Modeling binary-black hole signals for gravitational-wave astronomy:

Achievements, systematics and future challenges

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1 - Modeling binary black-hole signals:

- Main modeling frameworks
- Recent developments: orbital eccentricity [Planas+(incl. HE), arXiv:2503.13062 (2025), Gamboa+(incl. HE), PRD 112, 044038 (2025)] and equatorial asymmetries [HE+, arXiv:2506.19911 (2025)]

2 - New results from the O4 observing run:

- GWTC-4.0 [LVK+(incl. HE), arXiv:2508.18082, (2025)]
- ·Special events: broadening the parameter space, louder signals
- Systematic differences due to waveform models

3 - Expected systematic biases for improved detector sensitivities [Dhani+(incl. HE), PRX 15, 031036 (2025)]

- Biases for LVK-like population
- Biases across parameter space
- Effect on science goals with "golden binaries"

Inferring the source properties from GW observations

We want to infer the source properties of a signal detected on the data

$$d(t) = n(t) + h(t, \lambda)$$

Bayes rule to update prior probability of parameters with likelihood of data being described by the model with those parameters to obtain posterior probability of source parameters

$$p(\lambda \mid d, h) = \frac{p(d \mid h(\lambda)) p(\lambda, h)}{p(d, h)}$$

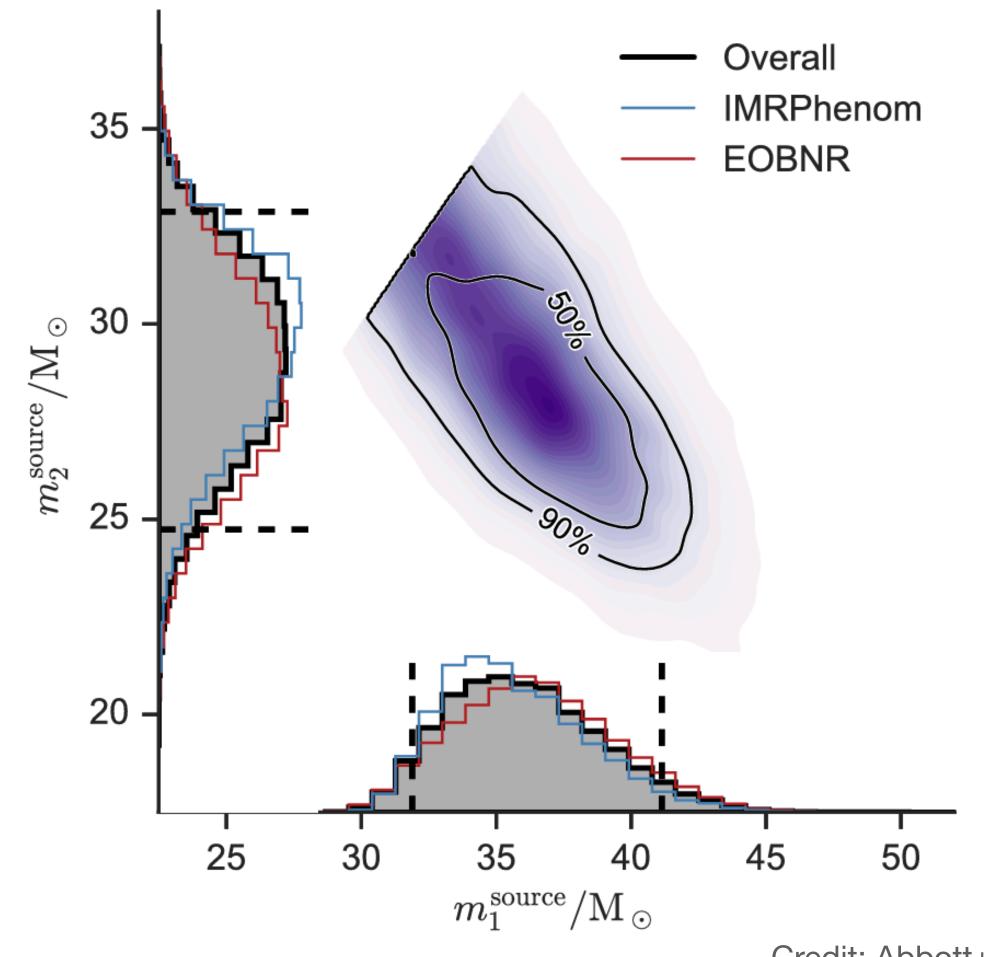
GW likelihood (assuming Gaussian and stationary noise):

$$\ln p(d \mid h(\lambda)) \propto - (d - h(\lambda) \mid d - h(\lambda)) \qquad (a \mid b) = 4 \operatorname{Re} \int_0^\infty \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)} df$$

We rely on the ability to model the signal!

Accurate models: avoid biases on inferred parameters

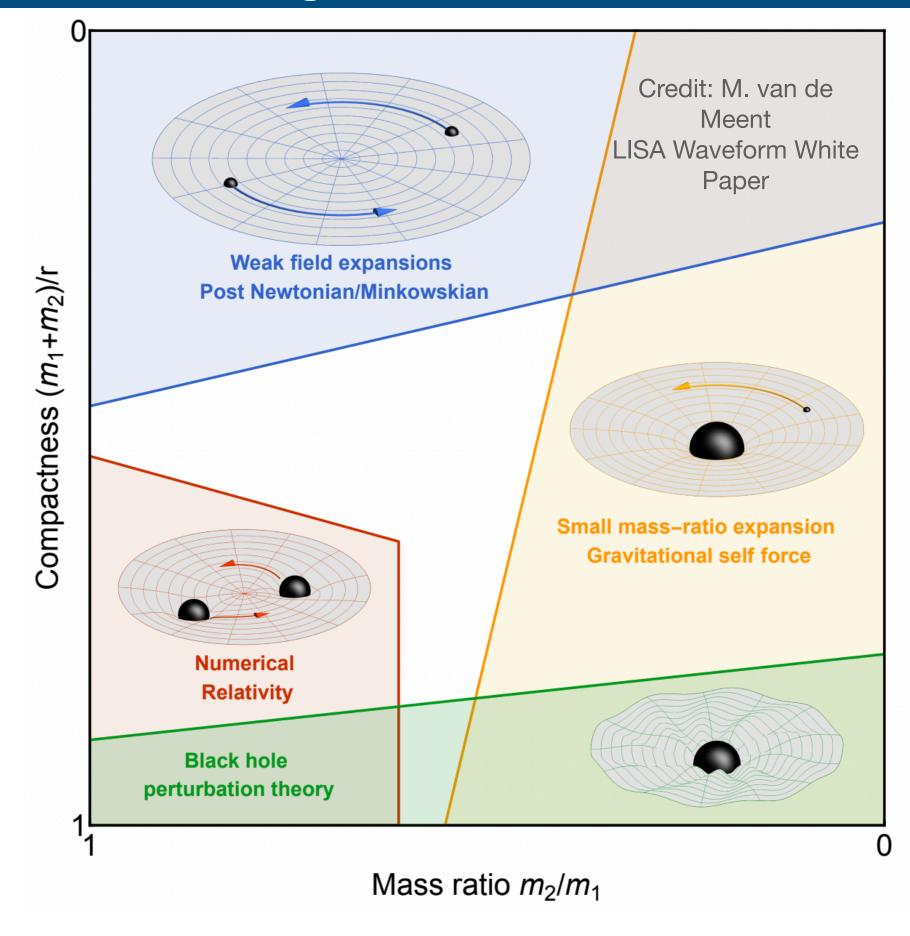
Efficient models: $10^7 - 10^8$ likelihood evaluations needed with standard stochastic sampling techniques

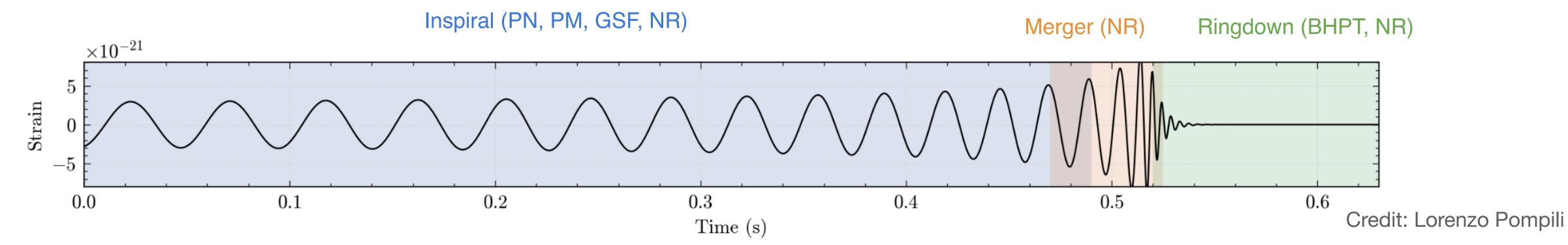


Modeling compact binary coalescences

Theoretical frameworks:

- Post-Newtonian theory: valid for slowly moving binaries (expansion in v/c) can describe inspiral stage.
- Post-Minkowskian theory: valid for weakly gravitating sources (expansion in G) can describe inspiral stage.
- Gravitational self-force: valid for high mass-asymmetric systems.
- Numerical relativity: numerical evolution of Einstein equations "exact" solutions, but computationally very expensive limitations in signal length and parameter space coverage.
- Black-hole perturbation theory: can describe ringdown emission of remnant black hole.





Inspiral-merger-ringdown Waveform models

Waveform modeling:

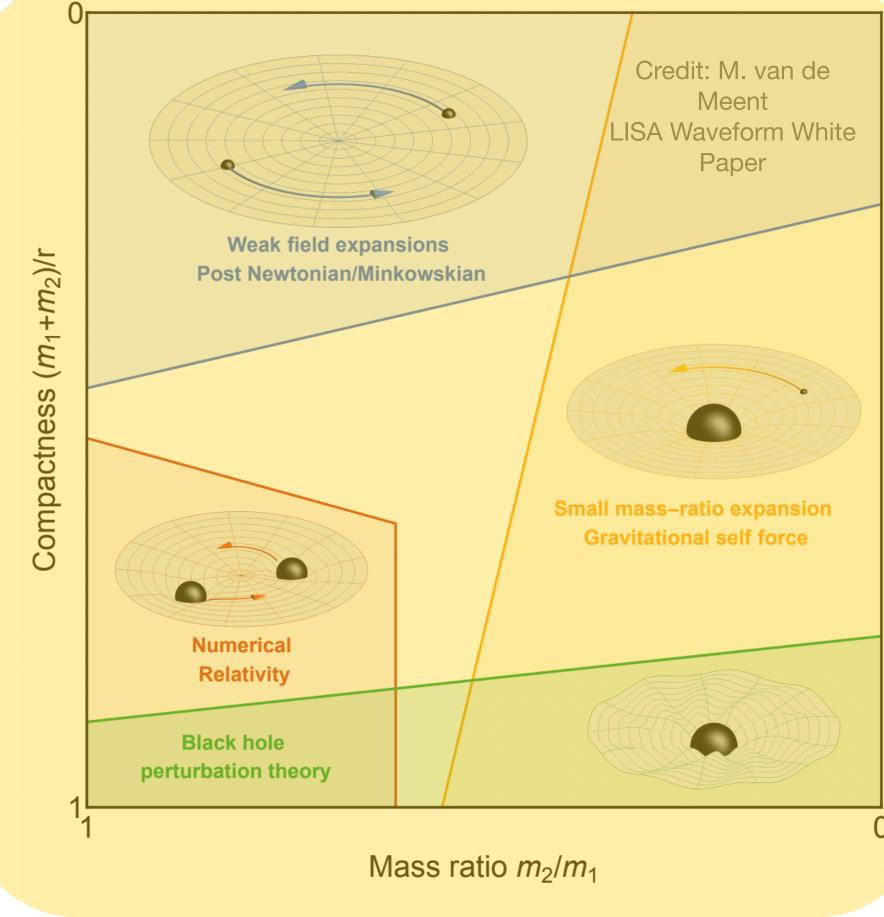
Combine information from numerical and perturbative approaches.

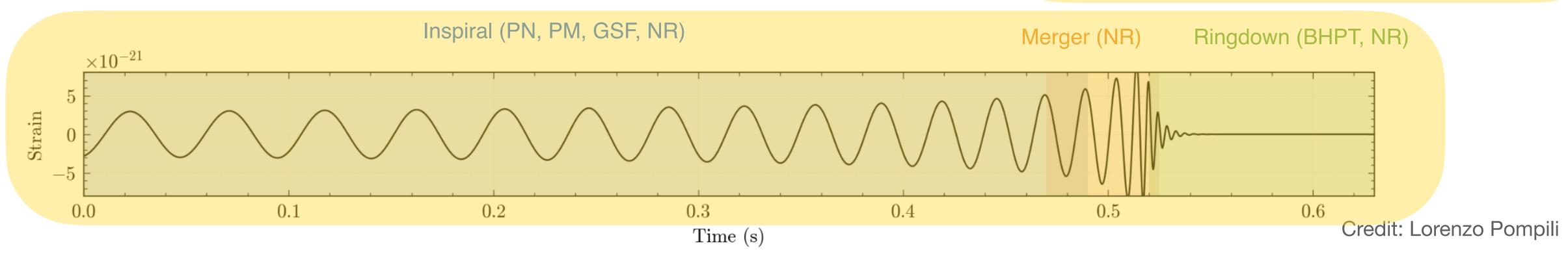
- Generality: Aims to cover the full parameter space of signals.
- Efficiency: Bayesian parameter source inference require $10^7 10^8$ waveform evaluations.
- Accuracy: Keep systematic errors below statistical uncertainty.

Main waveform model families: Different trade-offs between efficiency, accuracy and generality

- IMRPhenom (highly efficient)
- EOB (SEOBNR, TEOBResum) (more accurate but more expensive)
- NRSur (highly accurate, efficient, but limited in param space and length)

Waveform models





IMRPhenom: phenomenological modeling

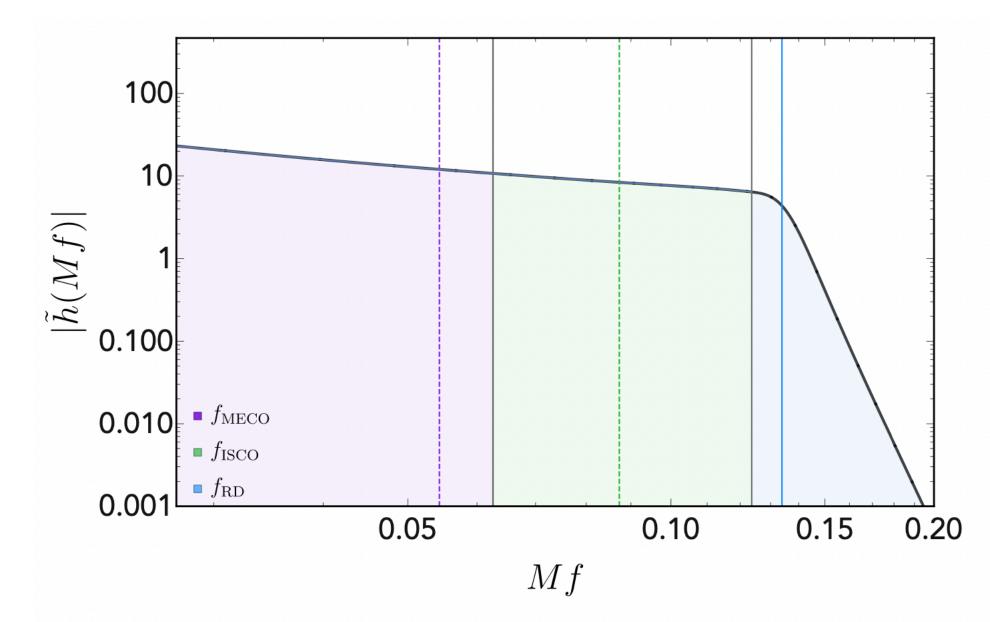
Phenomenological modeling of the amplitude and phase of the spin-weighted spherical harmonics of the signal. Arun+08, Santamaria+10, Husa+15, Khan+15

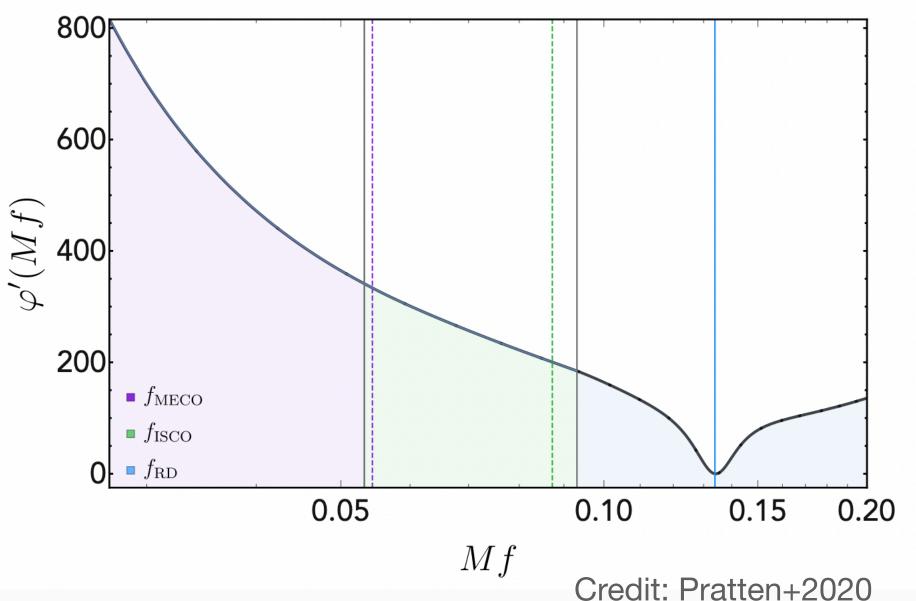
$$\tilde{h}_{+}(f) - i\tilde{h}_{\times}(f) = e^{i\alpha(f)} \sum_{\ell m} A_{\ell m}(f) e^{im\psi_{\ell m}(f)} {}_{-2}Y_{\ell m}[\beta(f), \gamma(f)]$$

Closed-form piece-wise expressions (in Fourier-domain or time-domain) for amplitude and phase of each harmonic.

Closed-form or numerical evolution of spin-precession of the orbital plane.

- 1. Inspiral using Post-Newtonian results augmented with calibration to Numerical Relativity and Teukolsky waveforms.
- 2. Intermediate using calibrated phenomenological expressions to describe the late-inspiral and plunge
- 3. Merger-ringdown using calibrated expressions including the decay time and asymptotic ringdown frequencies from perturbation theory.



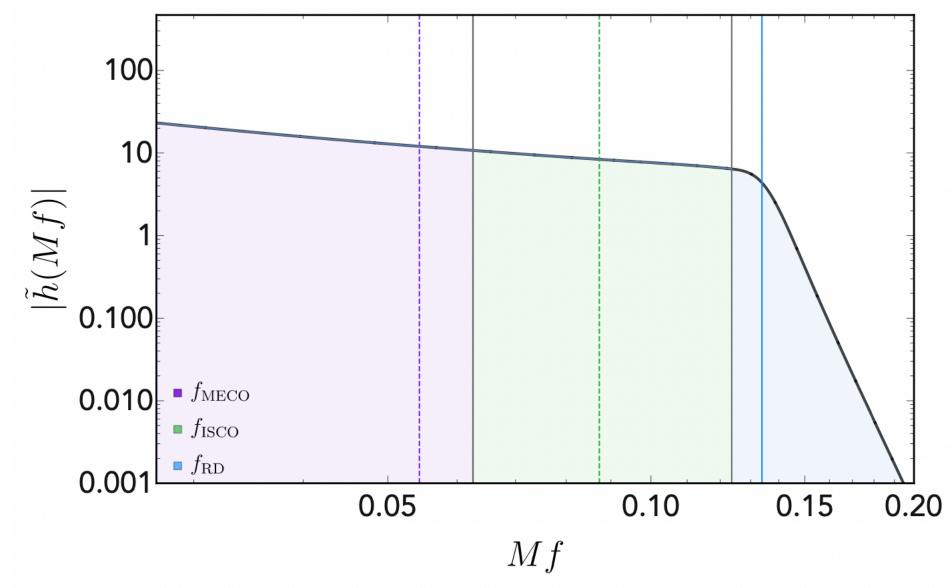


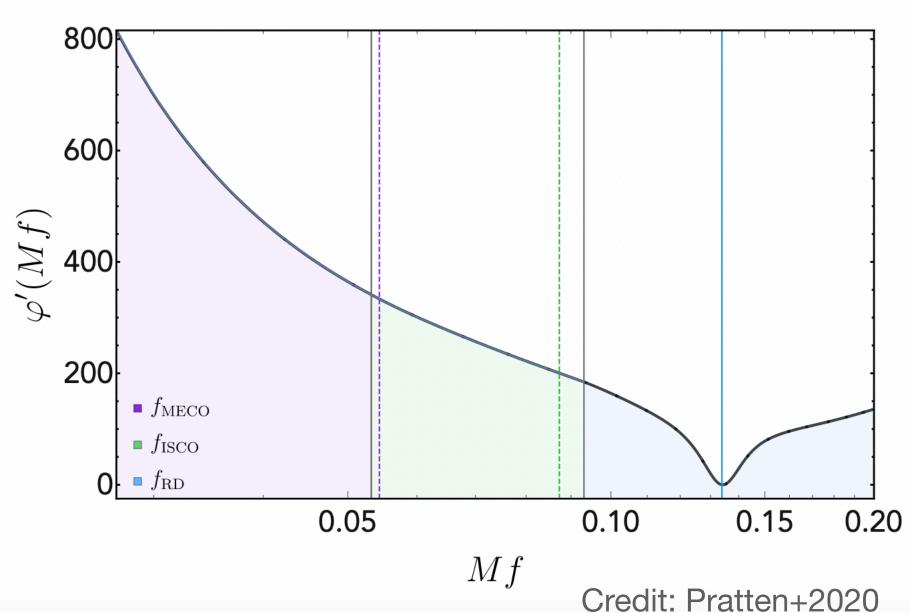
IMRPhenom: phenomenological modeling

Fourier-domain models: highly efficient, direct application in current dataanalysis methods, but shortcommings for modeling complex signals (eccentricity, spin-precession)

$$\tilde{h}_{+}(f) - i\tilde{h}_{\times}(f) = e^{i\alpha(f)} \sum_{\ell m} A_{\ell m}(f) e^{im\psi_{\ell m}(f)} _{-2} Y_{\ell m}[\beta(f), \gamma(f)]$$

- IMRPhenomXPHM[Pratten+(incl. HE), PRD 103, 104056 (2021)]: multipolar spin-precessing model, with closed-form double spin solution for the spin dynamics
- IMRPhenomXPHM-SpinTaylor [Colleoni+, PRD 111, 064025 (2025)]: improved spin-precessing during inspiral using numerical spin evolution
- IMRPhenomXO4a [Thompson+, PRD 109, 063012 (2024)]: Partial calibration to spin-precessing NR simulations, inclusion of dominant equatorial-asymmetry.
- IMRPhenomXPNR [Hamilton+, arXiv:2507.02604 (2025)]: Combination of SpinTaylor and XO4a improvements.





IMRPhenom: phenomenological modeling

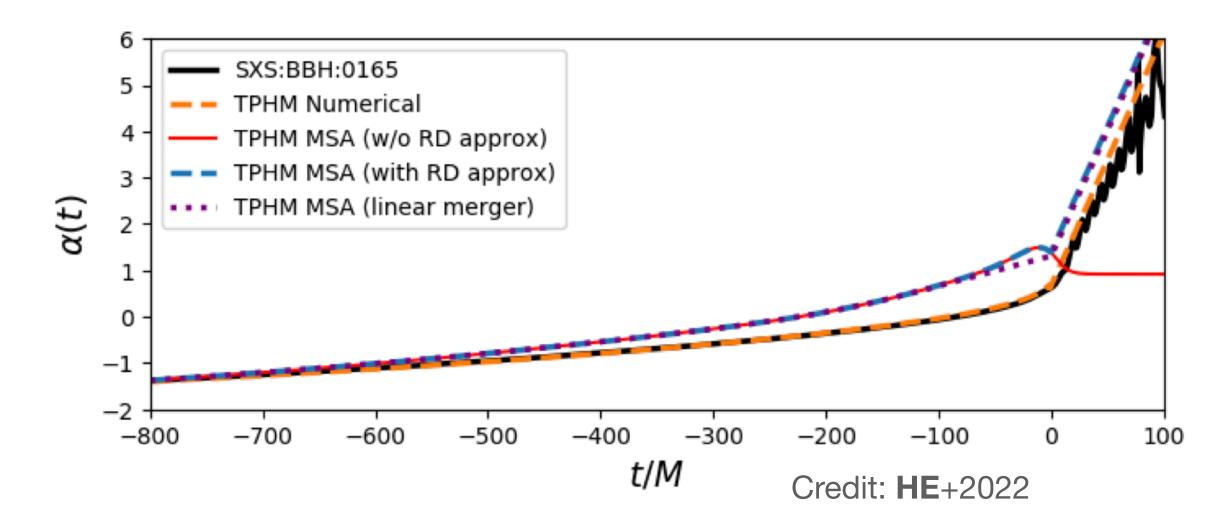
Time-domain models: still efficient, more direct connection with system dynamics (improved modeling of spin-precession and eccentricity), higher accuracy than Fourier-domain models.

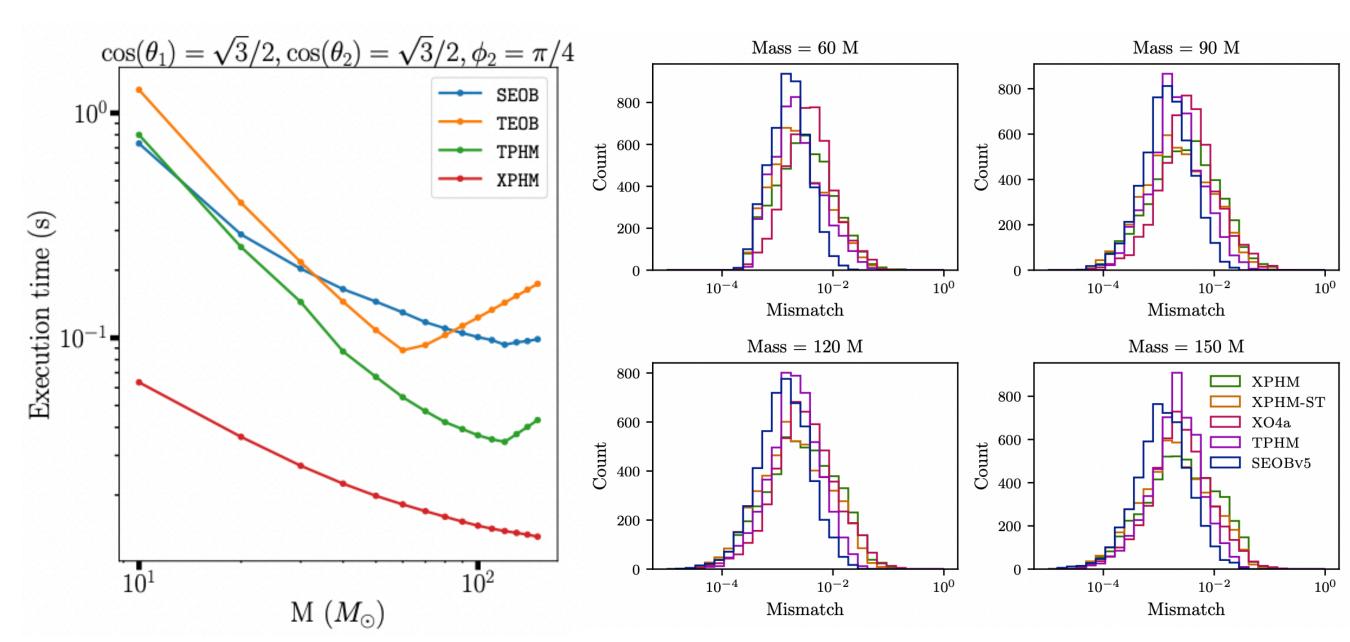
$$h_{+}(t) - ih_{\times}(t) = e^{i\alpha(t)} \sum_{\ell m} A_{\ell m}(t) e^{im\phi_{\ell m}(t)} {}_{-2}Y_{\ell m}[\beta(t), \gamma(t)]$$

- IMRPhenomTPHM [HE+, PRD 105, 084040 (2022)]:
 - Numerical evolution of spin-precession (more accurate than closed-form expressions, allowed the development of XPHM-SpinTaylor)
 - Improved merger-ringdown description of spin-precessing systems.
 - Fastest state-of-the-art time-domain model.

Recent improvements and extensions:

- IMRPhenomT(P)HM_20 [Rosselló+, PRD 110, 084074 (2024); Rosselló&Husa arXiv:2506.08888 (2025)]: inclusion of non-linear displacement memory and (2,0,0) QNM excitation.
- IMRPhenomTEHM [Planas+(incl. HE), arXiv:2503.13062 (2025)]: Model for systems in mild eccentric orbits.
- New efficient implementation in phenomxpy [García-Quirós+2025] with GPU support: employed for MBHB signals in LISA DDPC Mojito data challenge.





Credit: Thompson+2023

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Credit: Uilliam+2024

Modeling eccentric systems: IMRPhenomTEHM

IMRPhenomTEHM [Planas+(incl. HE), arXiv:2503.13062 (2025)]: Adding eccentric correction to IMRPhenomTHM (aligned-spin systems)

Orbital motion evolved with secular equations with eccentric corrections at 3PN:

 Quasi-circular limit from IMRPhenomTHM, calibrated to NR simulations

$$\dot{\Omega}(\Omega, e) = \dot{\Omega}^{\text{THM}}(\Omega) + \dot{\Omega}_{e\neq 0}^{3\text{PN}}(\Omega, e),$$

$$\dot{e}(\Omega, e) = \dot{e}^{3PN}(\Omega, e),$$

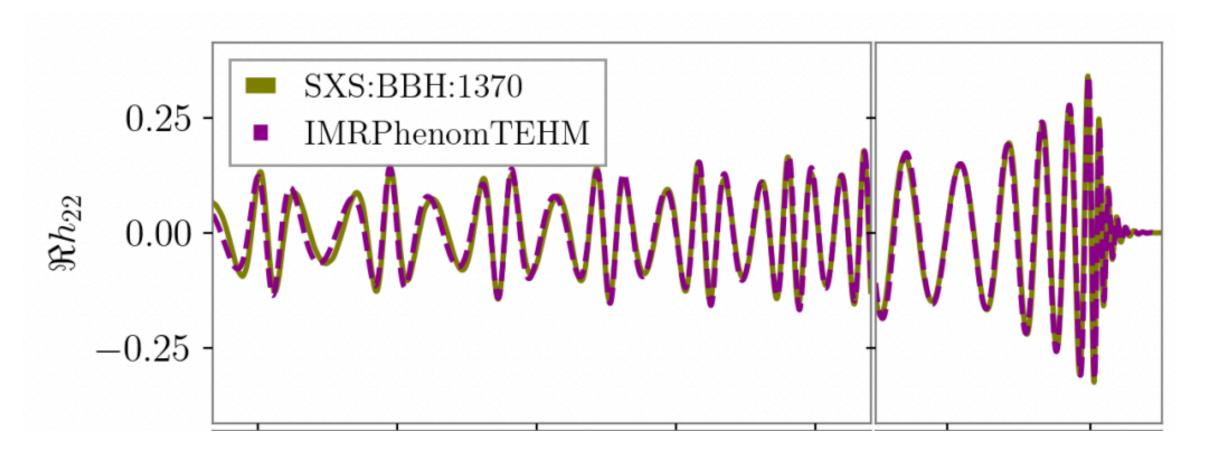
$$\dot{l}(\Omega, e) = \dot{l}^{3PN}(\Omega, e)$$

Eccentric corrections in waveform modes at 3PN non-spinning [Ebersold+2019] and 2PN spinning [Henry&Khalil 2023] (expanded to e^6):

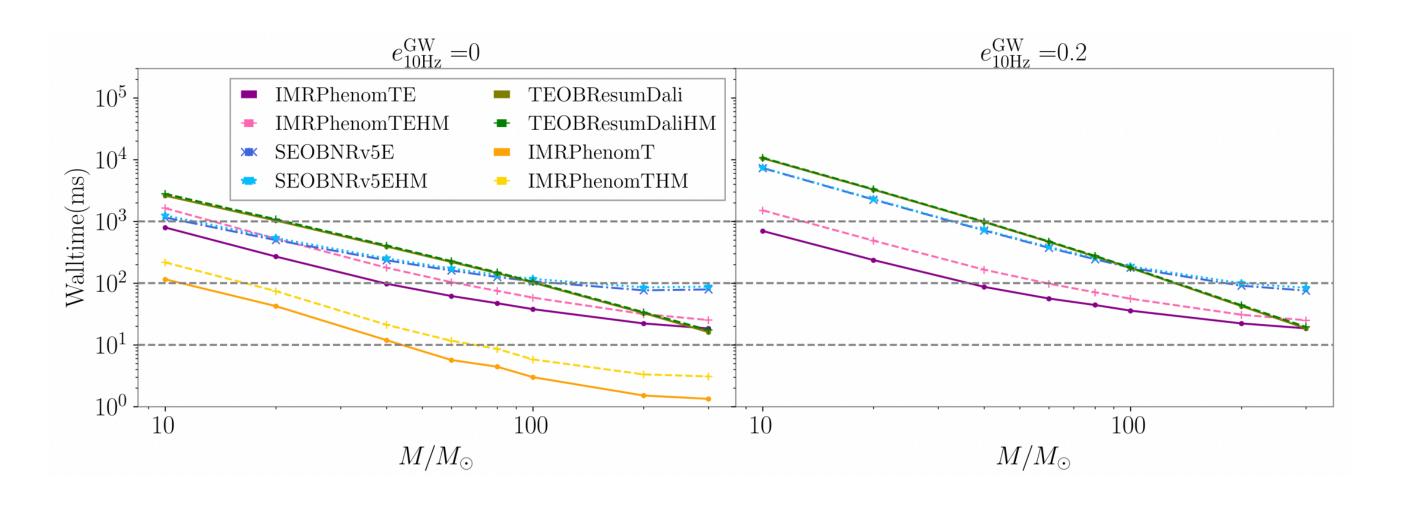
$$h_{lm}^{\rm TEHM} = \left(\tilde{h}_{lm}^{\rm THM} + \frac{8GM\eta}{c^2R} \sqrt{\frac{\pi}{5}} \left[xH^{lm}(x,e,l) - x^{e=0}H_{e=0}^{lm}(x^{e=0}) \right] e^{-im\lambda} \right) e^{-imW(l)}$$

Accurate for mild eccentricities

Most efficient eccentric IMR waveform model.



From Planas+2025a



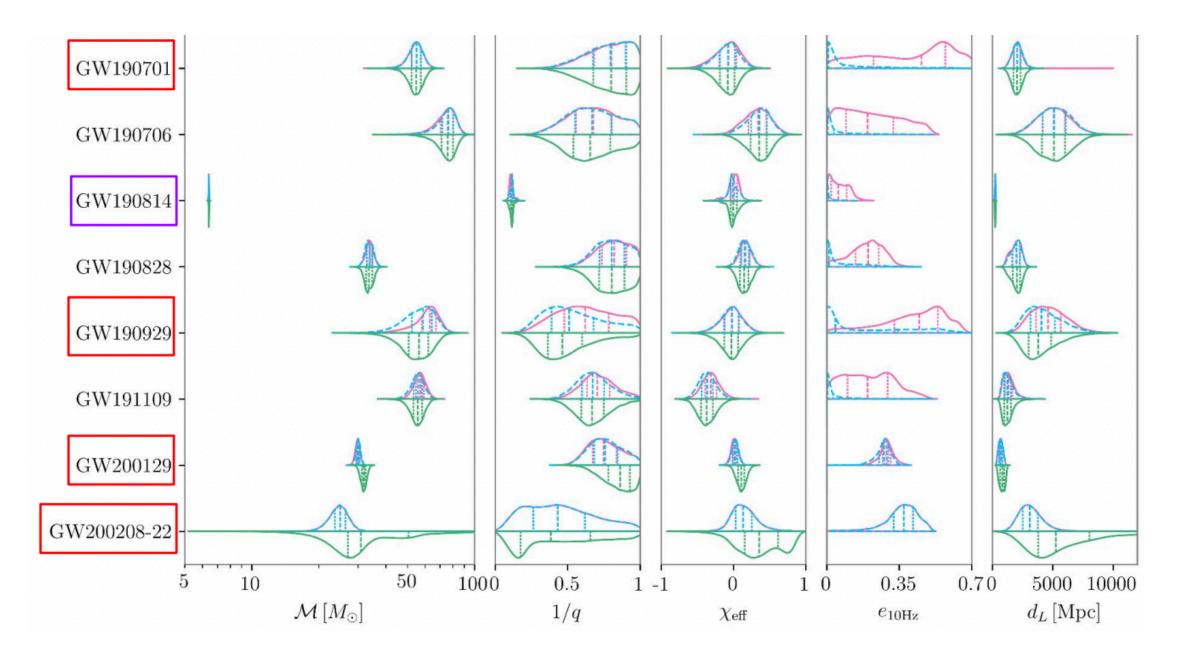
From Planas+2025a

Modeling eccentric systems: IMRPhenomTEHM

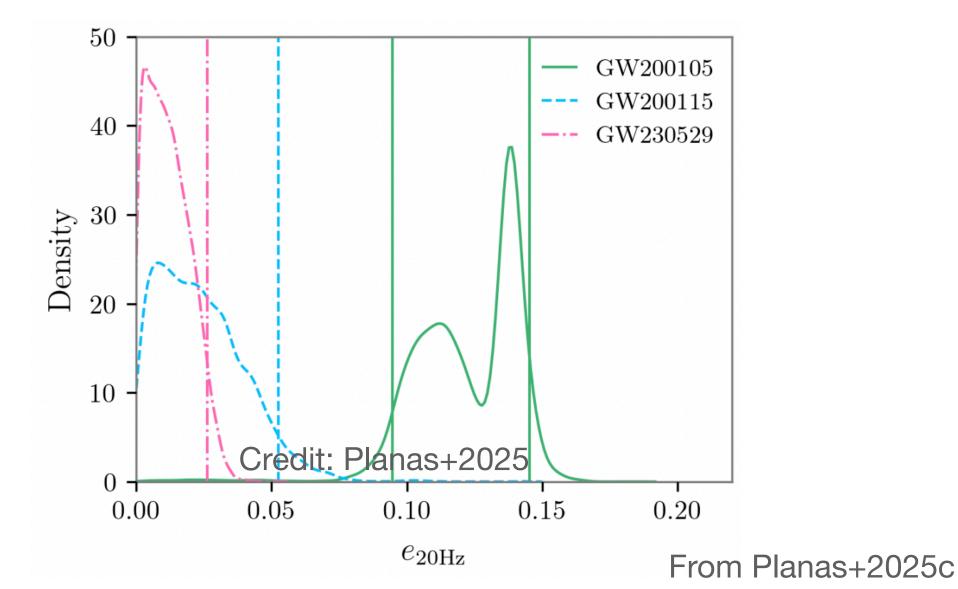
Eccentric re-analysis of GWTC-3 [Planas+(incl. HE), arXiv:2504.15833 (2025)]:

- BBH results consistent with [Gupta+2024], find 3 classes of events:
 - Compatibles with quasi-circular binaries (railing against e=0).
 - (GW190701, GW190929): PE prefers eccentricity, but high mass, short waveform, ... -> no claim of eccentric binary
 - GW200208_22: Low SNR, Uniform prior supports eccentricity, LogUniform lower support... No strong evidence.
 - GW200129: eccentricity is strongly preferred over precession claim
 - Bayes factors vs QC (log10): 1.3-4.0, vs NRSur: 0.13-1.87
 - Eccentricity robustly favored for different glitch subtractions.

Evidence for eccentricity in NSBH event GW200105 [Planas+, arXiv:2506.01760 (2025)] (confirming [Morras+2025])



From Planas+2025b

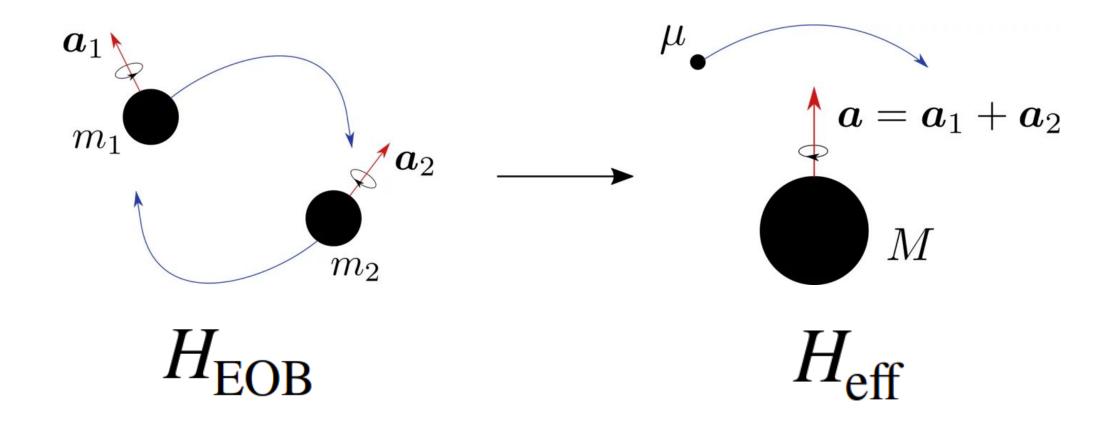


Effective-One-Body (EOB)

Two-body dynamics mapped into dynamics of one effective body moving in deformed BH spacetime, deformation being the mass ratio.

Main ingredients of EOB waveforms:

- 1. EOB Hamiltonian describing the conservative binary dynamics
- 2. Radiation reaction (RR) force to account for loss of energy and angular momentum via emission of GWs
- 3. Gravitational waveform modes for inspiral, merger, and ringdown



Credit: Lorenzo Pompili

$$H^{
m EOB} = M \sqrt{1 + 2
u igg(rac{H_{
m eff}}{\mu}igg)}$$

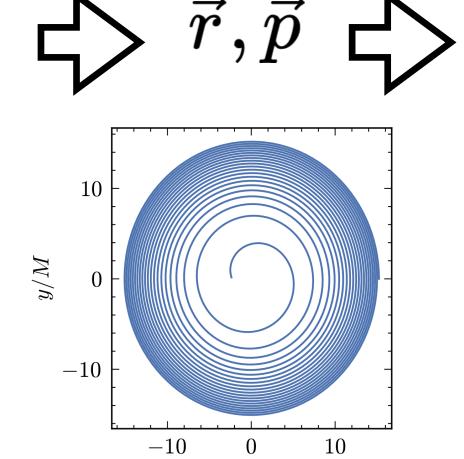
Radiation-reaction force

$$\mathscr{F} = rac{\Omega}{16\pi} rac{p}{L} \sum_{\ell,m} m^2 |h_{\ell m}|^2$$

Equations of motion (ODE)

$$egin{aligned} \dot{ec{r}} &= rac{\partial H^{ ext{EOB}}}{\partial ec{p}} \ \dot{ec{p}} &= -rac{\partial H^{ ext{EOB}}}{\partial ec{r}} + ec{\mathcal{F}} \end{aligned}$$

Orbital dynamics



Waveform modes

$$h_{\ell m}^{\rm insp-plunge} = h_{\ell m}^{\rm Newt} S_{\rm eff} T_{\ell m} f_{\ell m} e^{i\delta_{\ell m}} h_{\ell m}^{\rm NQC}$$

$$h_{\ell m}^{\text{merg-RD}} \tilde{A}_{\ell m} e^{i\tilde{\phi}_{\ell m}} e^{-i\sigma_{\ell m0}(t-t_{\text{peak}}^{22})}$$

$$h_{\ell m} = \begin{cases} h_{\ell m}^{\text{insp-plunge}}, & t < t_{\text{peak}}^{22}, \\ h_{\ell m}^{\text{merg-RD}}, & t > t_{\text{peak}}^{22}. \end{cases}$$

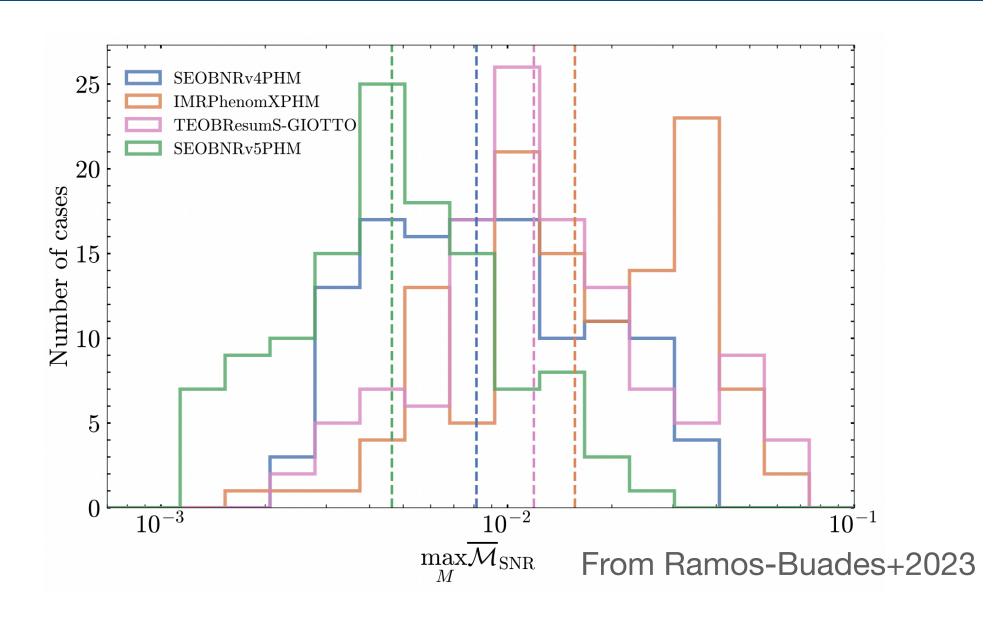
Effective-One-Body (EOB)

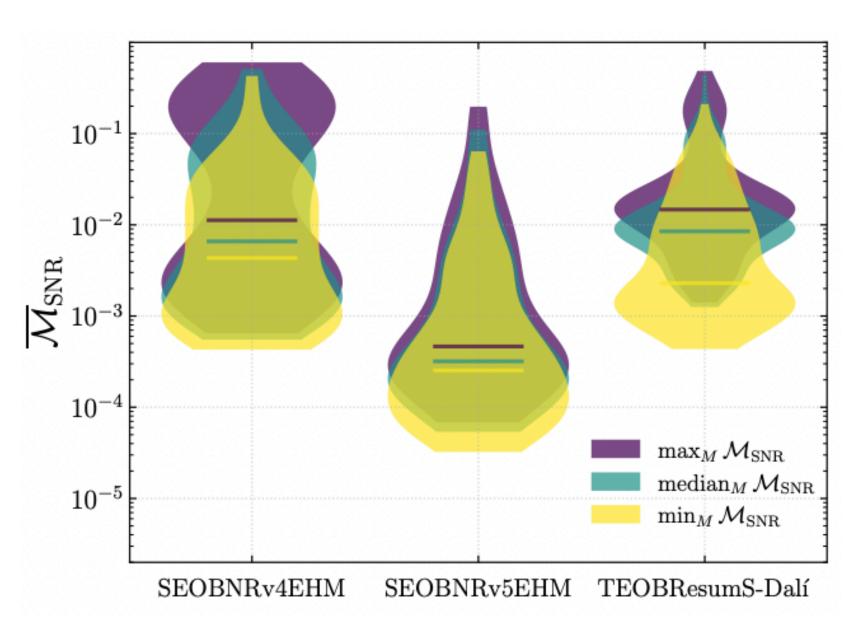
SEOBNRv5PHM: Latest generation of SEOBNR models for quasi-circular systems

- Updated PN information in the fluxes, waveform modes and Hamiltonian [Khalil+(incl. HE), PRD 108, 124036 (2023)]
- Inclusion of 2nd-order self-force results in the waveform modes [van de Meent+, PRD 108, 124038 (2023)]
- Fast spin evolution using Taylor-expanded PN equations (as in IMRPhenomTPHM) [Ramos-Buades+(incl. HE), PRD 108, 124037 (2023)]
- Efficient python implementation [Mihaylov+(incl. HE), Soft.X 30, 102080 (2025)]
- Inclusion of calibrated equatorial-asymmetric effects in the waveform modes [HE+, arXiv:2506.19911 (2025)]

SEOBNRv5EHM: Highly accurate aligned-spin model for systems on eccentric orbits [Gamboa+(incl. HE), PRD 112, 044038 (2025)]

- Eccentric effects up to 3PN in radiation-reaction and waveform modes
- High accuracy against eccentric NR simulations (despite lack of calibration)





Adding equatorial-asymmetric effects in SEOBNRv5PHM

Equatorial symmetry broken for spin-precessing systems [Boyle+2014]

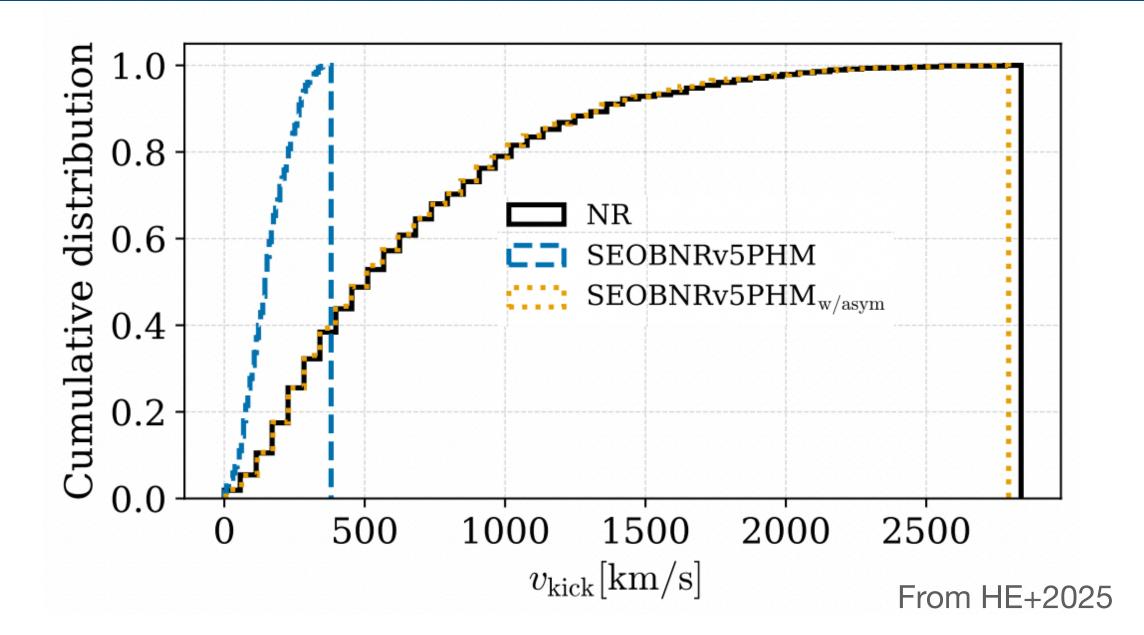
Antisymmetric contribution: asymmetry between positive and negative m-modes in the co-precessing frame.

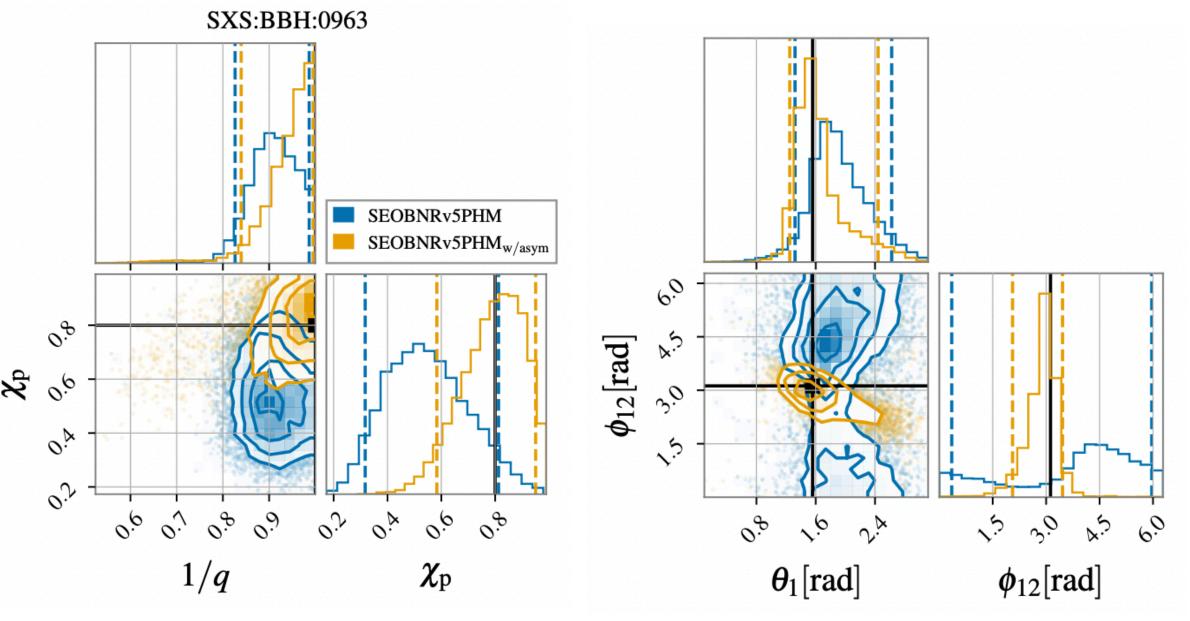
$$h_{lm}^{\text{asymm}} = \frac{h_{l,+m}^{\text{cop}} - (-1)^l h_{l,-m}^{\text{cop}}}{2}$$

We recently included [HE+, arXiv:2506.19911 (2025)] this contribution in SEOBNRv5PHM for the $\ell=m$ harmonics:

- Calibrated with ~1500 spin-precessing NR simulations and ~3000 Teukolsky geodesic waveforms
- 50% improvement of model accuracy for face-on systems.
- Crucial to explain high recoil velocities: good agreement with NR distribution.
- Reduction of inference biases in spin and mass parameters.

It will be employed for next catalog version GWTC-5.0

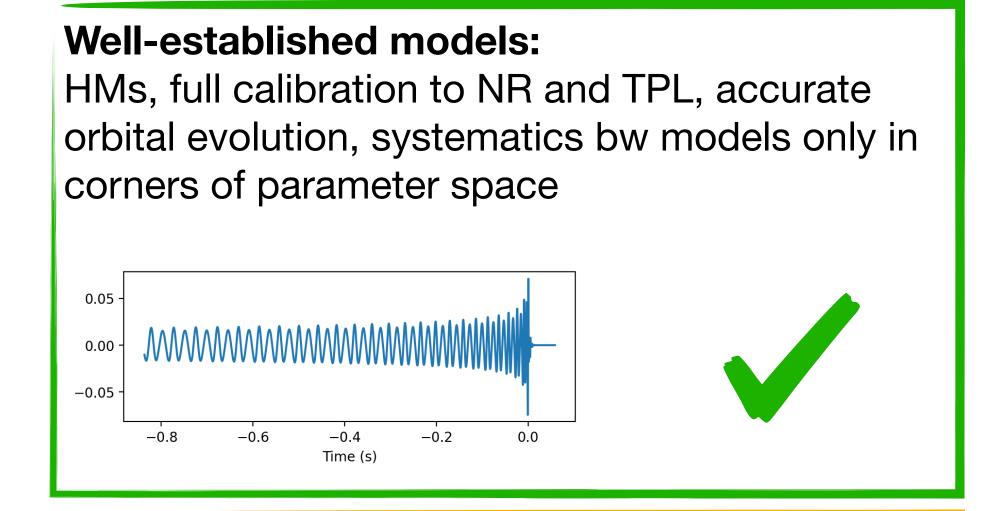




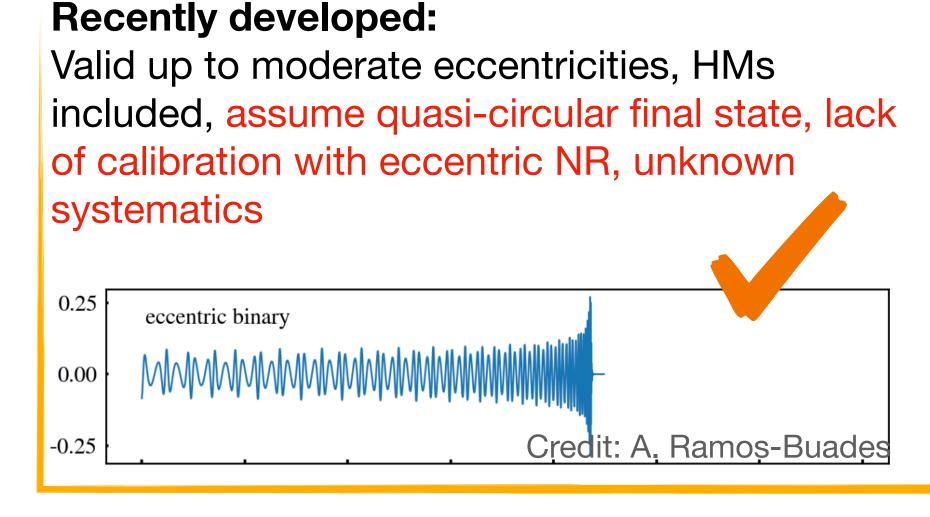
Modeling the BBH signal space

Spin-aligned

Quasi-circular orbits



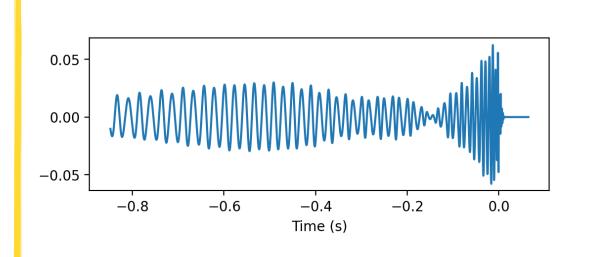
Eccentric orbits



Spin-precessing

Mature models:

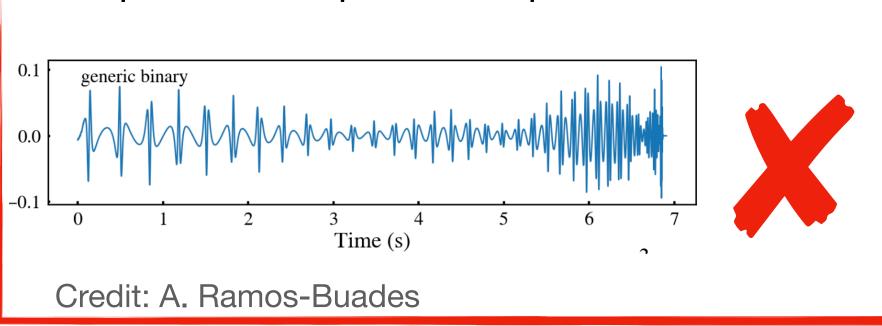
Accurate inspiral description of spin dynamics, lack of NR precessing calibration in some models, others with approximations (single-spin), missing physical effects in some models (equatorial asymmetry). Model systematics present even at moderate spins.





Starting to be investigated:

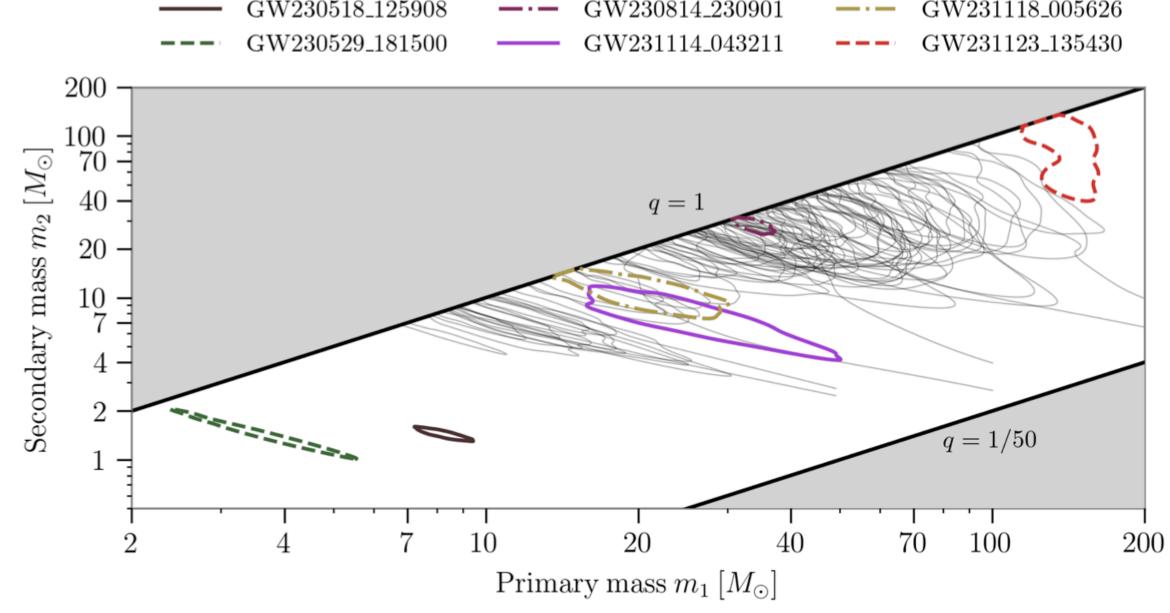
Initial studies on how to build these models, some proof-concept models presented.



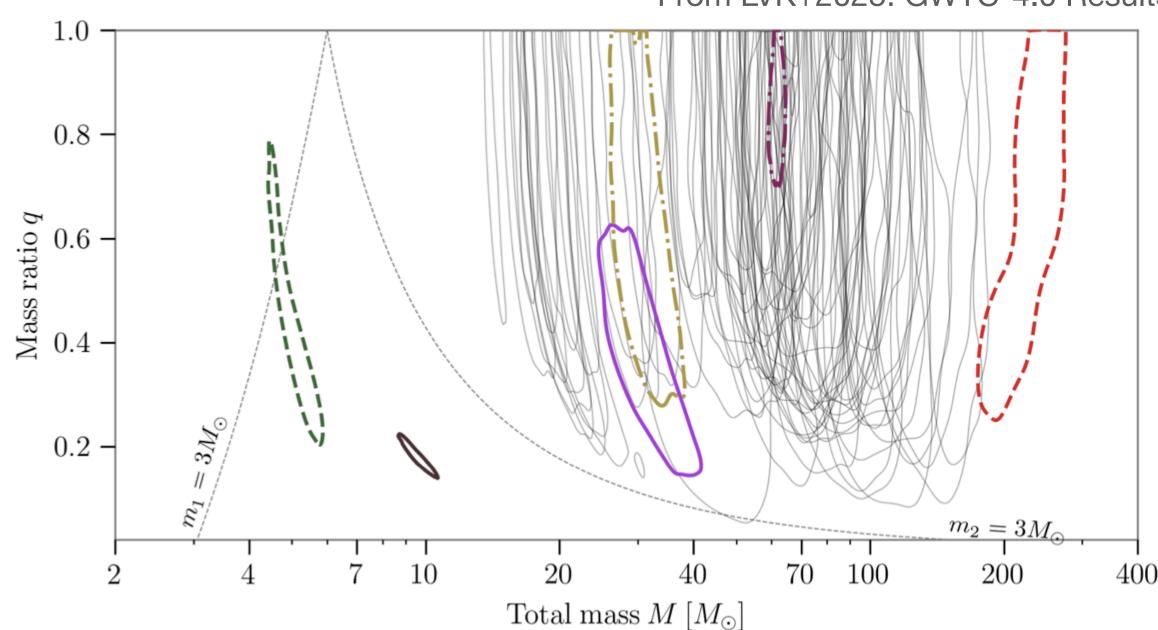
New results from O4a: GWTC-4.0

New update of the Gravitational Wave Transient Catalog (GWTC) with signals detected during the O4a observing run (May 23 - Jan 24)

- 128 new candidates with $p_{\rm astro} > 0.5$
- 86 with full inference of source parameters
- 2 NSBH candidates:
 - GW230518_125908
 - GW230529_181500 ($m_1 < 5M_{\odot}$ at 99% CL)
- 84 confident BBH candidates
 - Mass range $M \in [14,238] M_{\odot}$
 - Highes mass BBH to date: GW231123_135430 $(M = 238^{+28}_{-49} M_{\odot})$







Special events from O4: filling the mass gaps

GW230518_125908:

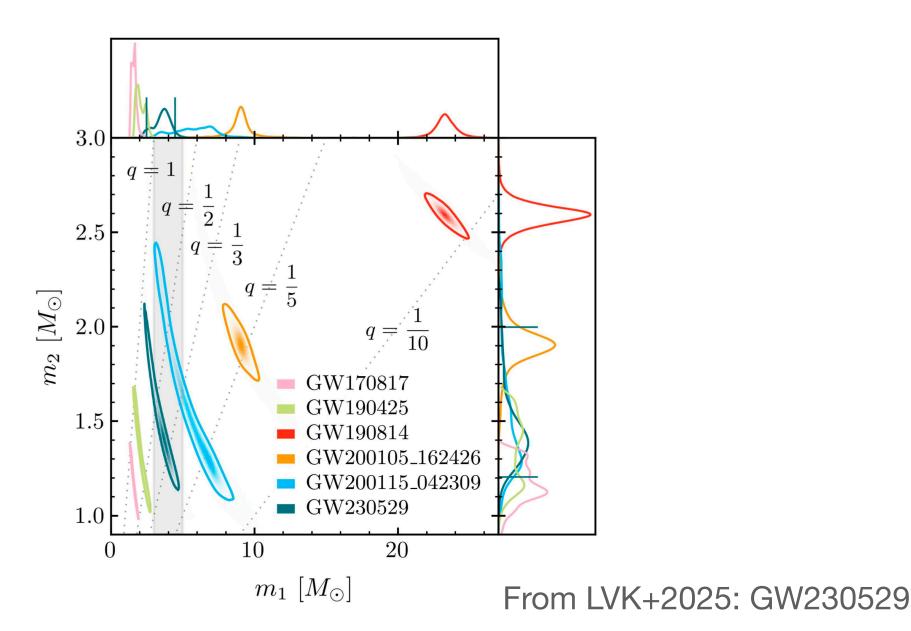
- NSBH with BH in the "low-mass gap" ($m_1 < 5 M_{\odot}$ at 99% CL)
- Comparable masses ($q \sim 0.4$), increased prospects for EM counterpart (but none observed).
- Increased rates of NSBH systems, systems with object in the low-mass gap, and EM bright NSBH systems.

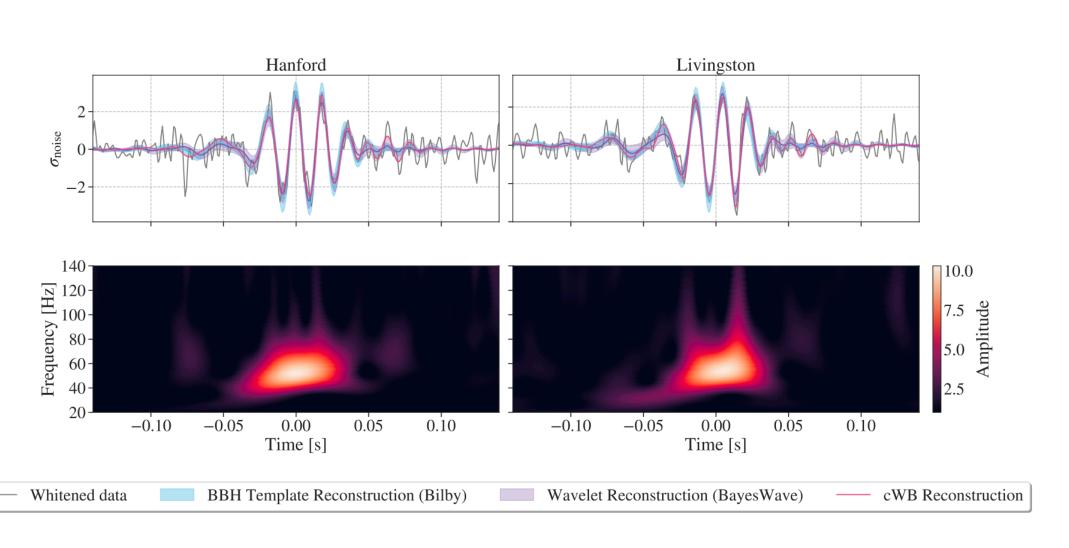
GW231123_135430:

- Very short signal ($\approx 0.1 s$, ~5 cycles)-> ringdown dominated morphology.
- Heaviest BBH to date. Inferred source:

$$m_1 = 137^{+22}_{-17} M_{\odot}, m_2 = 103^{+20}_{-52} M_{\odot}, \chi_1 \approx 0.9, \chi_2 \approx 0.8$$

- Masses lie in or across the **pair-instability-supernova** (PISN) mass-gap.
- High component spins -> herarchical-merger candidate?





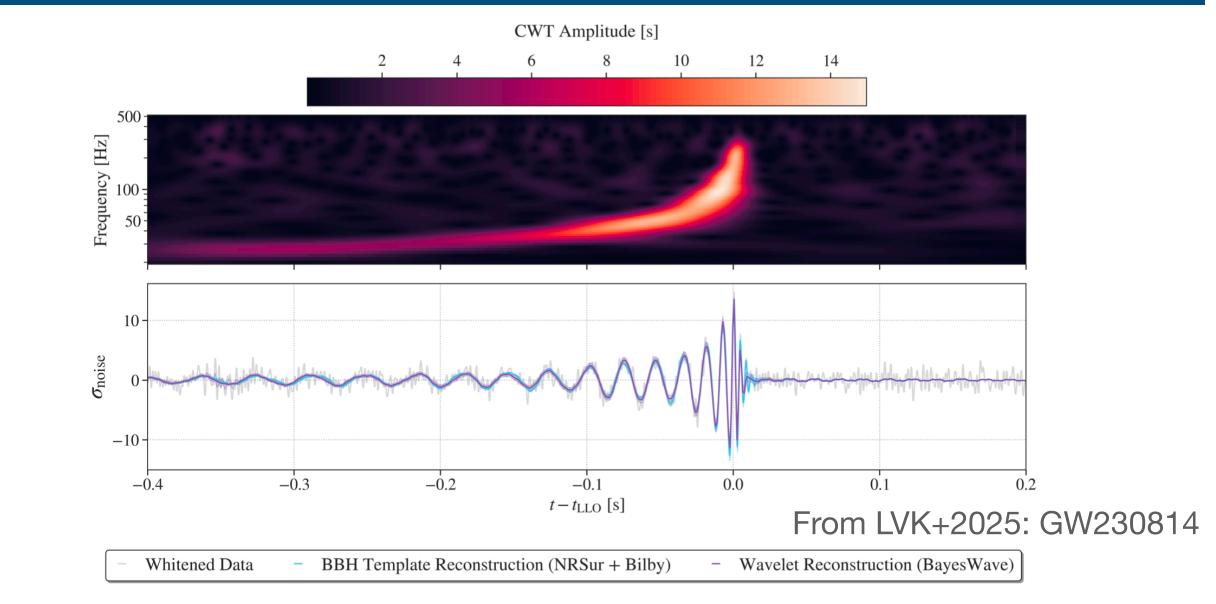
Special events from 04: testing fundamental physics with loud events

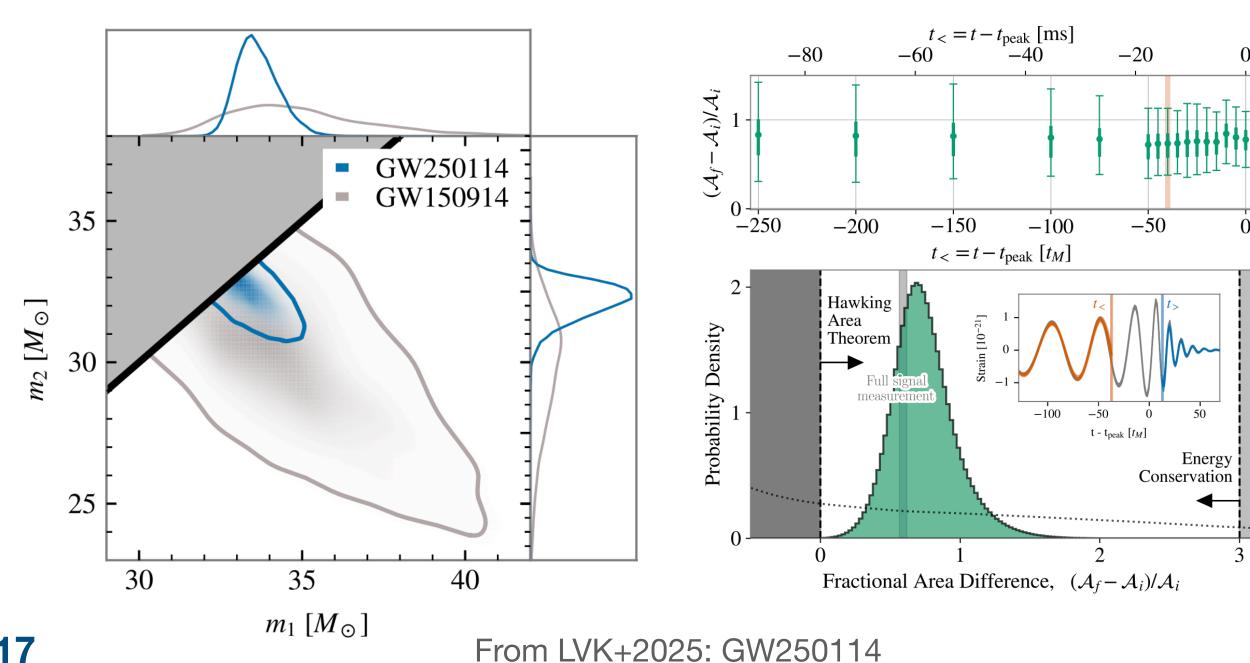
GW230814_23090:

- Highest SNR of O4a: $\rho = 42.13^{+0.07}_{-0.11}$
- Orientation likely edge-on, allowing subdominant mode visibility -> First confident detection of \emptyse =4 harmonic ($\rho_{\ell=4} \approx 3.3$)
- BH spectroscopy with ringdown signal -> mild deviation from GR observed, but attributed to noise and waveform systematics.

GW250114_082203:

- SNR: total ≈ 77 (network); ≈ 53 (LHO), ≈ 60 (LLO) -> Loudest GW signal to date!
- Source: near-equal-mass binary $m_1 \approx 34 M_{\odot}$, $m_2 \approx 34 M_{\odot}$, $\chi_{\rm eff} \approx 0$. GW150914-like with today's sensitivity!
- Black-hole spectroscopy (detect overtone n=1),
- Test of Hawking's area theorem.



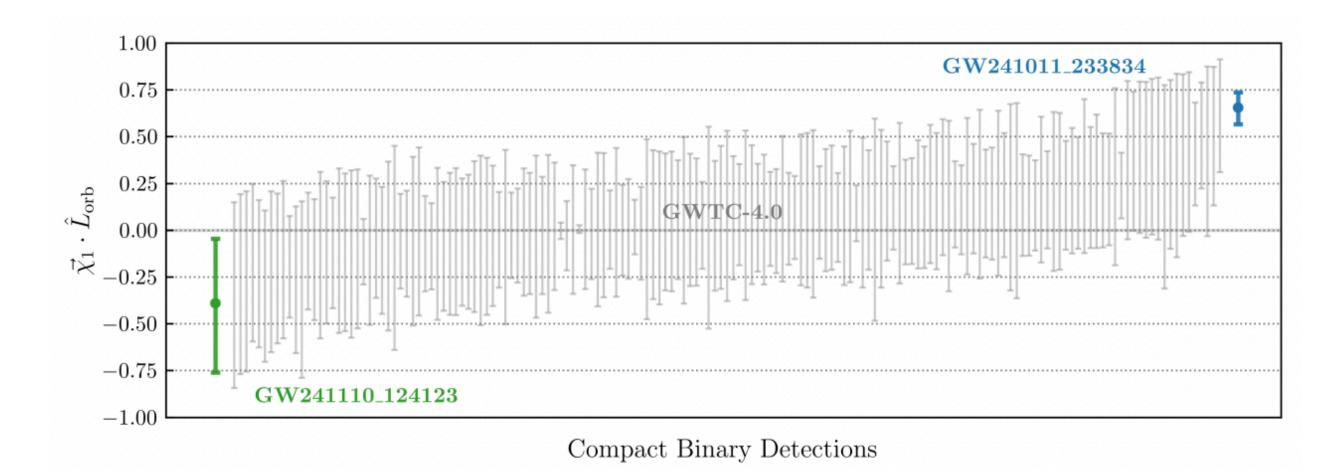


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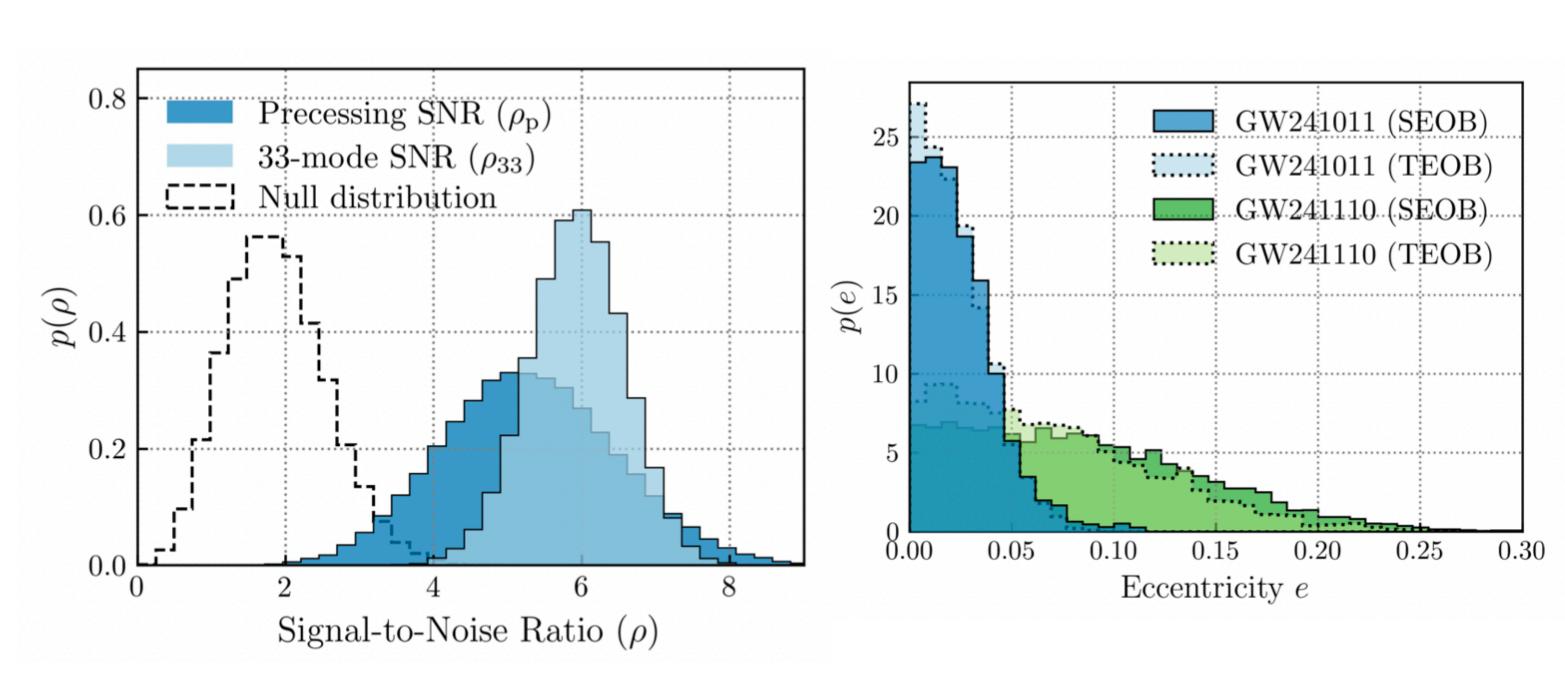
Special events from O4: highly-spinning & asymmetric

GW241011_233834 & GW241110_124123:

- Highly spinning, mass asymmetric BBHs. -> Candidate hierarchical mergers
- Measurement of spin-induced quadrupole moment
- Evidence of higher harmonics in the inspiral (33-mode) and spinprecession
- Eccentric analysis constrains orbital eccentricity of the systems.
- Measurement of spin-induced quadrupole moment -> three orders of magnitude improvement in the constraint of deviations from GR.



From LVK+2025: GW241011&GW241110



Models employed in GWTC-4.0: systematic differences

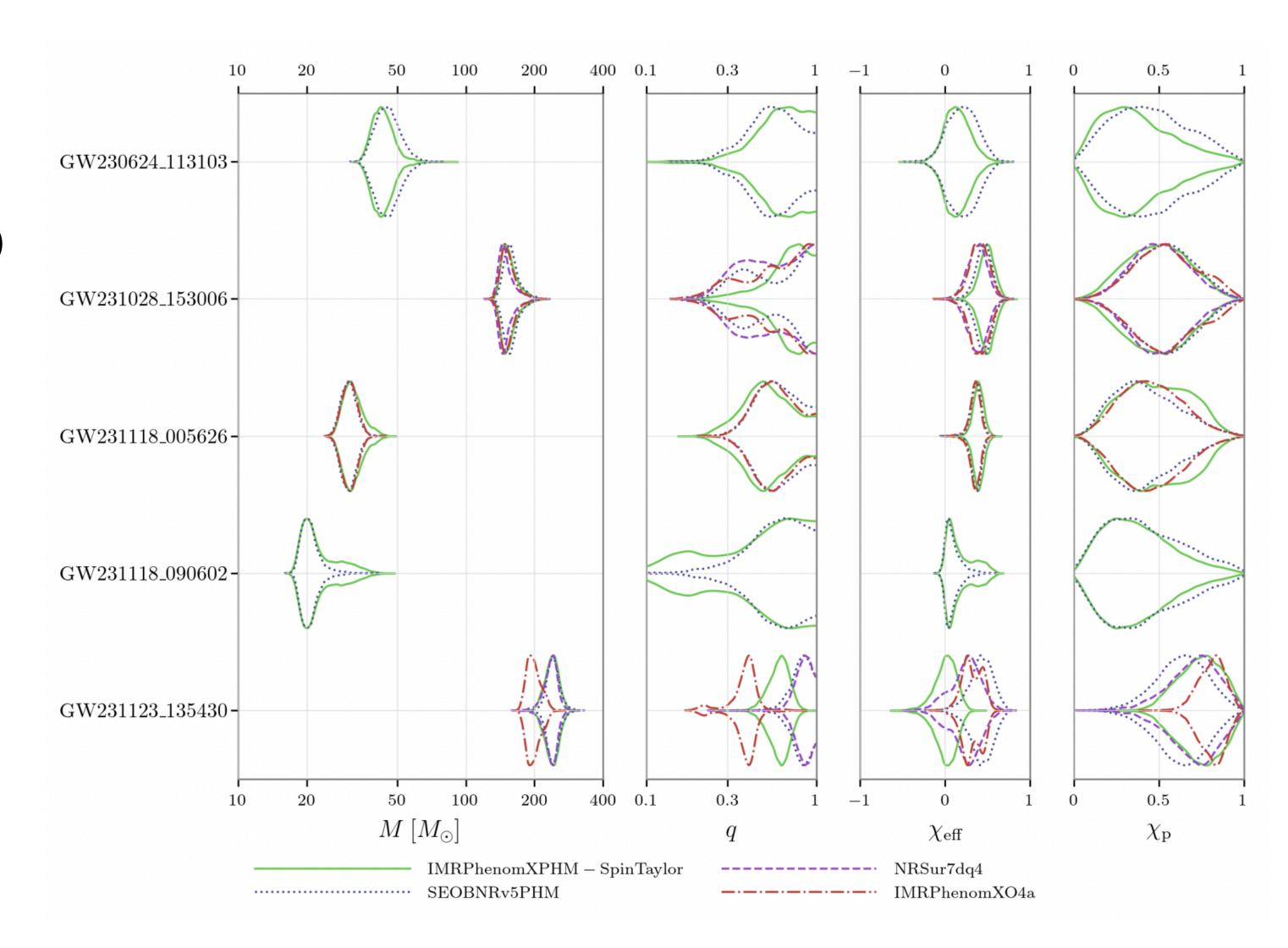
Waveform models employed in GWTC-4.0:

- IMRPhenomXPHM-SpinTaylor (all events)
- SEOBNRv5PHM (all events)
- NRSur7dq4 (events in its applicability region)
- IMRPhenomXO4a (high SNR, measurement of $\chi_{\rm p}$, or hints of systematic effects)
- IMRPhenomTPHM (for GW231123)

Most events show very consistent results for different models

But some show **systematic differences** due to **model inaccuracies**.

- High mass systems (hard for FD models)
- Unequal masses and/or high spins (limited calibration of the models)
- High SNR: shrink statistical error, while keeping systematic error.

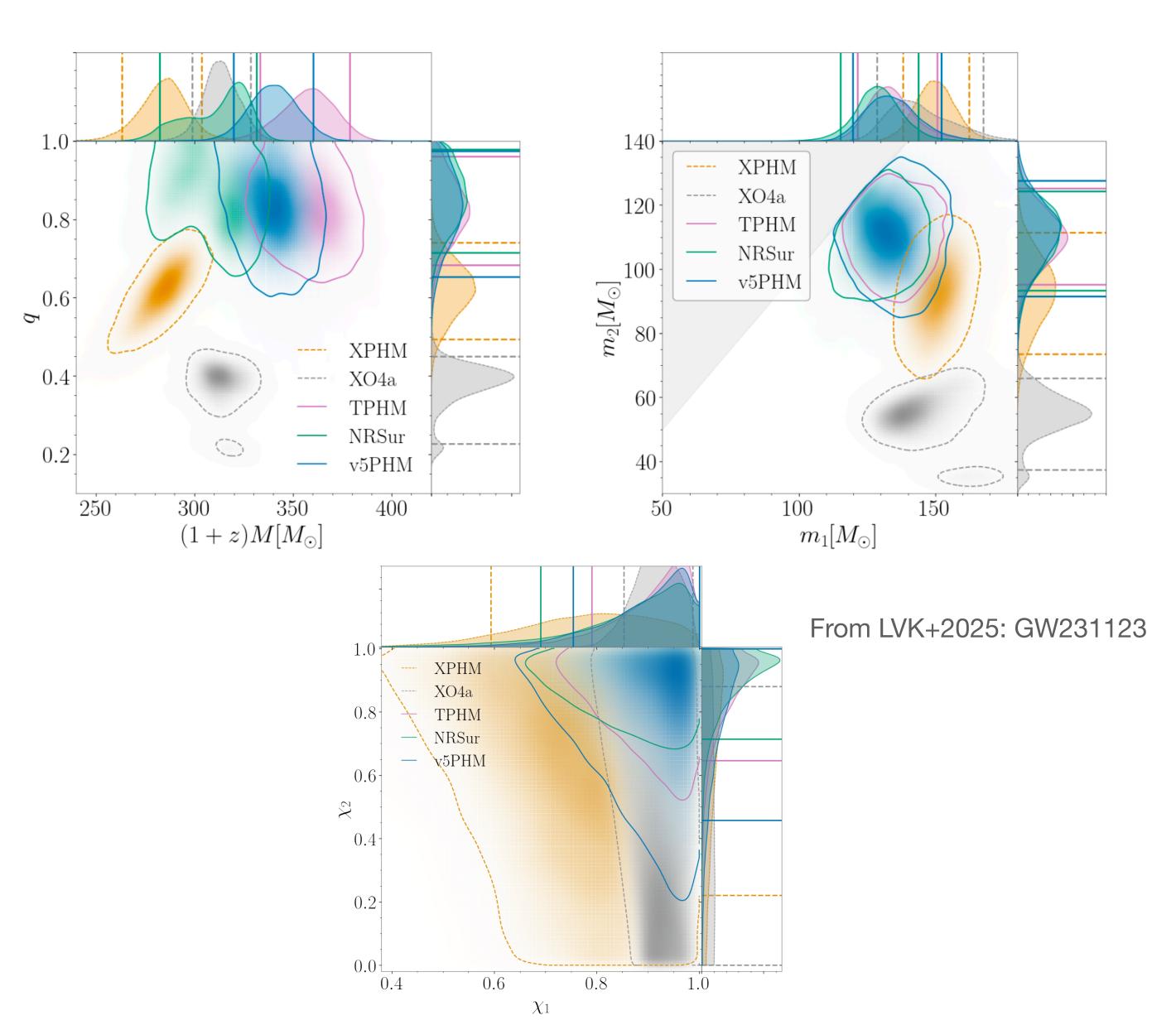


From LVK+2025: GWTC-4.0 Results

Systematics in challenging events

GW231123_135430

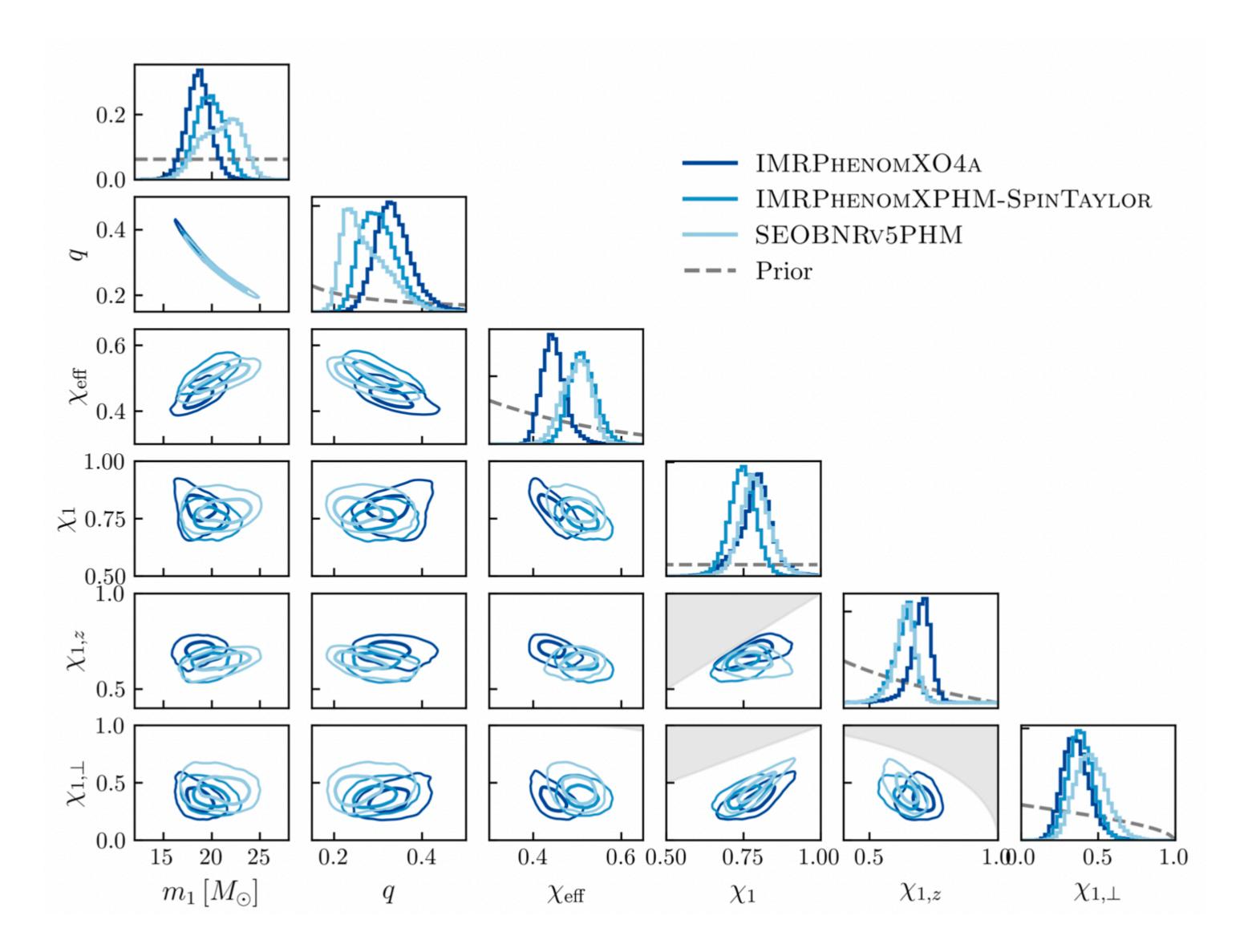
- Challenging region of parameter space:
 - Very high total mass (only merger-ringdown observed)
 - Large spins (Lack of NR simulations to inform models)
- Significant disagreement between models in total mass.
- Mass-ratio differences between time-domain models (near-equal-mass systems) and frequency-domain models (asymmetric masses).
- IMRPhenomXPHM not suited for this kind of event due to its approximations.



Systematics in challenging events

GW241011_233834

- Challenging region of parameter space:
 - Unequal masses
 - Large spins (Lack of NR simulations to inform models)
 - Spin-precession
- Slight disagreement between models in primary mass, primary spin and mass-ratio
- Different descriptions of spin-precession are probably the reason

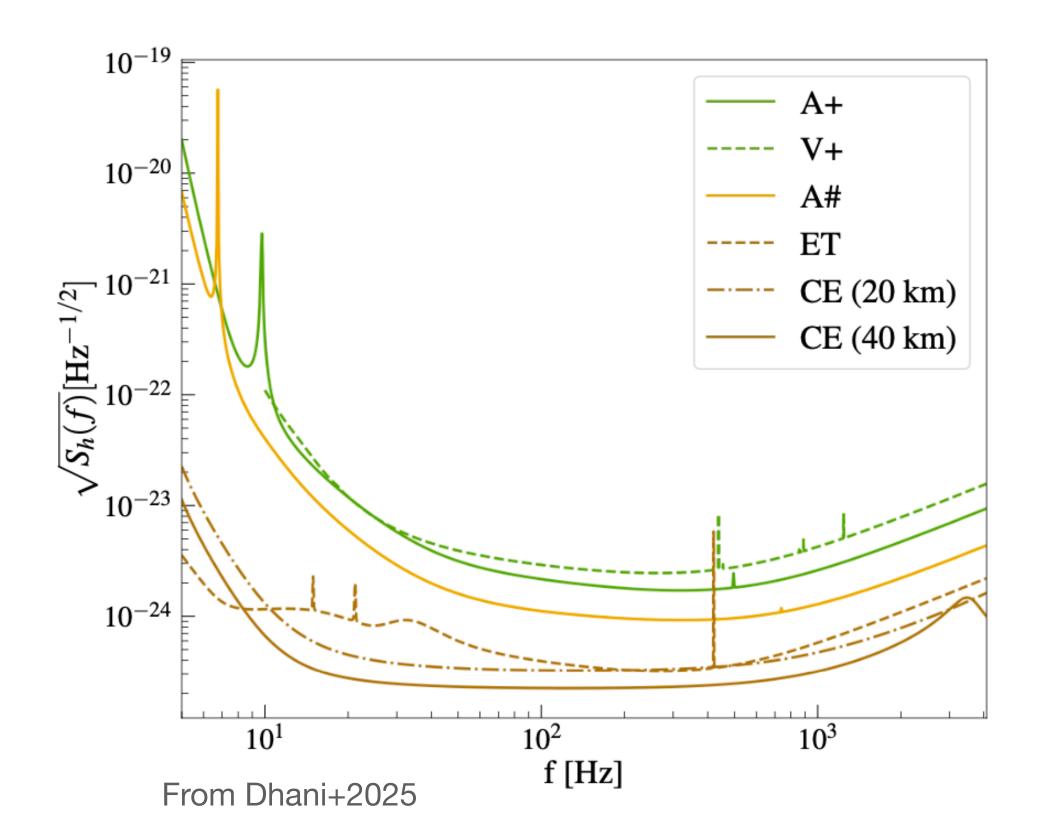


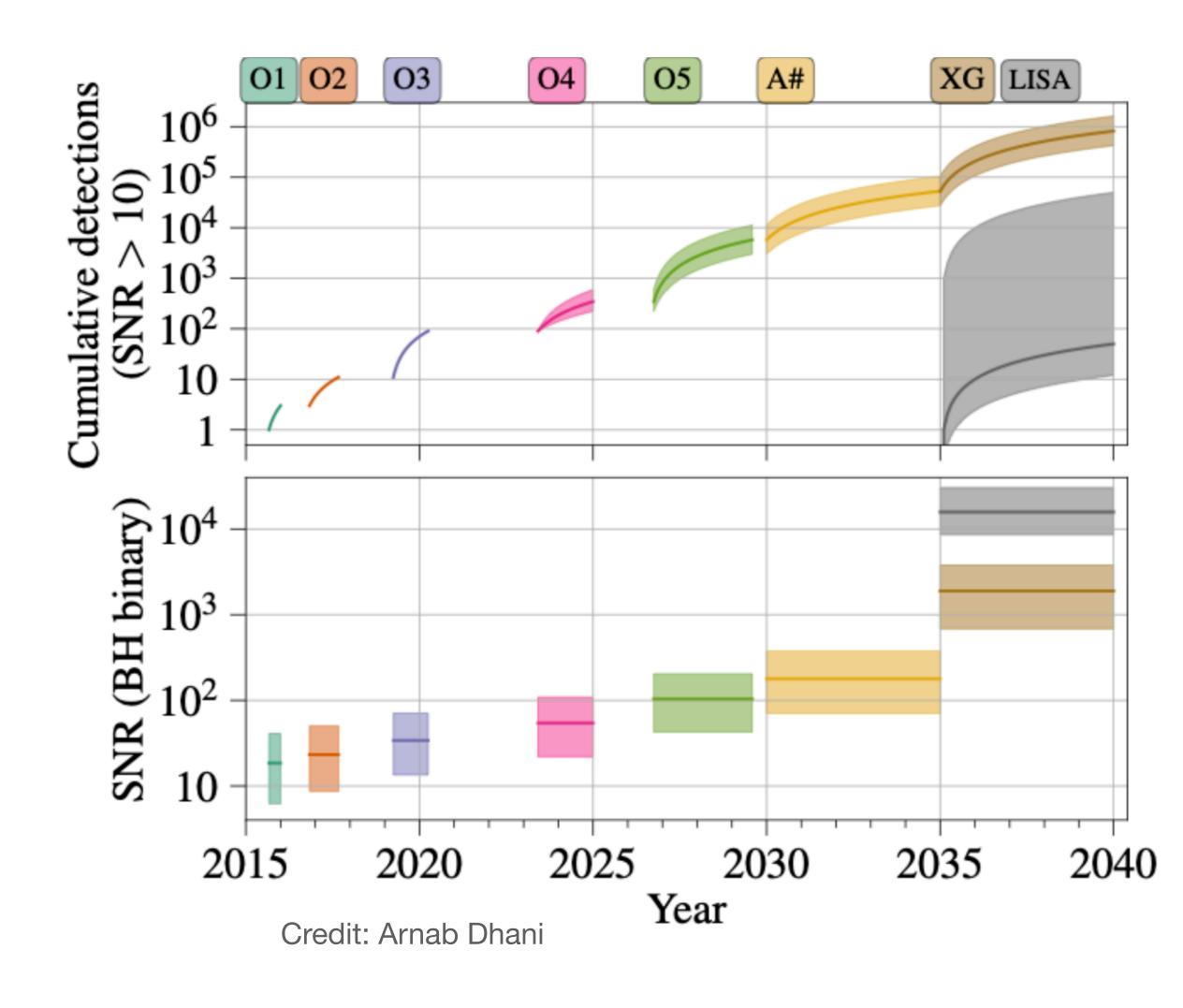
From LVK+2025: GW241011&GW241110

Accuracy challenge for next-generation detectors

Expected sensitivity improvements in current and future facilities:

- Dramatic increase in the number of events
- Higher SNR distribution: reduction of statistical uncertainty
- Will the systematic error due to inaccuracies in current waveform models bias the science outcomes?





Systematic biases due to inaccurate waveform models

Study **systematic biases** using two **state-of-the-art** waveform models for **spin-precessing** systems:

• Signal: **SEOBNRv5PHM**

Inference: IMRPhenomXPHM

1 - Biases across parameter space:

- Linear-signal-approximation for inference
- Three detector networks: O5 (2 aLIGO, aVirgo), A# (2 a#LIGO, aVirgo), XG (ET, 2 CE)
- Effect on population inference for an LVK-like mock population
- Bias horizon distance across parameter space

2 - Effect on science cases with "Golden" binaries:

- Full Bayesian analysis for selected cases
- Distance and sky-location biases: Hubble constant inference
- Primary-mass bias: Upper mass-gap limits
- Secondary-mass bias: Maximum NS mass

PHYSICAL REVIEW X 15, 031036 (2025)

Systematic Biases in Estimating the Properties of Black Holes Due to Inaccurate Gravitational-Wave Models

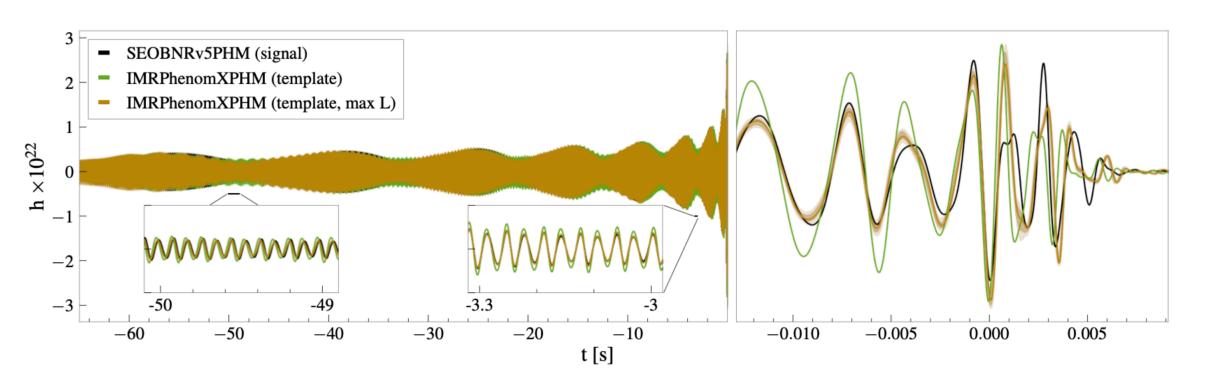
Arnab Dhani[®], ^{1,*} Sebastian H. Völkel[®], ¹ Alessandra Buonanno[®], ^{1,2} Hector Estelles[®], ¹ Jonathan Gair[®], ¹ Harald P. Pfeiffer, ¹ Lorenzo Pompili[®], ¹ and Alexandre Toubiana¹

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Department of Physics, University of Maryland, College Park, Maryland 20742, USA

GW190814-like, but strongly precessing: $q=m_1/m_2\sim 10$, $\chi_{\rm eff}=0.51$, $\chi_{\rm p}=0.45$

SNR (O5): ~75, **mismatch**(signal,template)~4%, **Biases**: $\delta \chi_p \sim 13\%$, $M\sim 3\%$, $q\sim 6\%$



Linear signal approximation

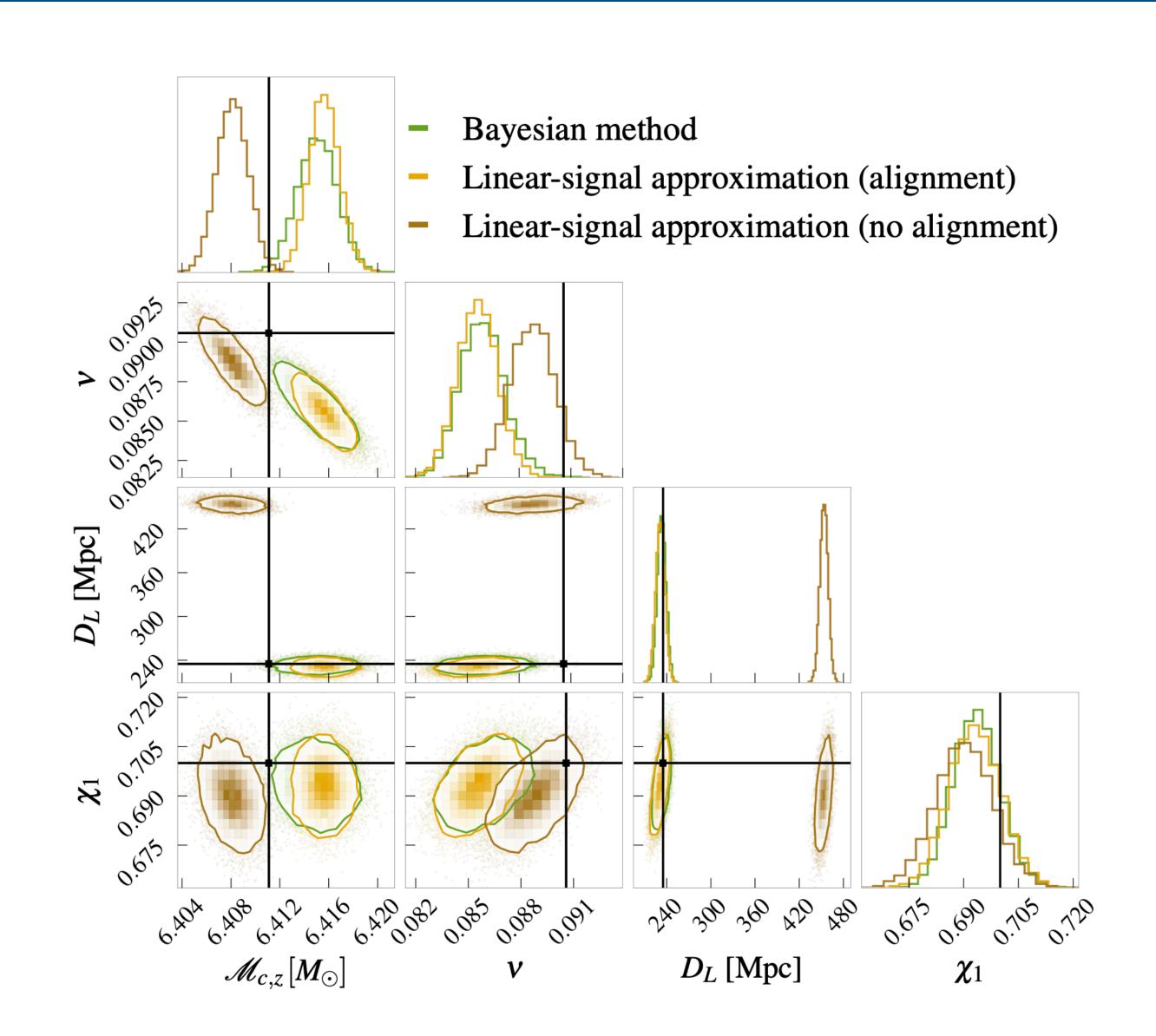
Linear signal approximation

- Fisher matrix: $F_{ij} = (\partial_{\theta_i} h(\pmb{\theta}) \,|\, \partial_{\theta_j} h(\pmb{\theta}))$
- Covariance matrix: $C_{ij} = (F_{ij})^{-1}$
- Statistical uncertainty: $\Delta \theta_i = \sqrt{C_{ii}}$
- Parameter bias: $\delta\theta_i = C^{ij}(\partial_{\theta_i} h(\pmb{\theta}) \,|\, \delta h)$

Flanagan-Hughes+98 Cutler-Vallisneri +07

Some extrinsic parameters do not agree completely between waveform models: alignment needed

$$\delta h \equiv \min_{t_c, \phi_{\text{ref}}, \psi, \phi_{\text{JL}}} (h_s - h)$$



Systematic biases in LVK-like population

Synthetic population

- Mass distribution: LVK GWTC-3
- **Spin distribution:** uniform magnitudes and isotropic directions
- Redshift: $z \in [0,3]$

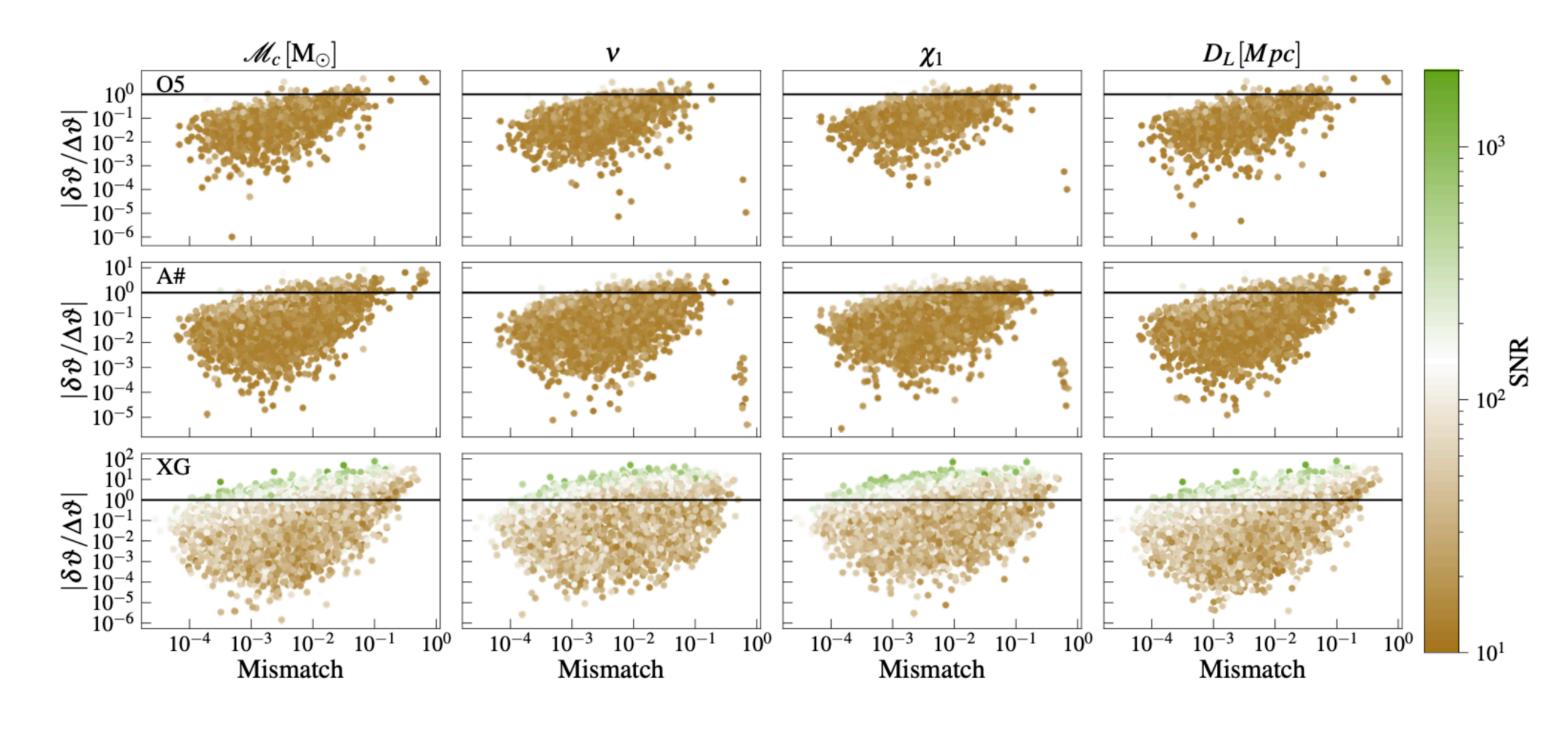
10⁴

series in the series of the series of

 $|\delta\theta/\Delta\theta| > 1$: Systematic bias greater than statistical error - parameter biased.

Percent biased:

- O5/A#: 2-3%
- XG: 10-25%
- Correlation with mismatch btw signal and template



Systematic biases across parameter space

Unphysical population

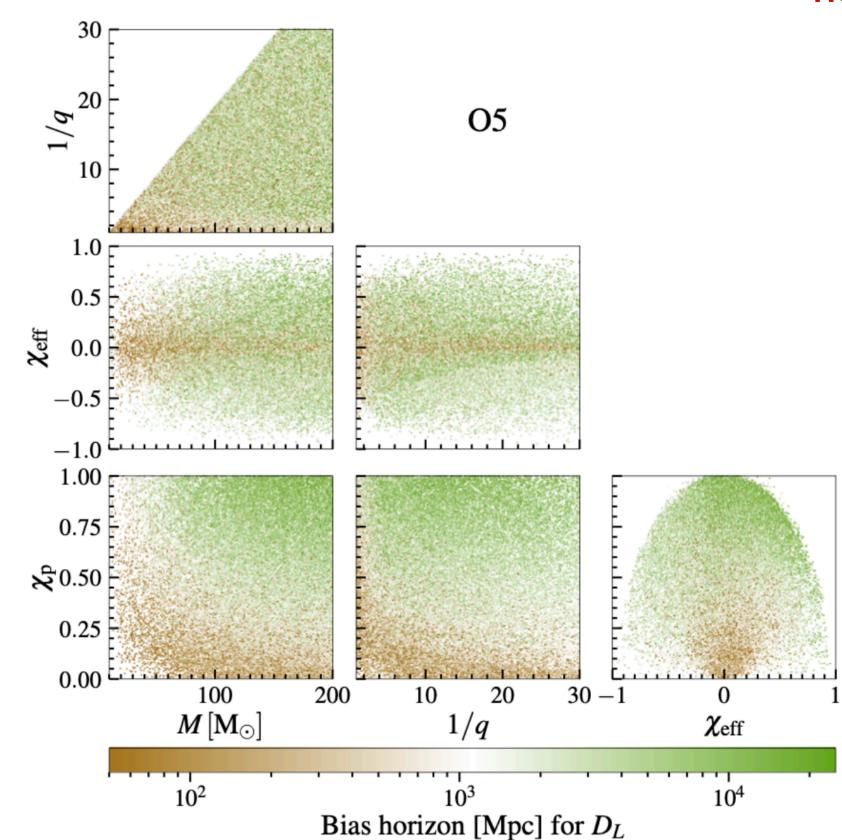
• Uniform distribution: $\chi_p \in [0,1]$

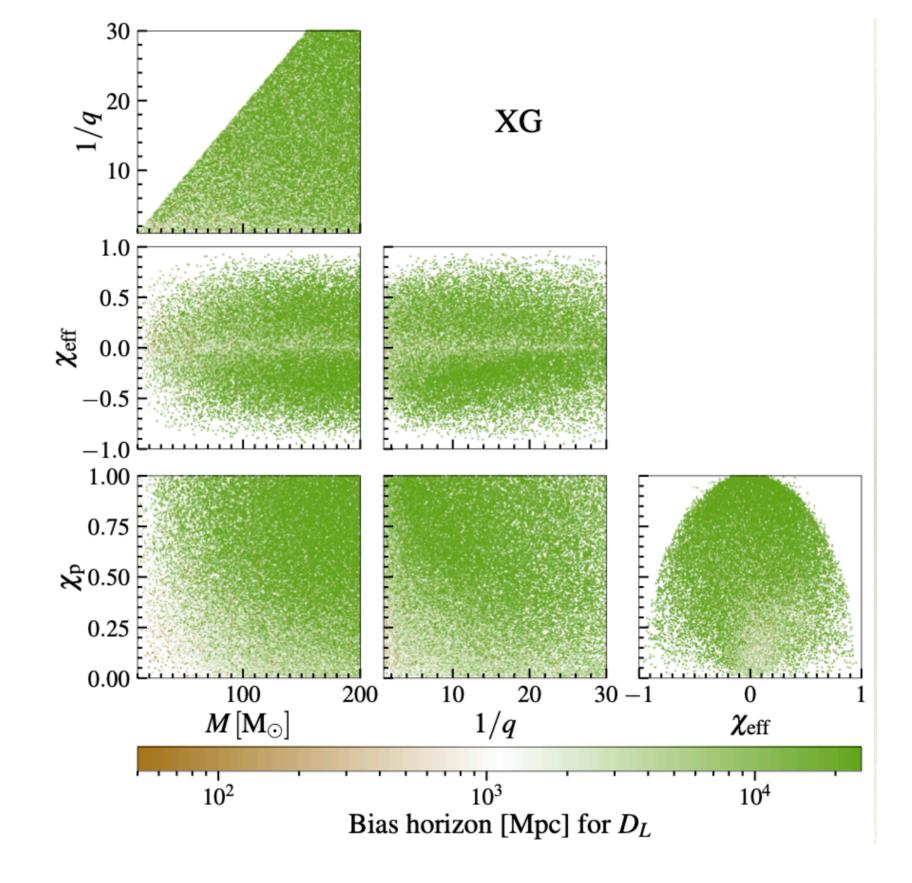
• Uniform distribution: $q = m_1/m_2 \le 30$

Goal: study biases across broader parameter space. Individual interesting binaries might be in challenging regions.

Bias horizon: minimum distance at which a given parameter is biased ($|\delta\theta/\Delta\theta| > 1$) (larger is worse - systematics appearing at lower SNRs)

Less systematics for smaller spins: improvements in spin-precession still needed



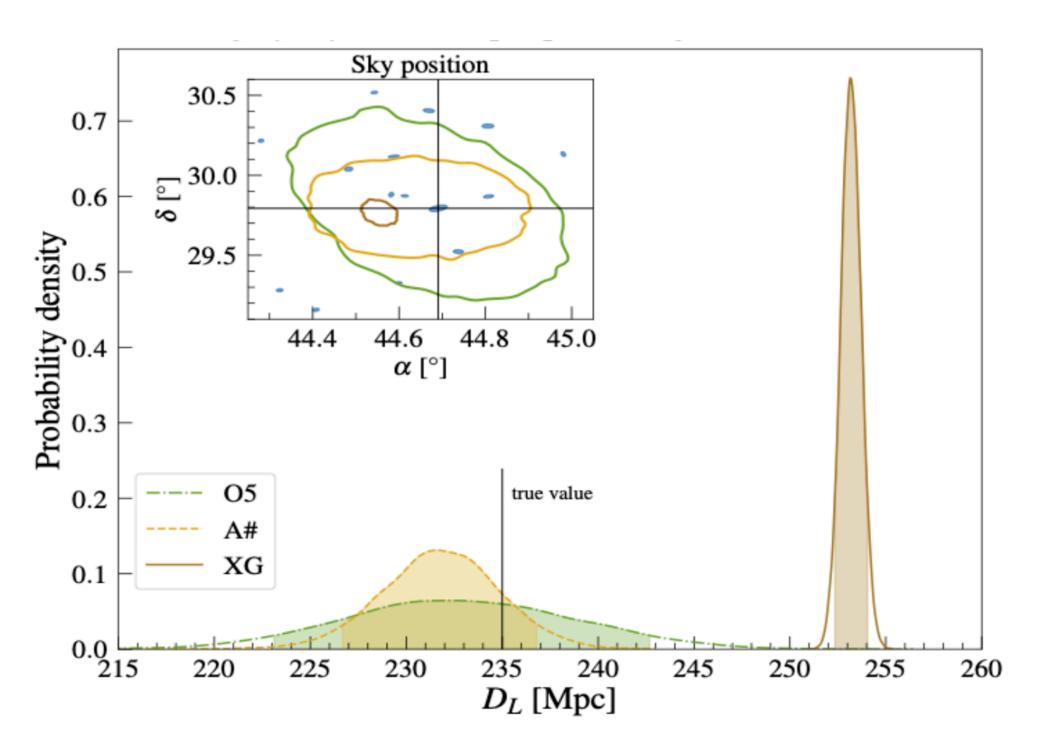


Effect on science goals: biased distance and sky-location

*
$$m_1 = 23.2 M_{\odot}, m_2 = 2.6 M_{\odot}$$

*
$$\chi_{eff} = 0.51, \chi_p = 0.45$$

$$SNR_{O5} = 75$$
, $SNR_{XG} = 1040$



Spin-precessing NSBH:

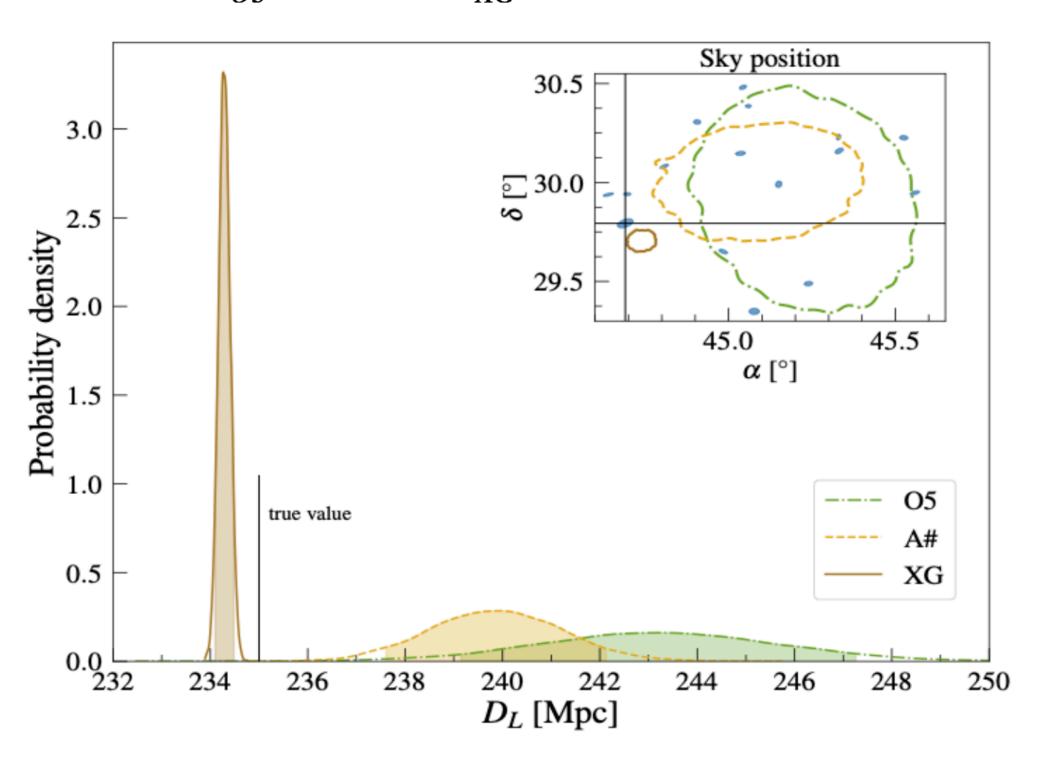
distance and sky-location biased for XG

Can affect Hubble constant estimation and the identification of EM counter-parts.

$$m_1 = 61.8 M_{\odot}, m_2 = 9.5 M_{\odot}$$

*
$$\chi_{eff} = -0.43, \chi_p = 0.77$$

*
$$SNR_{O5} = 119$$
, $SNR_{XG} = 2490$



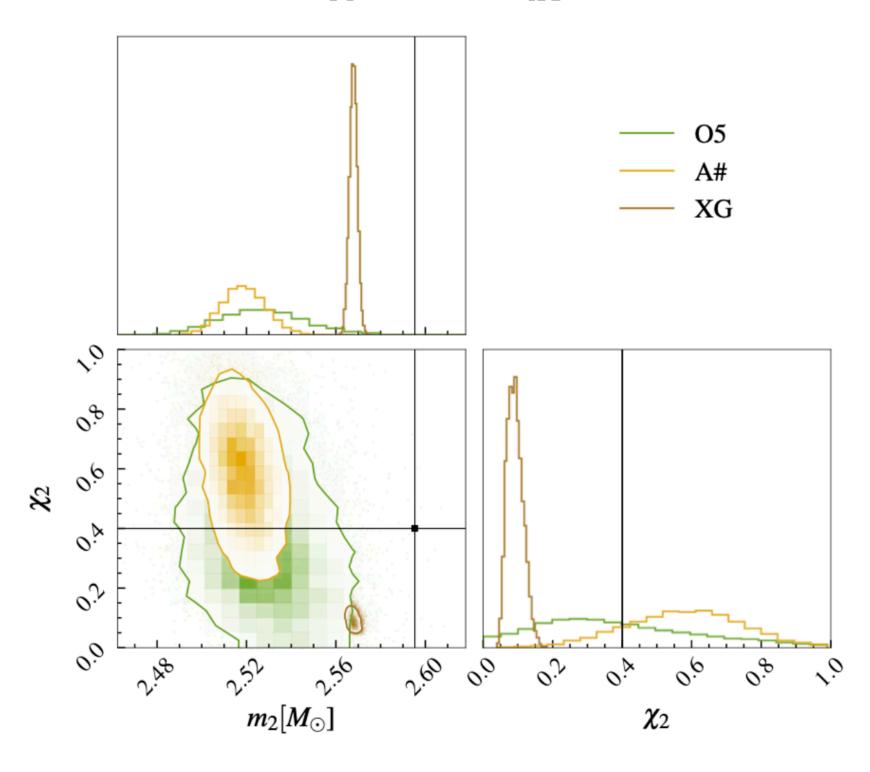
Highly asymmetric strongly precessing: distance and sky-location biased even for O5

Effect on science goals: biased component masses

*
$$m_1 = 23.2 M_{\odot}, m_2 = 2.6 M_{\odot}$$

*
$$\chi_{eff} = 0.51, \chi_p = 0.45$$

$$* SNR_{O5} = 75, SNR_{XG} = 1040$$



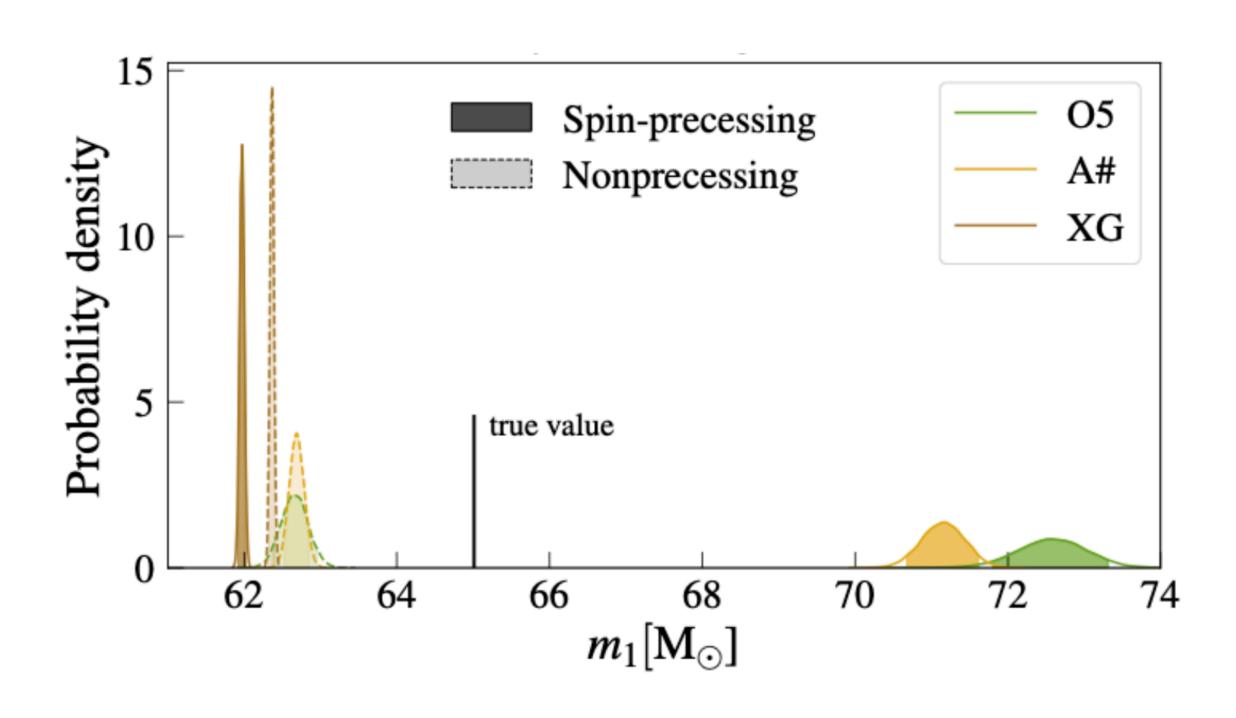
Spin-precessing NSBH: secondary-mass biased even for O5.

Can affect estimation of maximum NS mass.

$$m_1 = 61.8 M_{\odot}, m_2 = 9.5 M_{\odot}$$

*
$$\chi_{eff} = -0.43, \chi_p = 0.77$$
 and $\chi_{eff} = 0.89, \chi_p = 0$

*
$$SNR_{O5} = 119$$
, $SNR_{XG} = 2490$



Highly asymmetric strongly precessing (and non-precessing!): primary-mass biased

Can affect estimation of lower end of upper mass-gap.

Summary

- Active waveform development in the last years: mature models for quasi-circular systems and first models for eccentric systems
- New GW observations are increasing the parameter space and the SNR of detected systems: exciting new science outcomes.
- Waveform inaccuracies are starting to show systematic effects in the results.
- Current accuracy will not be enough for improve sensitivity of future detectors: biases in population and science goals with "golden binaries".
- More work needed to address the challenges of the future!