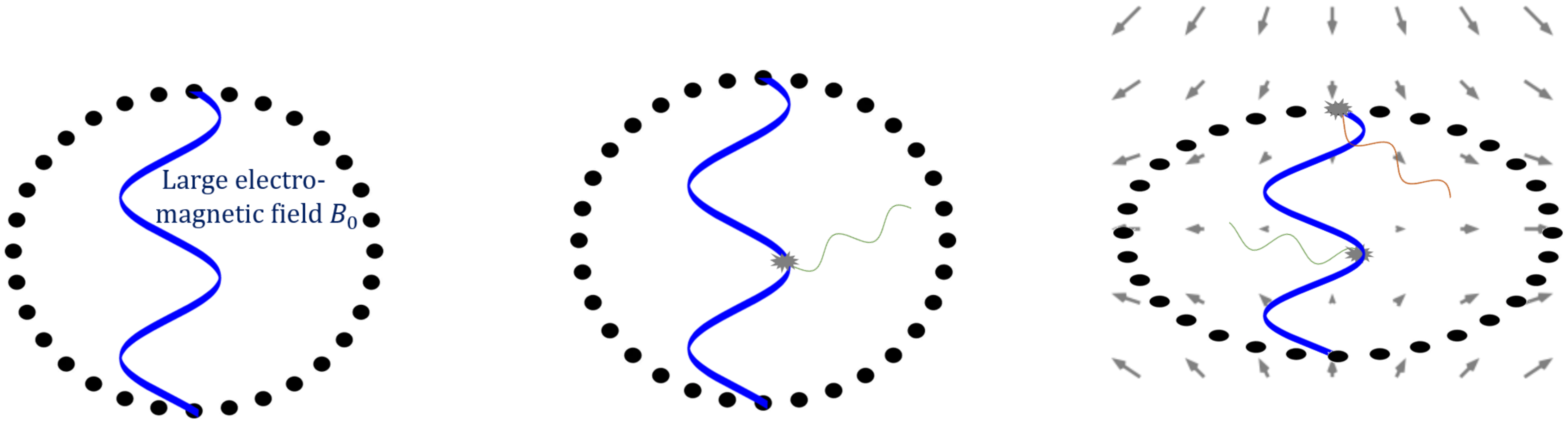

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Observables

- $\partial_\nu \delta F^{\mu\nu} = j^\mu_{\text{eff}}$
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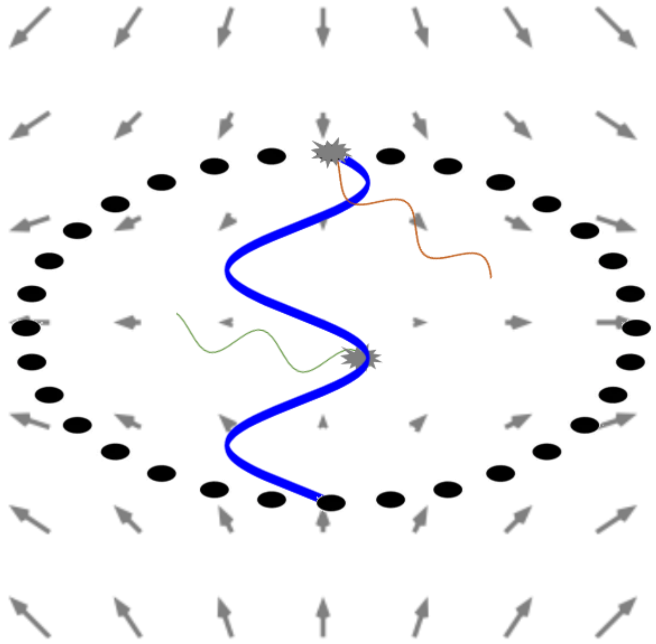
In general, a dipole antenna measures $\vec{E}$ at a point inside the cavity.
How should we define $\vec{E}$?



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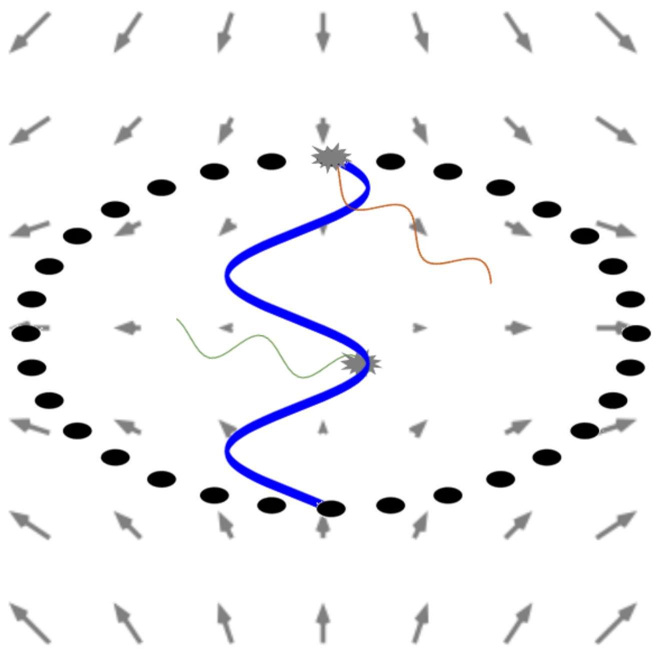

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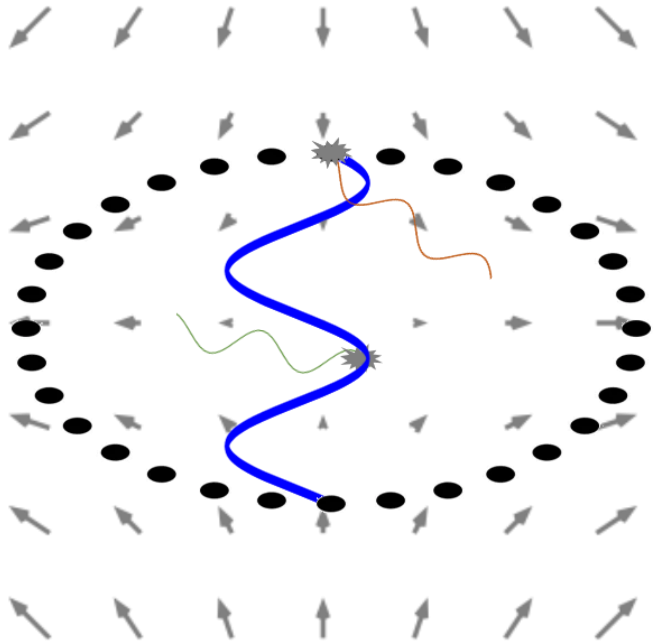

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$$\boldsymbol{E^{obs}_{\underline{a}} = F_{\mu\nu} u^\nu e^\mu_{\underline{a}}}$$

Infinitesimal coord. system
$g_{\mu\nu} e^\mu_{\underline{a}} e^\nu_{\underline{b}} = \eta_{\underline{ab}}$; $e^\nu_{\underline{0}} = u^\nu$
→ used to build a local Lorentz frame

Observer's 4-velocity

W. Ratzinger et al, JHEP ***2024*** *(2024)*

Linearizing, $\delta E^{obs}_{\underline{a}} = \delta F_{\mu\nu} e^\mu_{\underline{a}} u^\nu + F_{\mu\nu} \delta e^\mu_{\underline{a}} u^\nu + F_{\mu\nu} e^\mu_{\underline{a}} \delta u^\nu + \delta x^\rho (\partial_\rho F_{\mu\nu}) e^\mu_{\underline{a}} u^\nu$

→ δF_{a0} is **not** the observed field in curved spacetime

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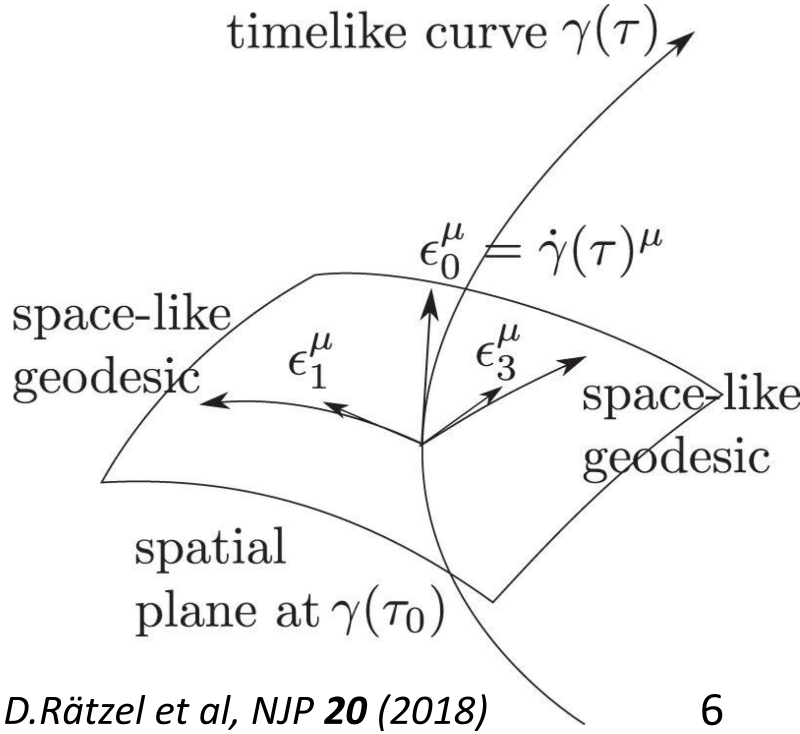
- Proper Detector (PD) Frame : coordinate system built by extending observer's tetrads into geodesics

→ More intuitive : GW acts as a Newtonian force on rigid bodies

→ Metric perturbation more involved



*From D.Rätzel et al, NJP **20** (2018)*

Which frame should we use ?

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What about axion haloscopes ?

In conductor, $v_s \sim 10^{-5}$, i.e for a cavity with $L \sim 0.1$ m, and $\omega_g \sim$ GHz, $\omega_g L/v_s \gg 1$

→ TT more convenient

$$\delta E_{\underline{a}}^{TT,FF} = \delta F_{a0} + \cancel{F_{\mu 0}\delta e_{\underline{a}}^{\mu}} + \cancel{F_{a\nu}\delta u^{\nu}} + \cancel{\delta x^{\rho}(\partial_{\rho}F_{a0})} = \delta F_{a0}$$

Signal power

→ In TT, at high frequency, $\partial_\nu \delta F^{\mu\nu} = j^{\mu}_{\text{eff}}$ solved by expanding δF_{a0} in cavity eigenmodes.

On resonance, the signal power in a mode $\vec{E}_n$ is given by

$$P_{sig} = \frac{1}{2} Q \omega_g V \eta_g^2 h^2 \left|\vec{B}\right|^2$$

Adapted from A. Berlin et al, PRD ***105*** *(2022)*

with the coupling

$$\eta_g = \frac{\left|\int dV\, \vec{E}_n . \hat{j}_{\text{eff}}\right|}{\sqrt{V \int dV \left|\vec{E}_n\right|^2}}$$

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The signal power from axion DM is $P^a_{sig} = \frac{1}{2} g^2_{a\gamma} Q\omega_a V \eta_a^2 a^2 \left|\vec{B}\right|^2$ with $\eta_a = \frac{\left|\int dV \vec{E}_n . \hat{B}\right|}{\sqrt{V \int dV \left|\vec{E}_n\right|^2}}$

*P. Sikivie, RMP **93** (2021)*

→Up to $\mathcal{O}(0.1)$ couplings, we have $h = a g_{a\gamma} \sim 10^{-22}$

Gauge invariance

Considering $\omega_g \sim \mathcal{O}(\mathrm{GHz})$, and background magnetostatic field, the observed electric field is

- In TT, $\delta E_{\underline{a}}^{obs} = \delta F_{a0}^{TT}$ and $\partial_\nu \delta F^{\mu\nu,TT} = j_{\mathrm{eff}}^{\mu,TT}$
- In PD, $\delta E_{\underline{a}}^{obs} = \delta F_{a0}^{PD} + F_{ai}\delta u_i^{PD}$ and $\partial_\nu \delta F^{\mu\nu,PD} = j_{\mathrm{eff}}^{\mu,PD}$

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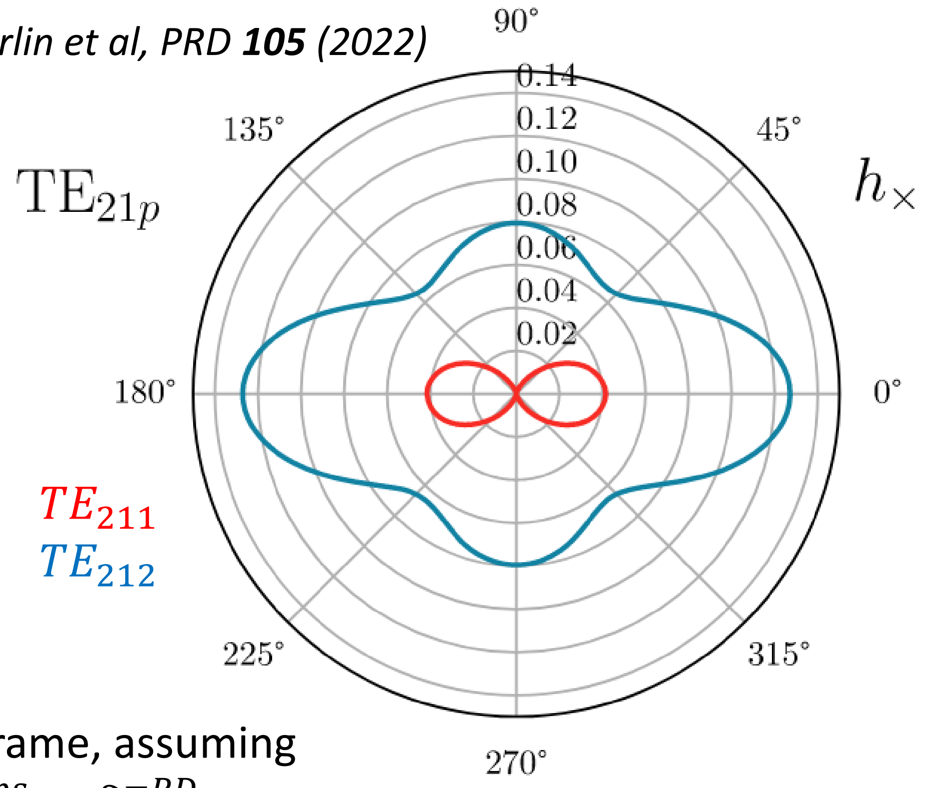
A. Berlin et al, PRD ***105*** *(2022)*



$\eta_g(\beta)$

Incoming GW latitude

PD frame, assuming $\delta E^{obs}_{\underline{a}} = \delta F^{PD}_{a0}$

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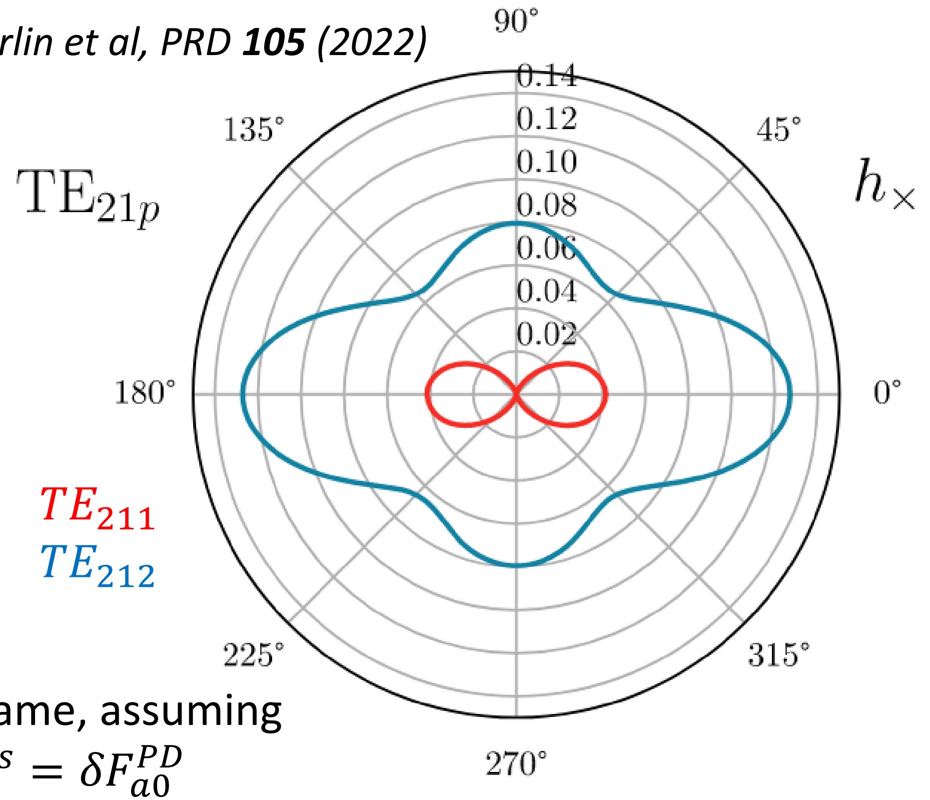

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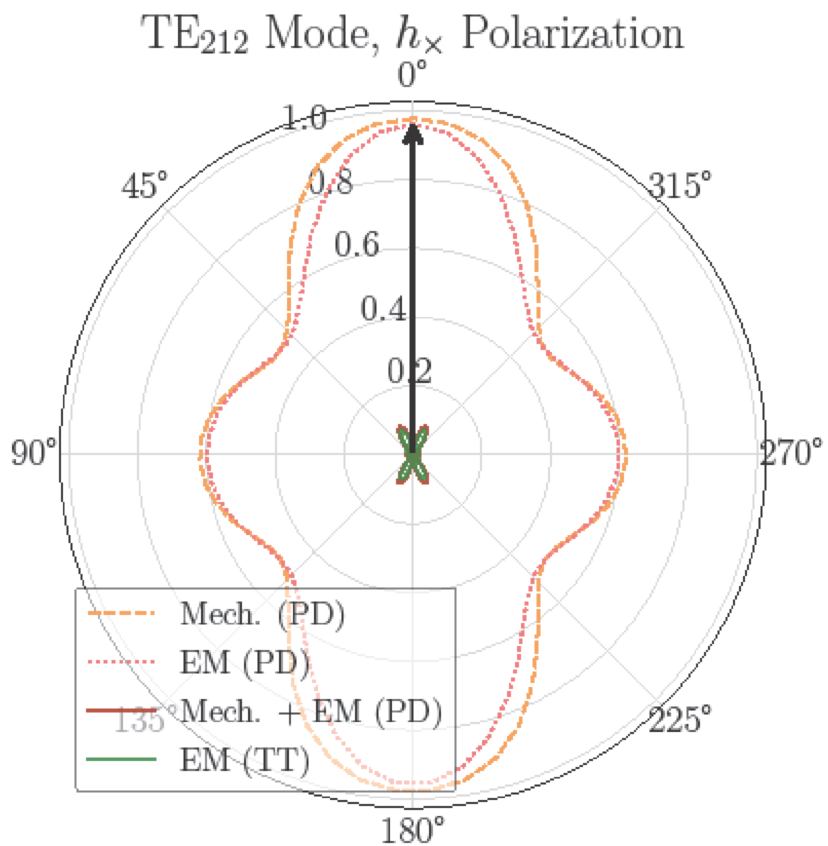
T. Krokotsch, JG, in preparation



→ Signal gauge invariant as expected, provided all terms are taken into account.

What is going on now ?

Build covariant framework to compute the full signal in PD/TT at any frequency

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- Heterodyne/Homodyne setups

Conclusion

- Microwave cavities are powerful probes of monochromatic HFGW ($h \sim 10^{-22}$)

- GW couples to all types of energy,
  care must be taken to model all effects

- With current quantum technology,
  this is not enough to probe cosmological GW

→ Cross correlate multiple cavities : GravNet ERC

→ Use of Earth modulation for persistent signals

→ Quantum enhancement techniques (e.g. squeezing)