GRAVITATIONAL WAVES AS A PROBE OF THE EARLY UNIVERSE

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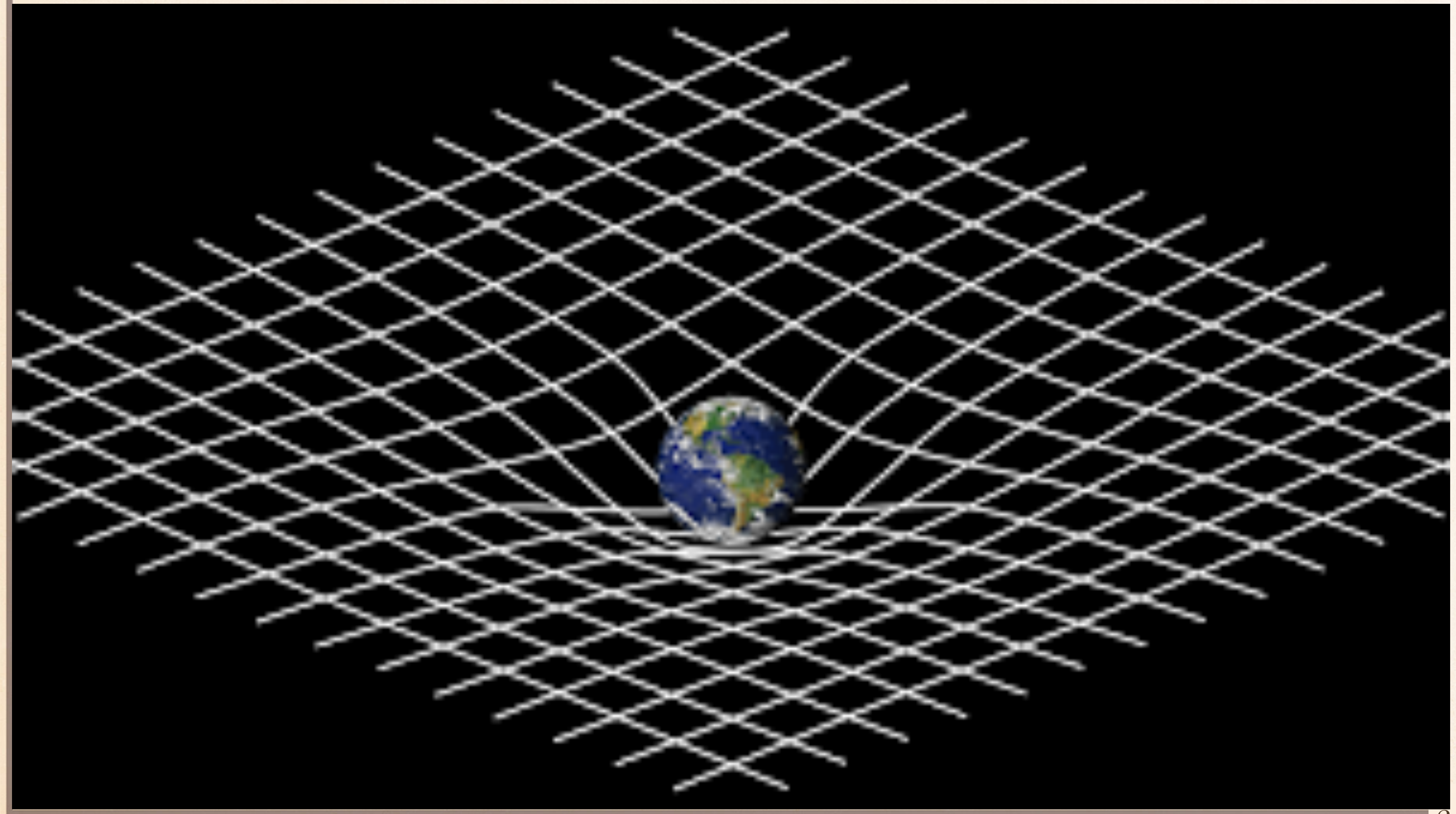
Based on:
G. Domènech, S. Pi, **AW** 251x.xxxxx

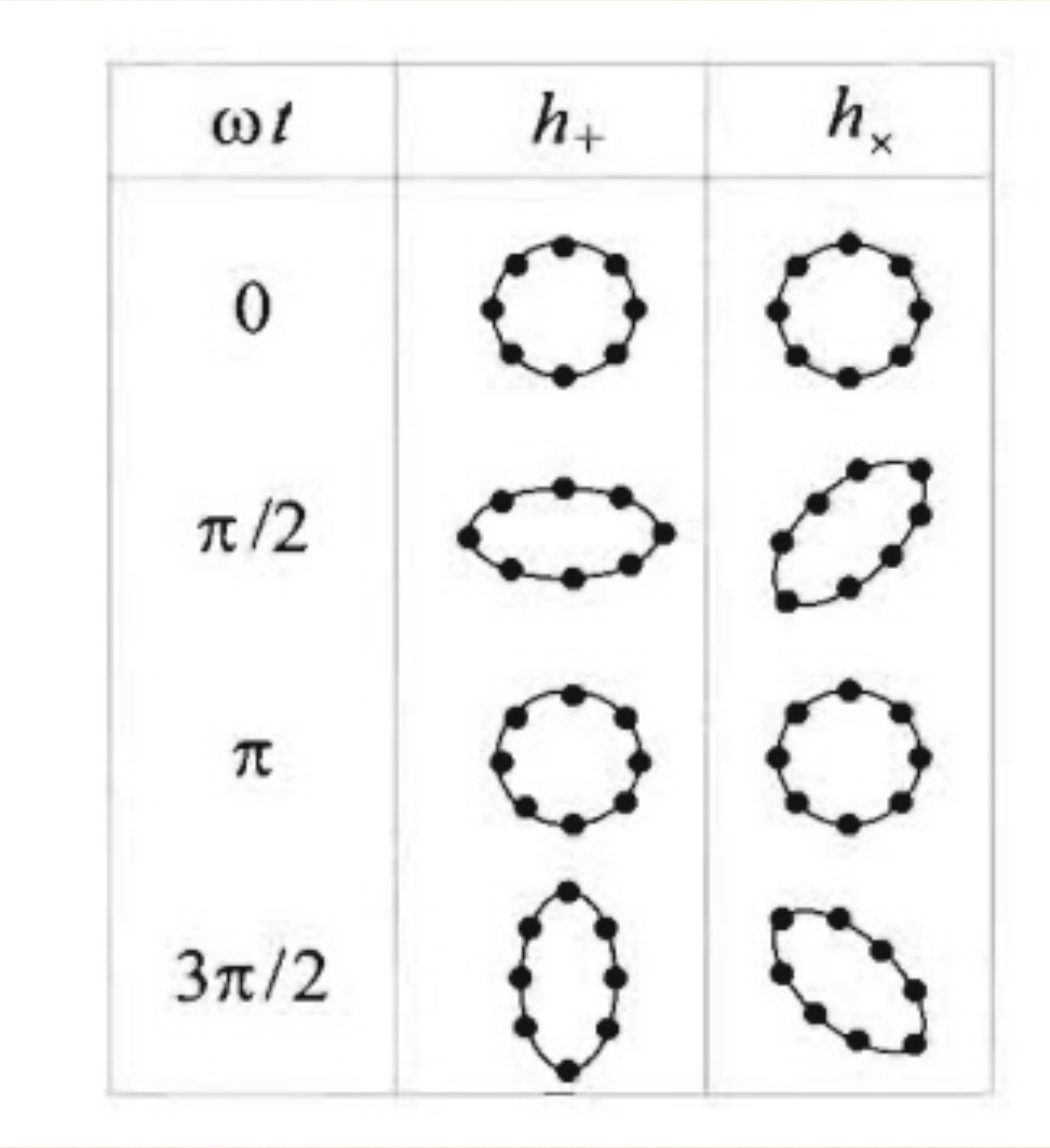
Nov.06.2025, BIG meeting @ ICCUB

WHAT IS GW?

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$g_{\mu\nu}$: 10 components → 2 DoF





Michele Maggiore, Gravitational Waves

GW EFFECT

Geodesic deviation equation

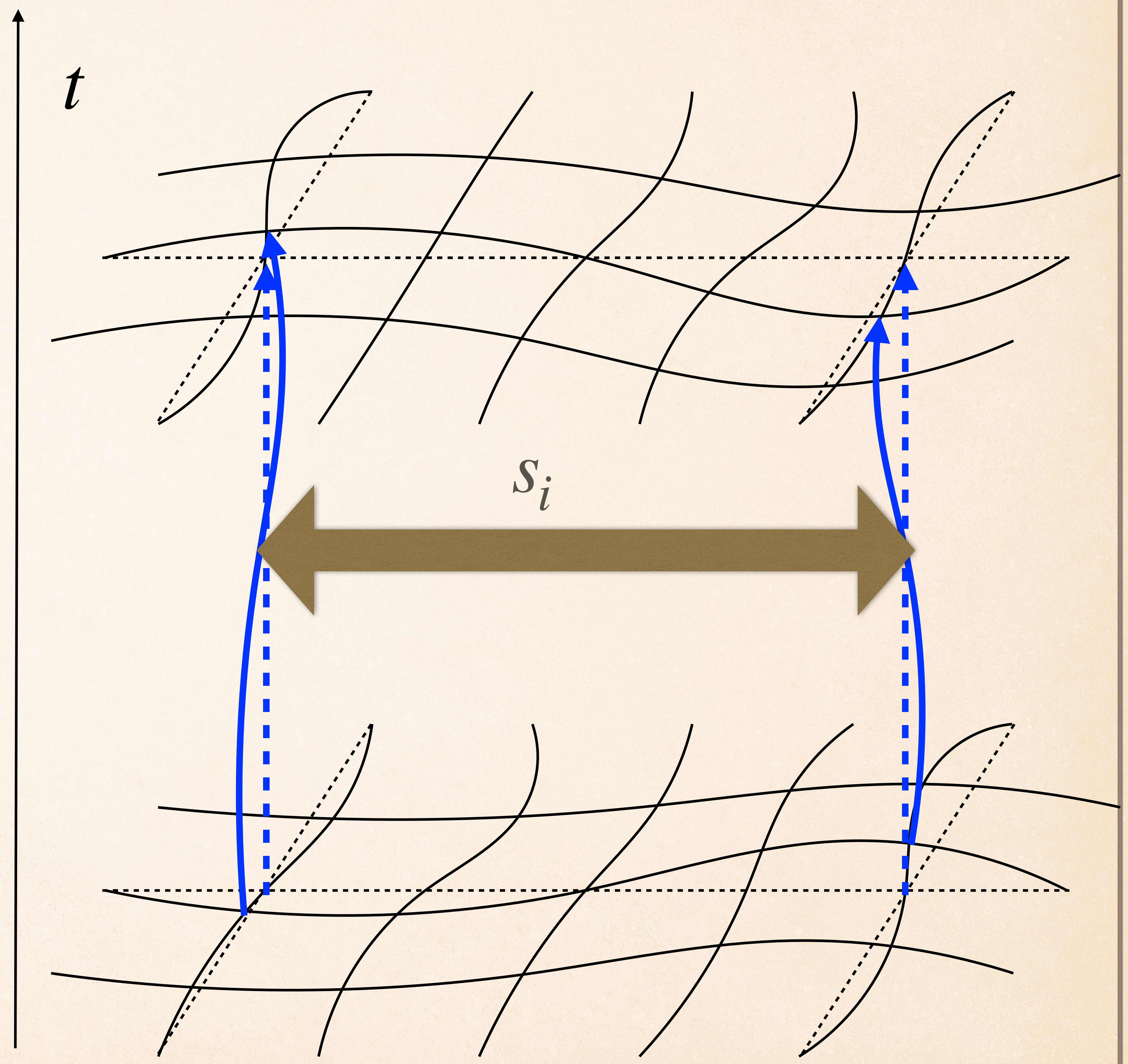
$$\ddot{s}_i = -\frac{1}{2}\ddot{h}_{ij}s_j \qquad h_{ij} = \Lambda_{ij}^{kl} g_{kl}$$

at rest $dx^i/d\tau = \mathcal{O}(h)$

scale $|s| \ll L$

first-order perturbation

Michele Maggiore, Gravitational Waves



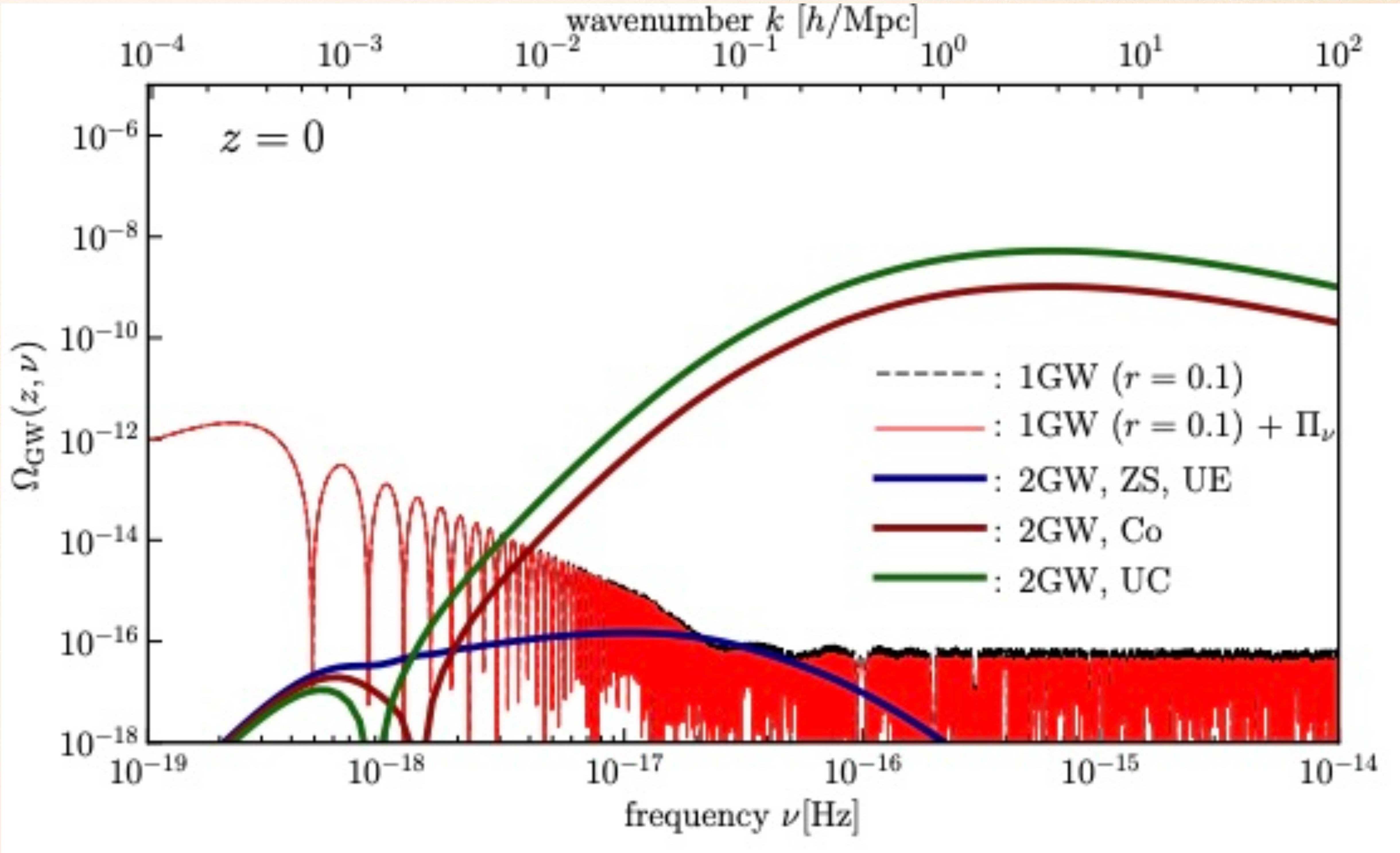
GAUGE ISSUE?

$$ds^2 = a(\eta)^2\Big(-(1+2\Phi)\eta^2 + ((1-2\Psi)\delta_{ij} + h^N_{ij})\mathrm{d}x^i\mathrm{d}x^j \Big)$$

$$h^G_{ij} = h^N_{ij} - \Lambda^{ab}_{ij}\left\{\partial_a T_G \partial_b T_G + \partial_a\partial_k L_G \partial_b \partial_k L_G\right\}$$

Comoving slicing $\quad T_C = V_N$

$$\partial_i T_G \partial_i T_G > h^N_{ij}$$



J. Hwang, D. Jeong, & H. Noh, 2017

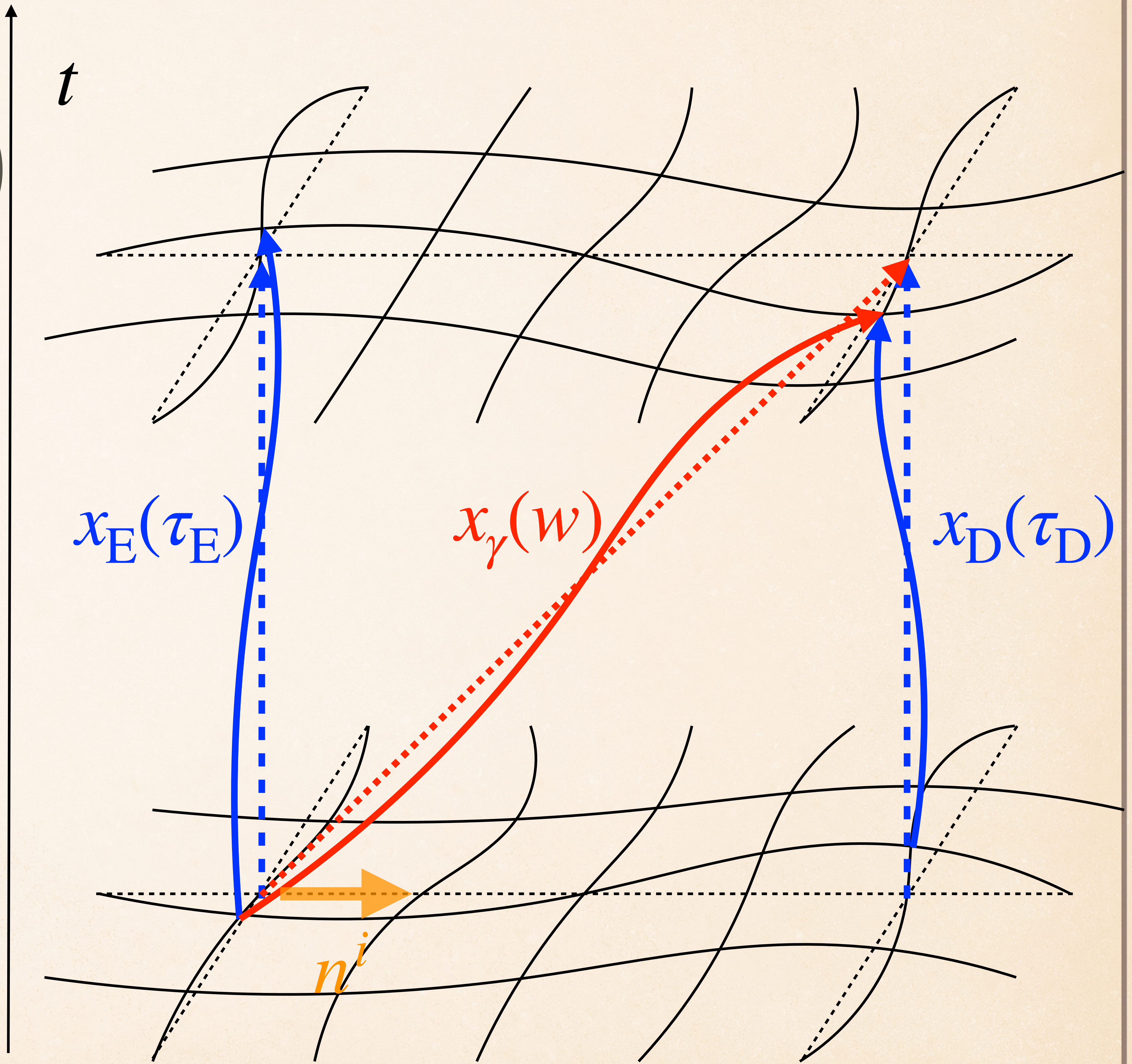
OUTLINE

- First-order effect
- Second-order effect

GW EFFECT

$$ds^2 = a(\eta)^2\Big(-N^2 \mathrm{d}\eta^2 + g_{ij}(\mathrm{d}x^i + N^i d\eta)(\mathrm{d}x^j + N^j d\eta)\Big)$$

$\tau_{\mathrm{E\,i}}$ $\quad$ $x_{\mathrm{D}}(\tau_{\mathrm{D\,f}}) = x_\gamma(w_{\mathrm{f}})$

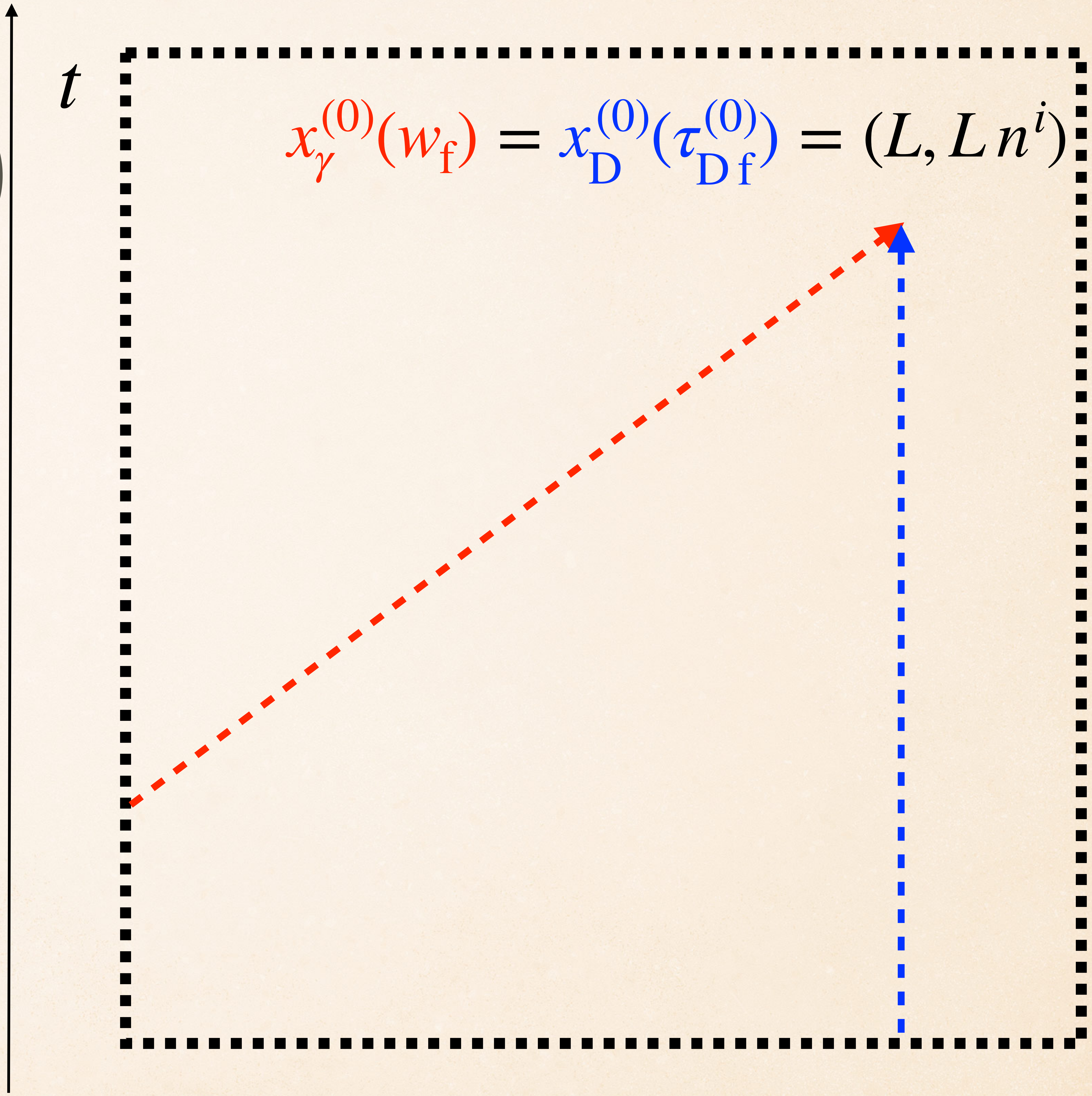


GW EFFECT

$$ds^2 = a(\eta)^2\left(-N^2 d\eta^2 + g_{ij}(dx^i + N^i d\eta)(dx^j + N^j d\eta)\right)$$

$\tau_{\mathrm{E\,i}}$ $\quad x_{\mathrm{D}}(\tau_{\mathrm{D\,f}}) = x_\gamma(w_{\mathrm{f}})$

$$x^{(0)}_{\mathrm{D}}(\tau^{(0)}_{\mathrm{D\,f}}) = x^{(0)}_\gamma(w_{\mathrm{f}})$$



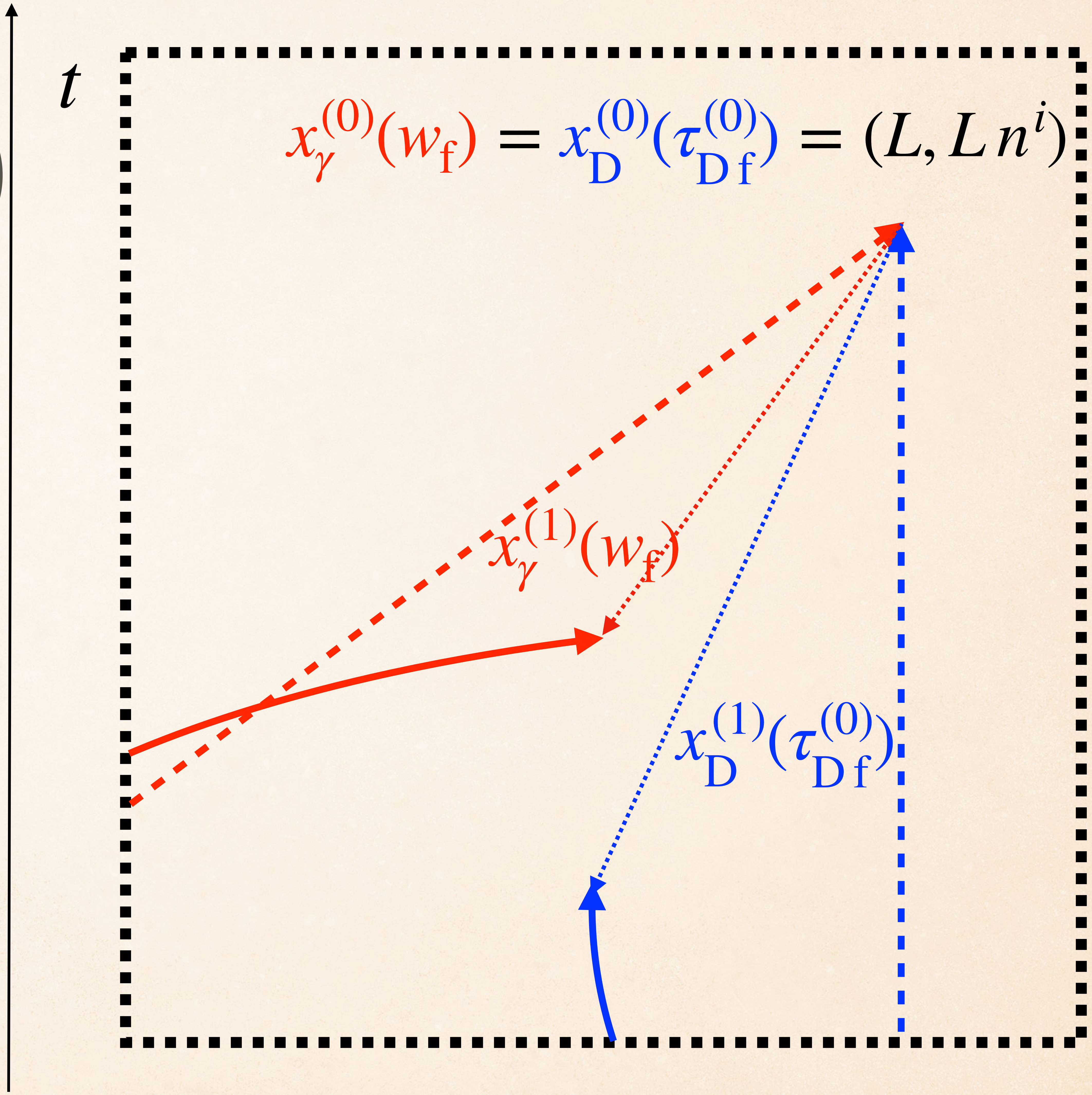
GW EFFECT

$$ds^2 = a(\eta)^2\Big(-N^2 \mathrm{d}\eta^2 + g_{ij}(\mathrm{d}x^i + N^i d\eta)(\mathrm{d}x^j + N^j d\eta)\Big)$$

$\tau_{\mathrm{E\,i}}$ $\quad x_{\mathrm{D}}(\tau_{\mathrm{D\,f}}) = x_\gamma(w_{\mathrm{f}})$

$$x_{\mathrm{D}}^{(0)}(\tau_{\mathrm{D\,f}}^{(0)}) = x_\gamma^{(0)}(w_{\mathrm{f}})$$

$$x_{\mathrm{D}}^{(1)}(\tau_{\mathrm{D\,f}}^{(0)}) \neq x_\gamma^{(1)}(w_{\mathrm{f}})$$



GW EFFECT

$$ds^2 = a(\eta)^2\Big(-N^2 \mathrm{d}\eta^2 + g_{ij}(\mathrm{d}x^i + N^i d\eta)(\mathrm{d}x^j + N^j d\eta)\Big)$$

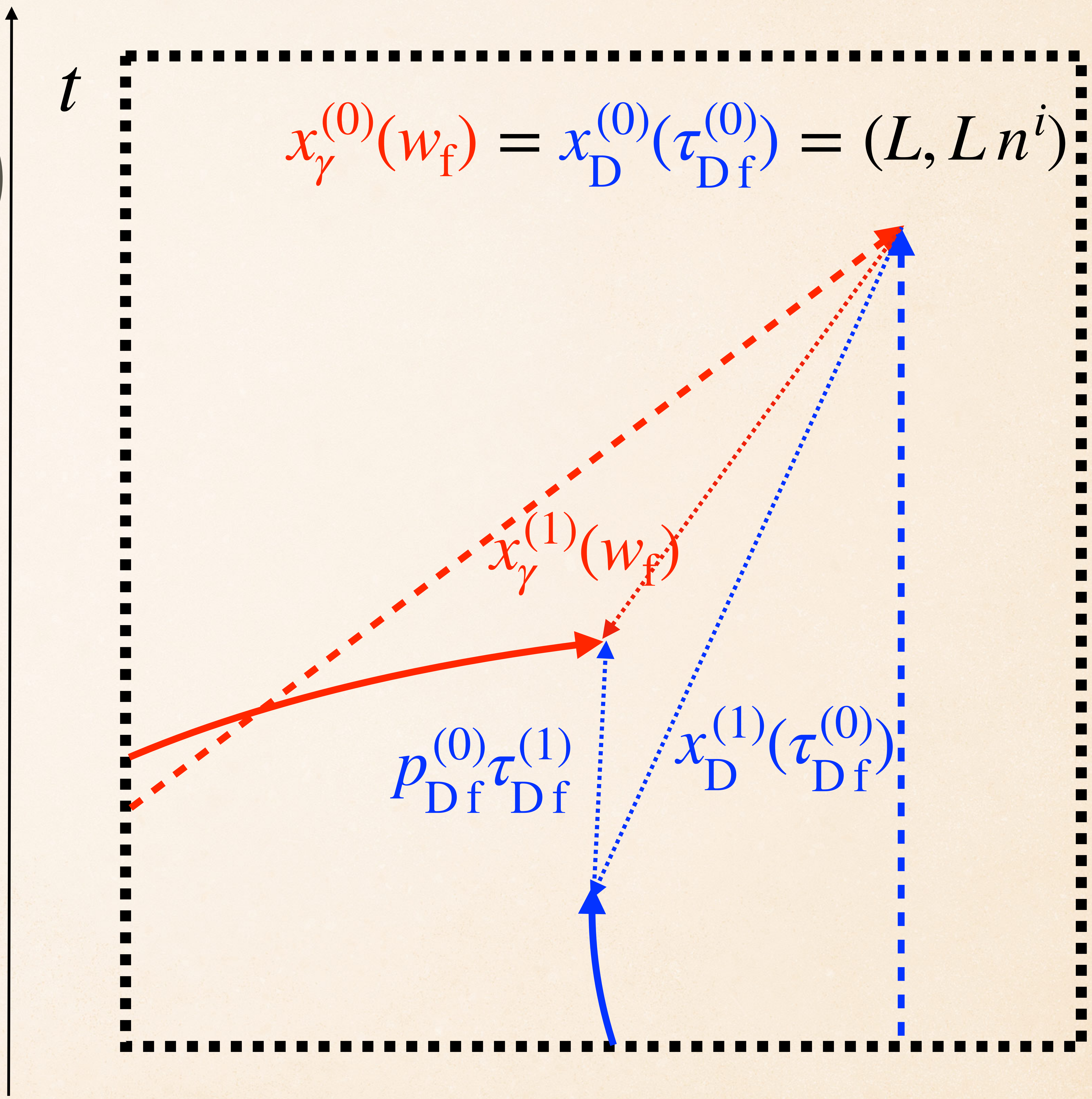
$$\tau_{\mathrm{E\,i}} \qquad x_{\mathrm{D}}(\tau_{\mathrm{D\,f}}) = x_\gamma(w_{\mathrm{f}})$$

$$x_{\mathrm{D}}^{(0)}(\tau_{\mathrm{D\,f}}^{(0)}) = x_\gamma^{(0)}(w_{\mathrm{f}})$$

$$x_{\mathrm{D}}^{(1)}(\tau_{\mathrm{D\,f}}^{(0)}) \neq x_\gamma^{(1)}(w_{\mathrm{f}})$$

$$\tau_{\mathrm{D\,f}}^{(1)} = \frac{1}{p_{\mathrm{D\,f}}^{(0)}}\left(x_\gamma^{(1)}(w_{\mathrm{f}}) - x_{\mathrm{D}}^{(1)}(\tau_{\mathrm{D\,f}}^{(0)})\right)$$

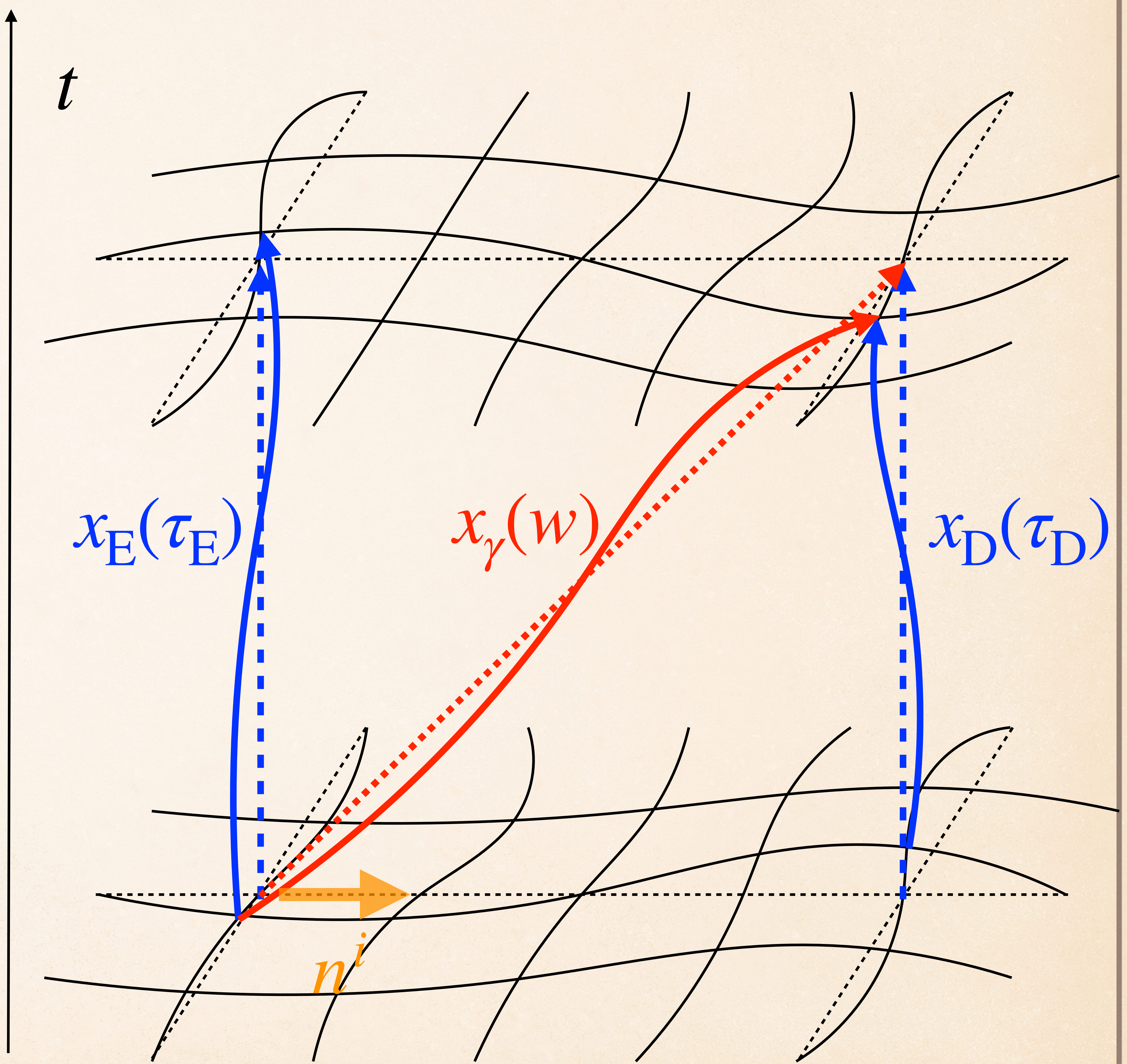
$$p_{\mathrm{D}}^{(0)\mu} = \frac{\mathrm{d}x_{\mathrm{D}}^{(0)\mu}}{\mathrm{d}\tau_{\mathrm{D}}} = \frac{1}{a}(1,\vec{0})$$



PHOTON MOTION

$$\left(x_\gamma^{(1)0} - n^i x_\gamma^{(1)i}\right)\Big|_0^{w_\mathrm{f}} = \frac{1}{2}\int_0^{w_\mathrm{f}} \mathrm{d}w\, g_{//}^{(1)},$$

$$g_{//}^{(1)} = k_\gamma^{(0)\mu} g_{\mu\nu}^{(1)} k_\gamma^{(0)\nu} \qquad k_\gamma^{(0)\mu} = \frac{\mathrm{d}x_\gamma^{(0)\mu}}{\mathrm{d}w} = \frac{1}{a^2}(1, n^i)$$



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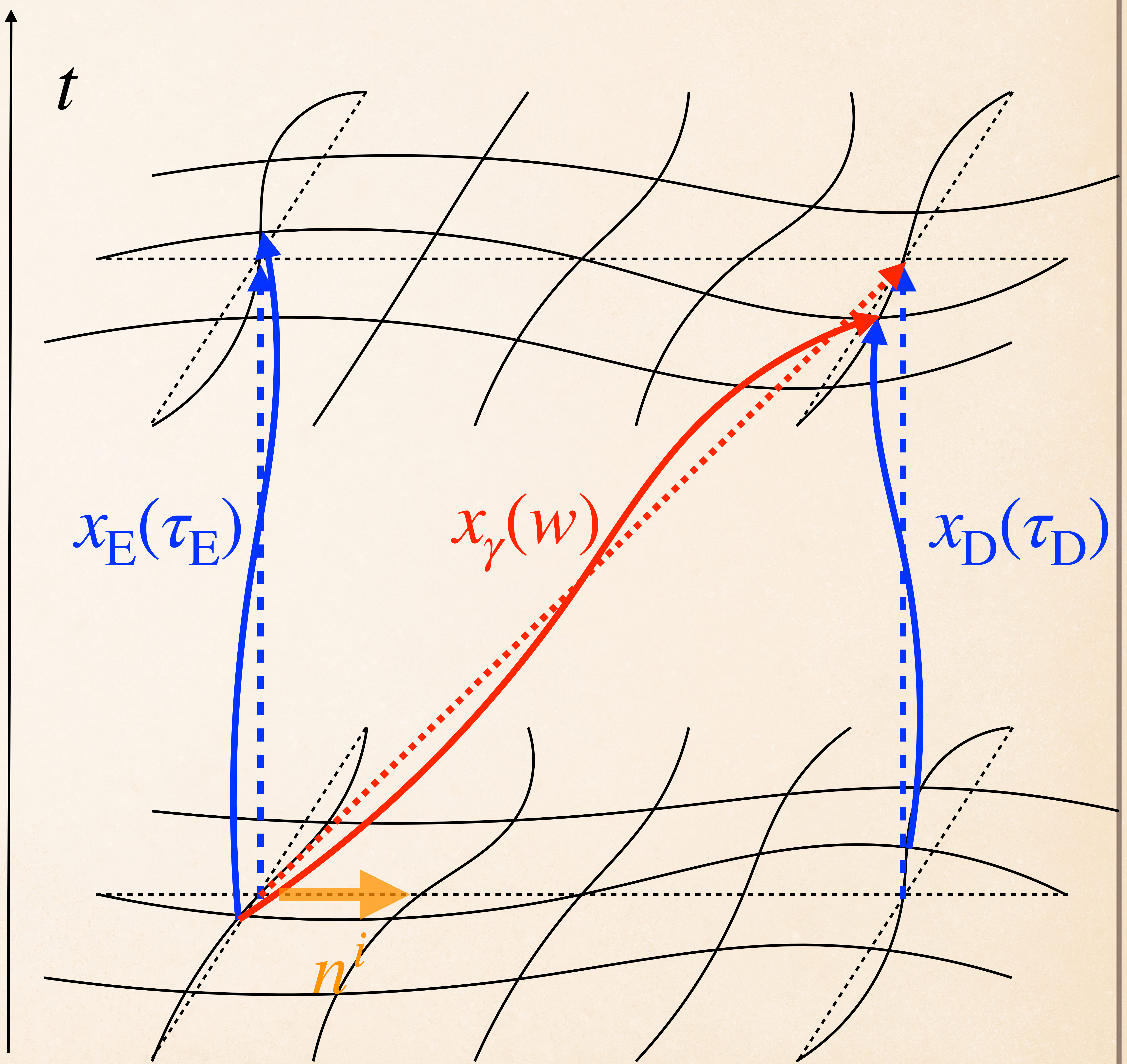
$$\frac{1}{2}\int_0^{w_\mathrm{f}} \mathrm{d}w\, g_{//}^{(1)} = \frac{1}{2} n^i n^j \int_0^L \mathrm{d}\lambda\, h_{ij}^{(1)}$$

$$+ n^i E_{,i}^{(1)}\Big|_0^{w_\mathrm{f}}$$

$$- \int_0^L \mathrm{d}\lambda \left(\alpha^{(1)} - \mathscr{R}^{(1)} + \sigma^{(1)\prime}\right) + \sigma^{(1)}\Big|_0^{w_\mathrm{f}}$$

$$x_\gamma^{(0)\mu}(w) = x_{\gamma\,\mathrm{i}}^{(0)\mu} + \lambda\,(1, n^i)$$



PHOTON MOTION

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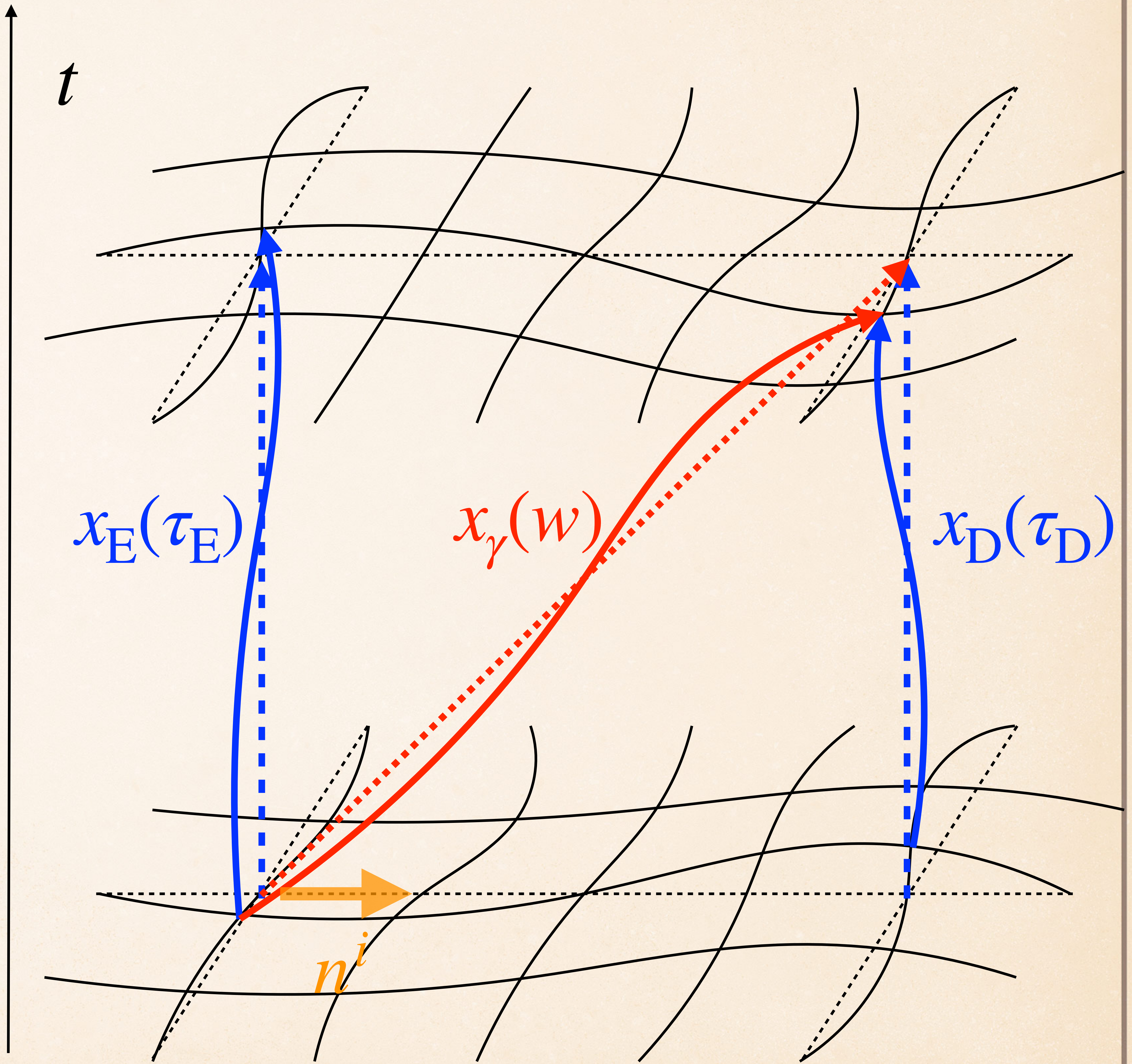
$$+ n^i E_{,i}^{(1)}\Big|_0^{w_\mathrm{f}}$$

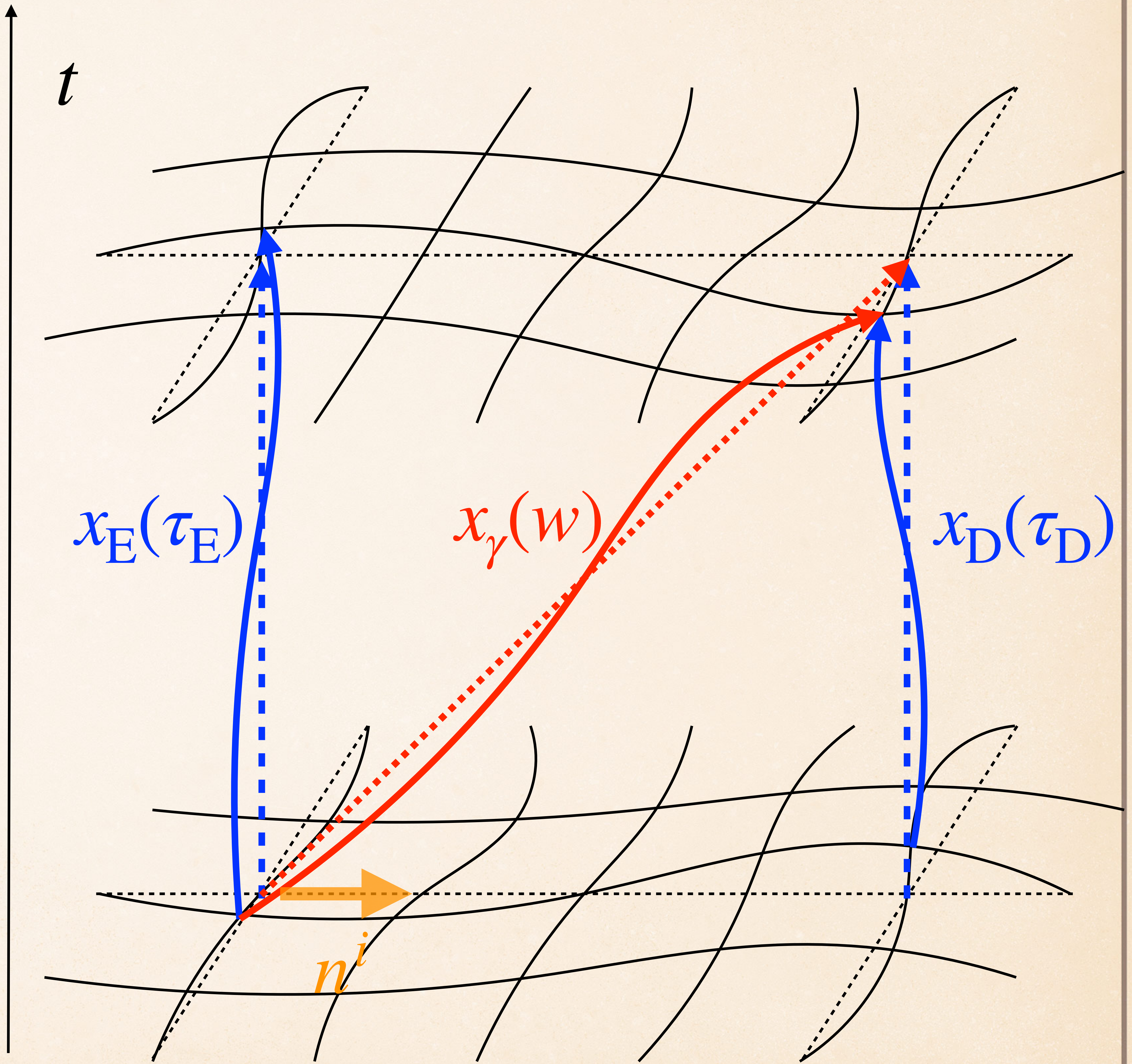
$$-\int_0^L \mathrm{d}\lambda\, \underbrace{\left(\alpha^{(1)} - \mathscr{R}^{(1)} + \sigma^{(1)\prime}\right)}_{\Phi^{(1)} + \Psi^{(1)}} + \sigma^{(1)}\Big|_0^{w_\mathrm{f}}$$

$$x_\gamma^{(0)\mu}(w) = x_{\gamma\,\mathrm{i}}^{(0)\mu} + \lambda\,(1, n^i)$$



OBSERVERS MOTION

$$a\left(\tau_{\mathrm{D}}\right) x_{\mathrm{D}}^{(1)0}\left(\tau_{\mathrm{D}}\right) = a_{\mathrm{i}} x_{\mathrm{Di}}^{(1)0} - \int_0^{\tau_{\mathrm{D}}} \mathrm{d}\tilde{\tau}_{\mathrm{D}} \alpha^{(1)}\left(\tilde{\tau}_{\mathrm{D}}\right)$$



OBSERVERS MOTION

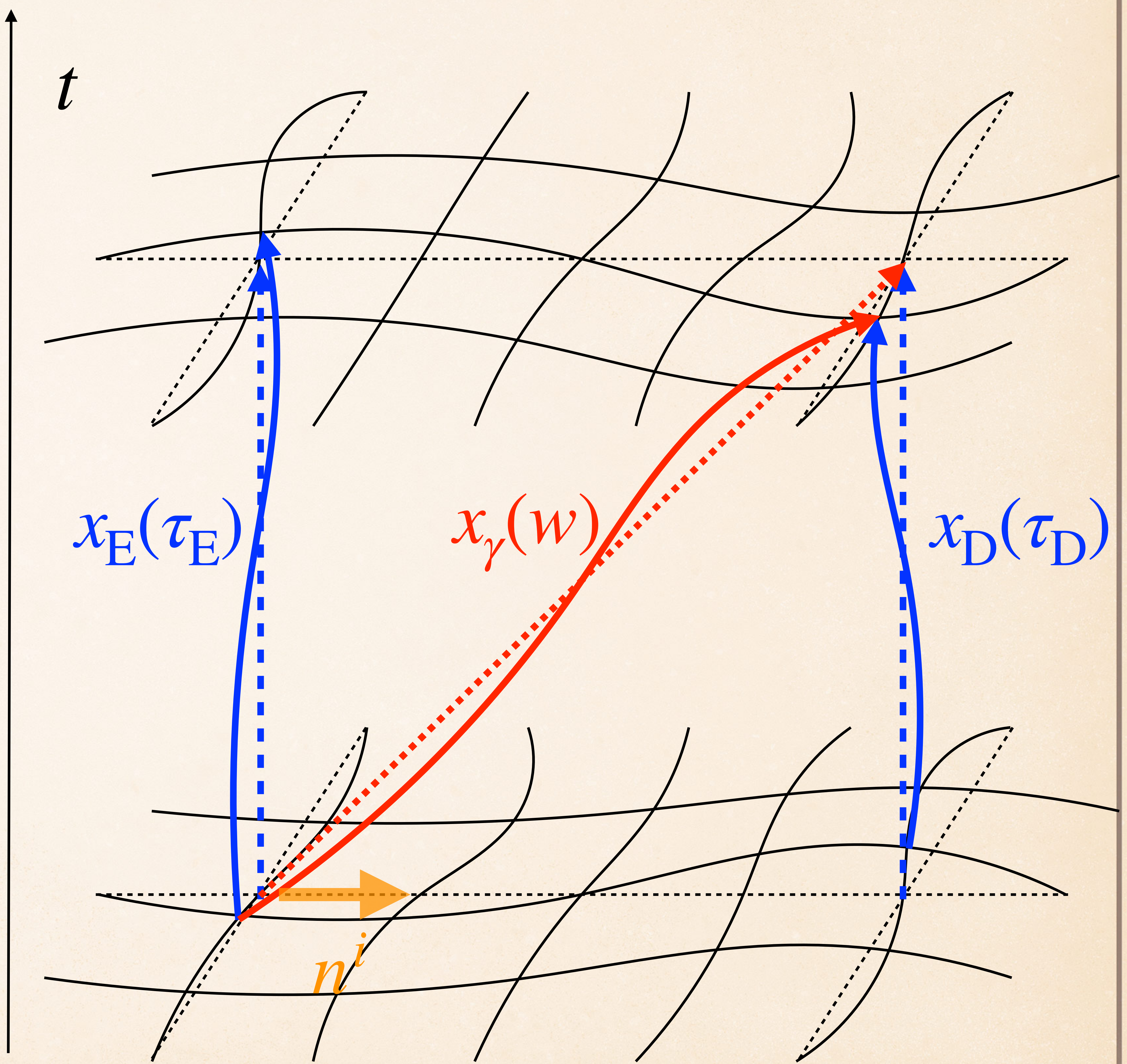
$$a\left(\tau_{\rm D}\right) x_{\rm D}^{(1)0}\left(\tau_{\rm D}\right) = a_{\rm i} x_{\rm Di}^{(1)0} - \int_0^{\tau_{\rm D}} {\rm d}\tilde{\tau}_{\rm D} \alpha^{(1)}\left(\tilde{\tau}_{\rm D}\right)$$

$$x_{\rm D}^{(1)i}\left(\tau_{\rm D}\right) = x_{\rm Di}^{(1)i} - E_{,i}^{(1)}\Big|_0^{\tau_{\rm D}}$$

$$+\int_0^{\tau_{\rm D}} \frac{{\rm d}\tilde{\tau}_{\rm D}}{a^2\left(\tilde{\tau}_{\rm D}\right)}\left(a_{\rm i} V_{{\rm Di},i}^{(1)} + a\left(\tilde{\tau}_{\rm D}\right) V_{{\rm ac},i}^{(1)}\left(\tilde{\tau}_{\rm D}\right)\right)$$

$$v_{{\rm D},i}^{(1)} \equiv p_{\rm D}^{(1)i} / p_{\rm D}^{(0)0} \qquad V_{\rm Di}^{(1)} = v_{\rm Di}^{(1)} + E_{\rm Di}^{(1)\prime}$$

$$a\left(\tilde{\tau}_{\rm D}\right) V_{{\rm ac},i}^{(1)} = -\int_0^{\tau_{\rm D}} {\rm d}\tilde{\tau}_{\rm D}\, \Phi^{(1)}(\tilde{\tau}_{\rm D})$$



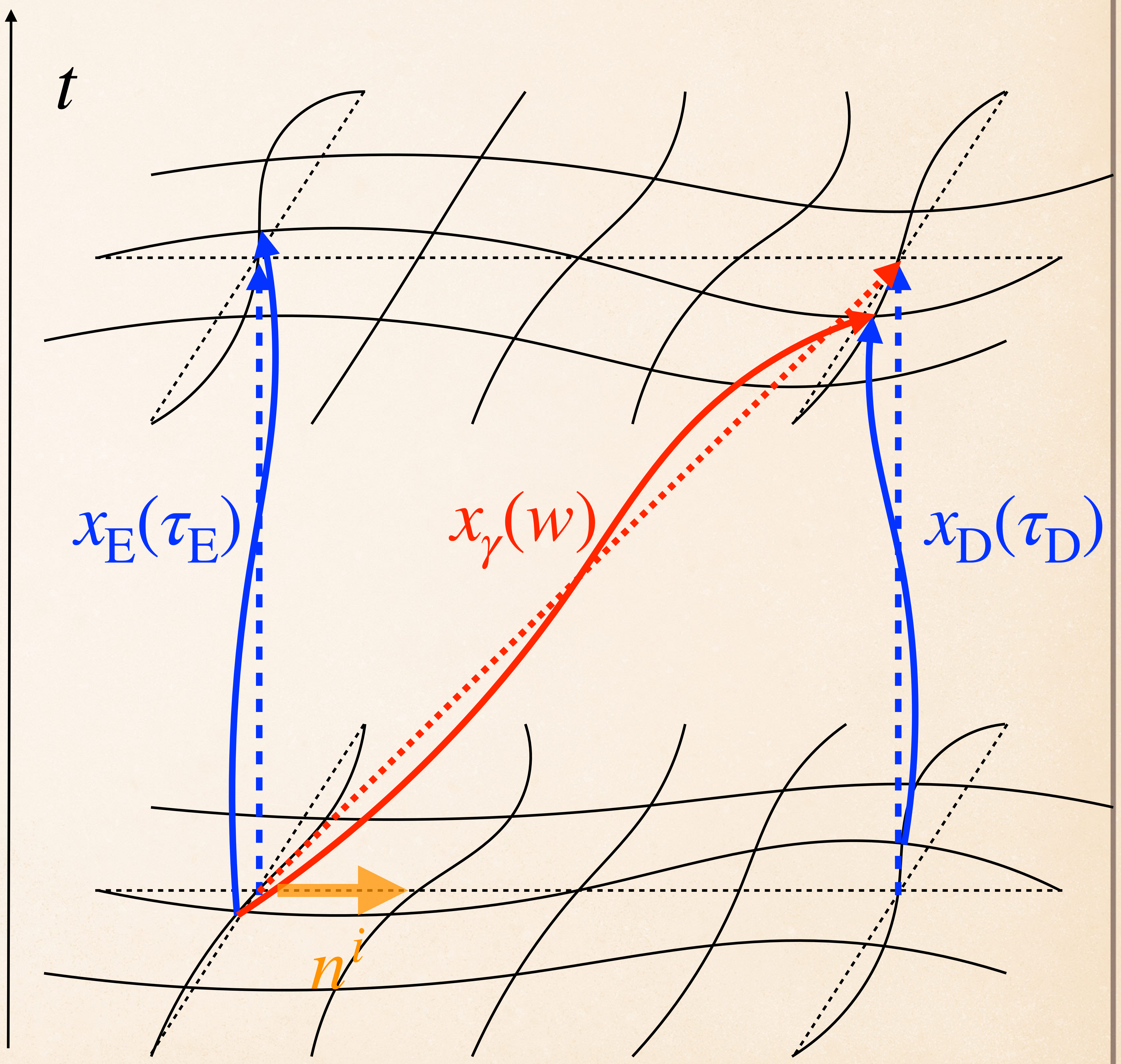
TIME DELAY

$$\tau^{(1)}_{\mathrm{D\,f}} = \frac{1}{p^{(0)}_{\mathrm{D\,f}}}\left(x^{(1)}_{\mathrm{D}}(\tau^{(0)}_{\mathrm{D\,f}}) - x^{(1)}_{\gamma}(w_{\mathrm{f}})\right)$$

$$= a_{\mathrm{f}}\left[\frac{1}{2}n^i n^j \int_0^L \mathrm{d}\lambda\, h^{(1)}_{ij}\right.$$

$$+n^i\left(X^{(1)i}_{\mathrm{D\,f}} - X^{(1)i}_{\mathrm{E\,i}}\right)$$

$$\left.-\int_0^L \mathrm{d}\lambda\,(\Phi^{(1)} + \Psi^{(1)}) + X^{(1)0}_{\mathrm{E\,i}} - X^{(1)0}_{\mathrm{D\,f}}\right]$$



TIME DELAY

$$\tau^{(1)}_{\mathrm{D\,f}} = \frac{1}{p^{(0)}_{\mathrm{D\,f}}}\left(x^{(1)}_{\mathrm{D}}(\tau^{(0)}_{\mathrm{D\,f}}) - x^{(1)}_{\gamma}(w_{\mathrm{f}})\right)$$

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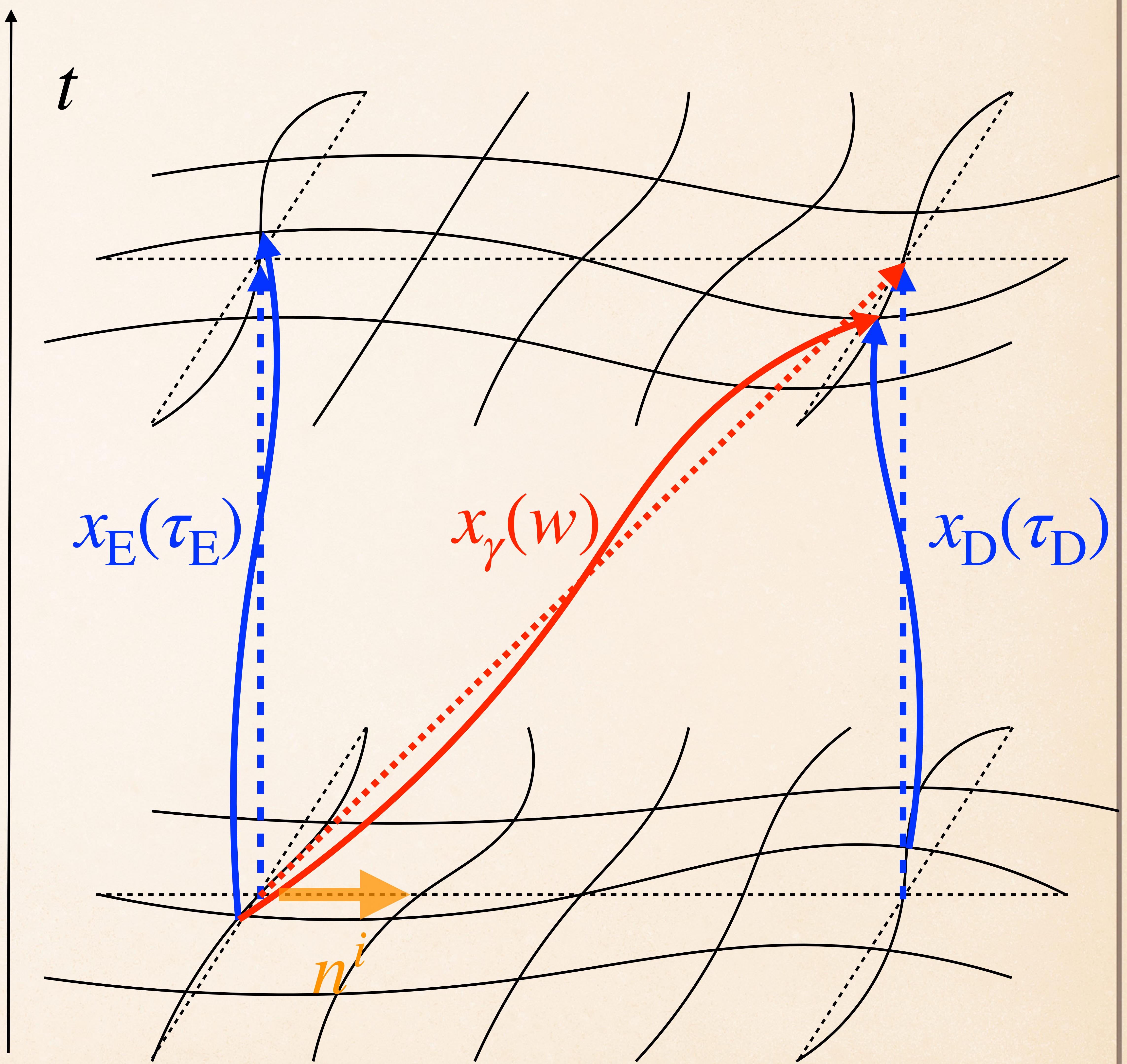
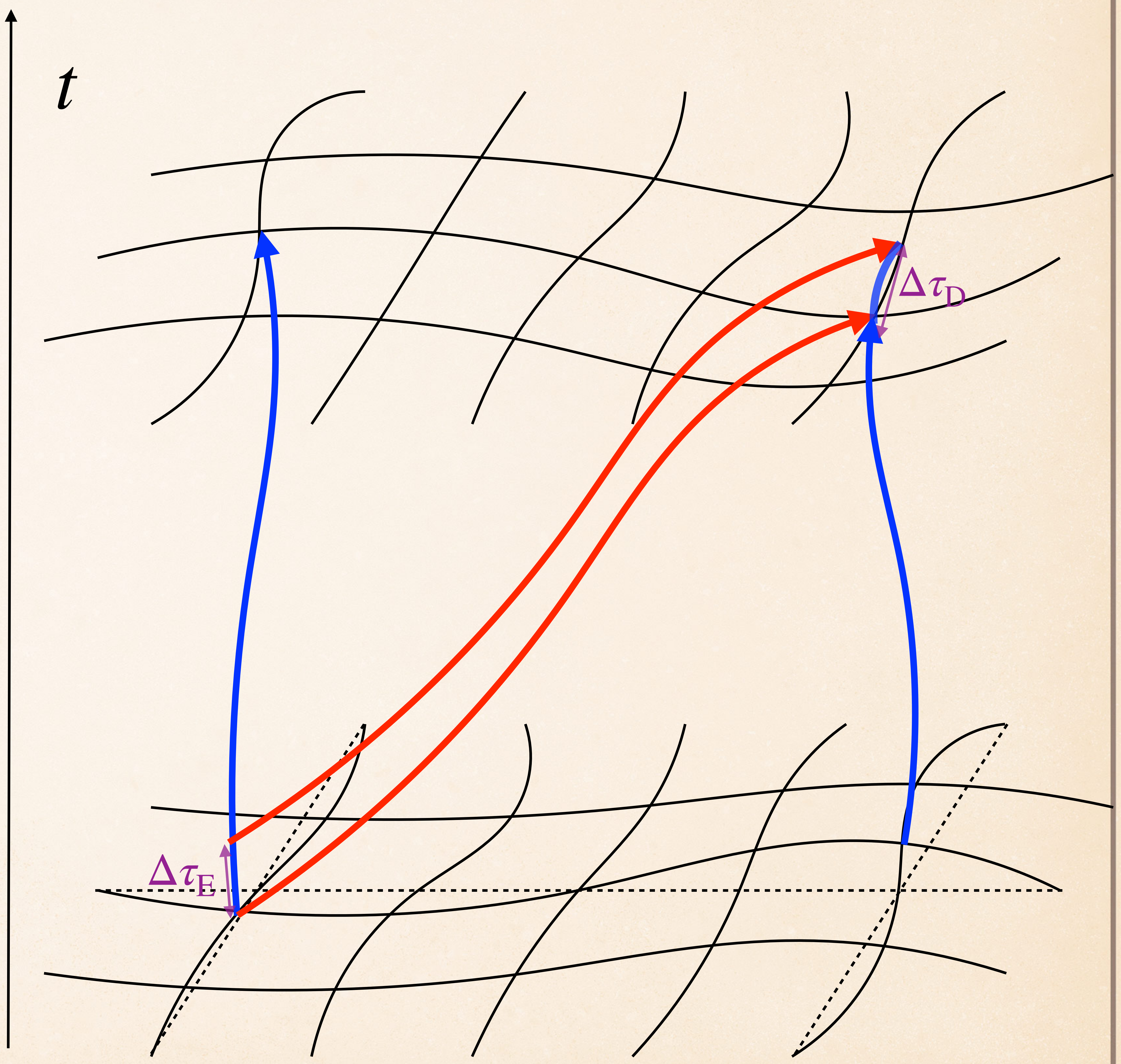
$$+n^i\left(X^{(1)i}_{\mathrm{D\,f}} - X^{(1)i}_{\mathrm{E\,i}}\right)$$

$$\left. -\underbrace{\boxed{\int_0^L \mathrm{d}\lambda\,(\Phi^{(1)} + \Psi^{(1)})}}_{\textcolor{blue}{\text{Shapiro}}} + X^{(1)0}_{\mathrm{E\,i}} - X^{(1)0}_{\mathrm{D\,f}}\right]$$



GRAVITATIONAL REDSHIFT

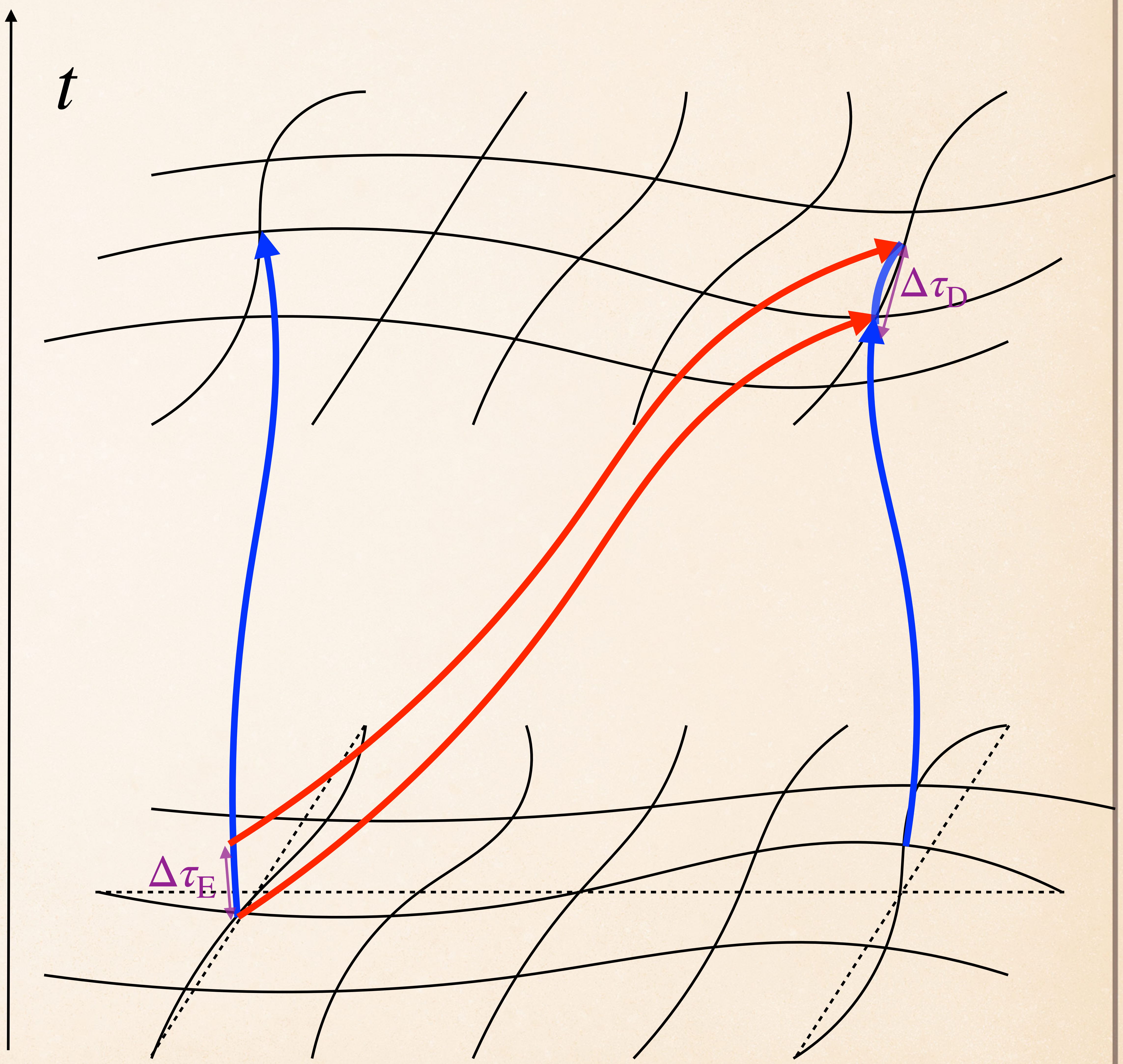
Redshift $$1 + z = \frac{\Delta\tau_D}{\Delta\tau_E}$$



GRAVITATIONAL REDSHIFT

Redshift $$1 + z = \frac{\Delta\tau_D}{\Delta\tau_E}$$

$f\,\Delta\tau_E \ll 1$

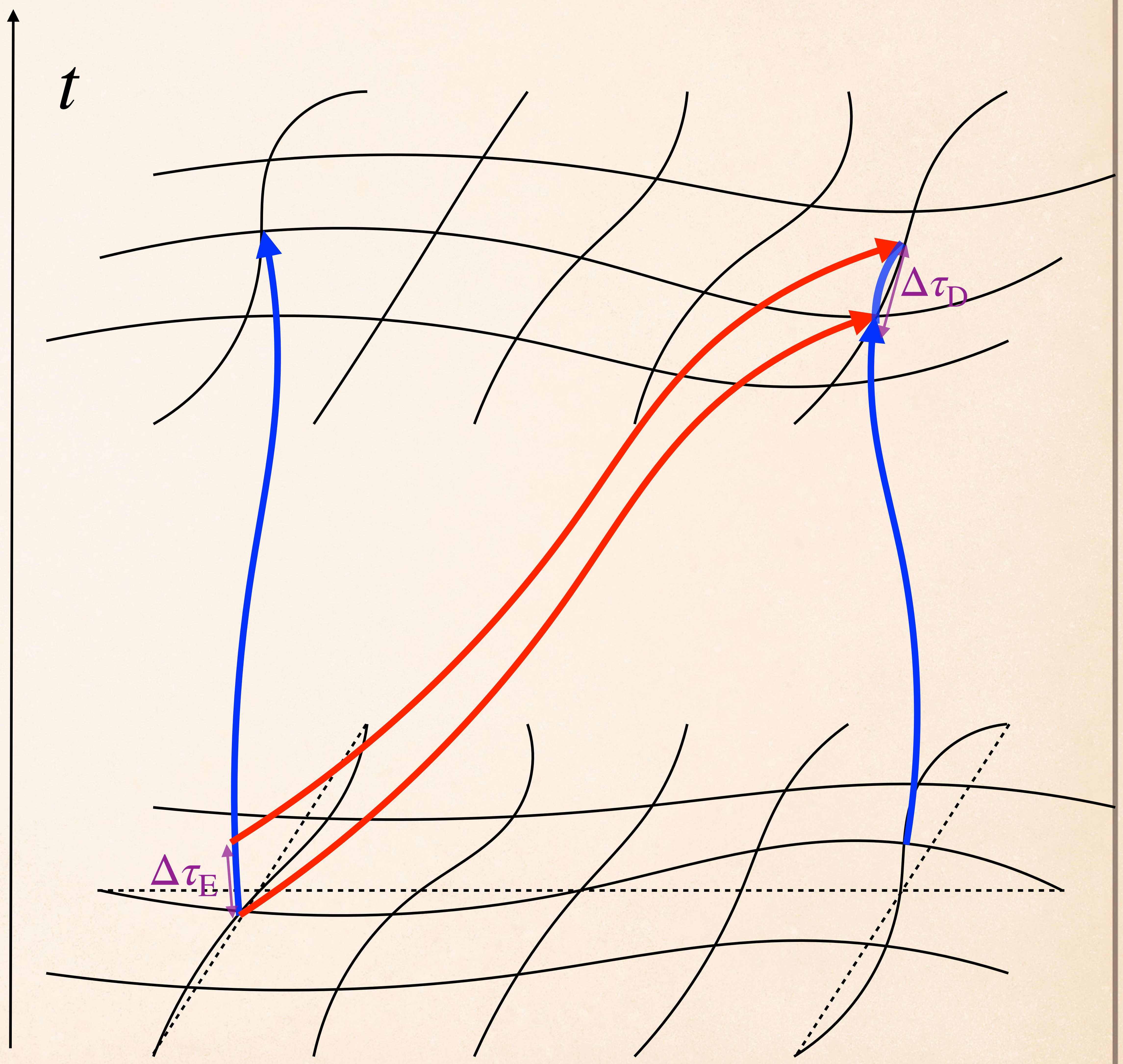


GRAVITATIONAL REDSHIFT

Redshift $$1 + z = \frac{\Delta\tau_{\mathrm{D}}}{\Delta\tau_{\mathrm{E}}}$$

$f\,\Delta\tau_{\mathrm{E}} \ll 1$

$$\begin{aligned} z^{(1)} = \frac{a_{\mathrm{f}}}{a_{\mathrm{i}}} \Bigg(& n^i n^j \int_0^L \mathrm{d}\lambda \left(h_{ij}^{(1)}\right)' \\ & + n^i \left(V_{\mathrm{D},i}^{(1)}\left(\tau_{\mathrm{Df}}\right) - V_{\mathrm{E},i}^{(1)}\left(\tau_{\mathrm{Ei}}\right) \right) \\ & + \Phi_{\mathrm{f}}^{(1)} - \Phi_{\mathrm{i}}^{(1)} - \int_0^L \mathrm{d}\lambda \left(\Phi^{(1)} + \Psi^{(1)} \right)' \\ & + \left(\frac{a'}{a} X_{\mathrm{D}}^{(1)0} \right)\left(\tau_{\mathrm{Df}}\right) - \left(\frac{a'}{a} X_{\mathrm{E}}^{(1)0} \right)\left(\tau_{\mathrm{Ei}}\right) \Bigg) . \end{aligned}$$



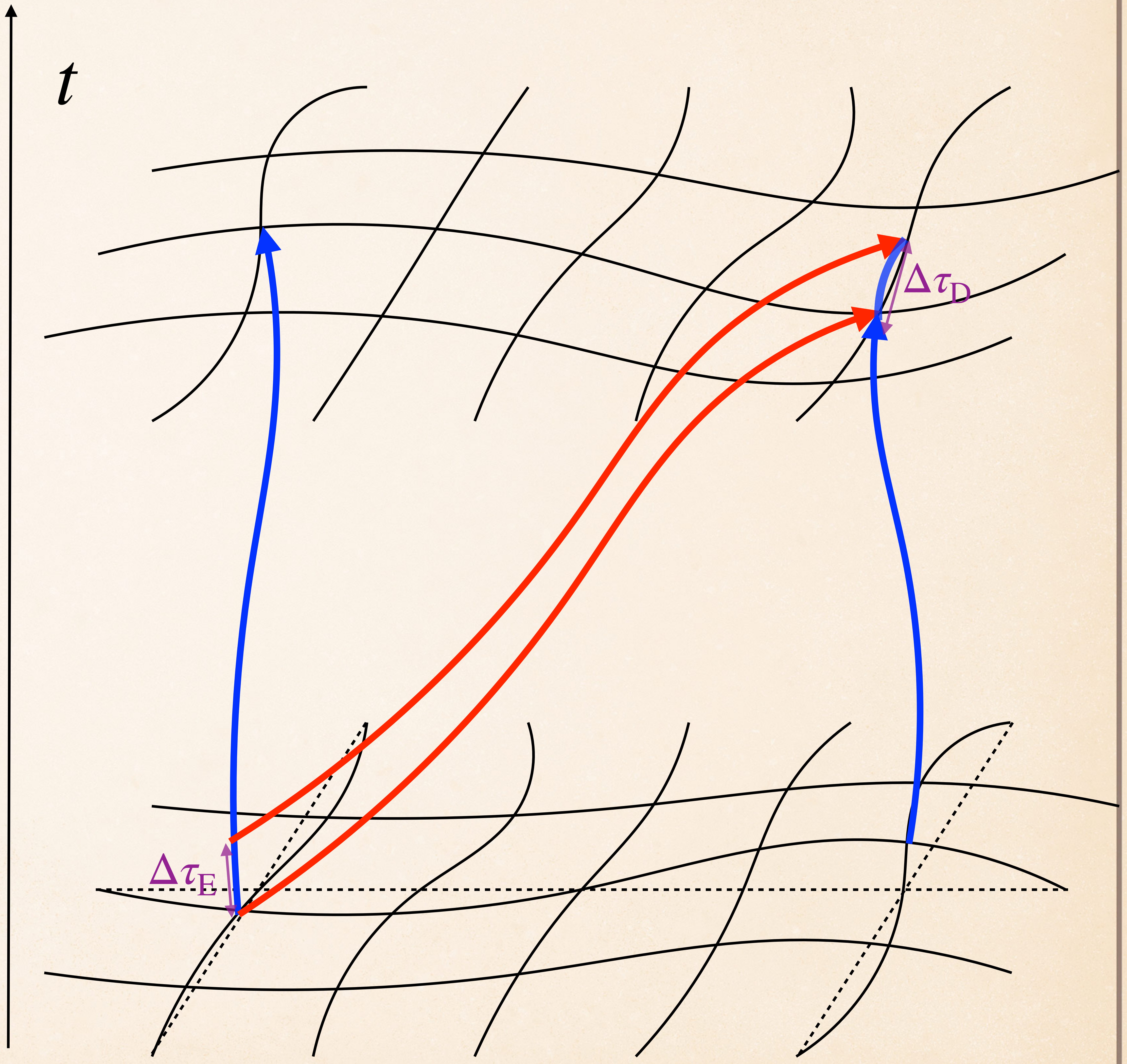
GRAVITATIONAL REDSHIFT

Redshift $$1+z=\frac{\Delta\tau_{\mathrm{D}}}{\Delta\tau_{\mathrm{E}}}$$

$$f\,\Delta\tau_{\mathrm{E}} \ll 1$$

$$\begin{aligned}
z^{(1)} = \frac{a_{\mathrm{f}}}{a_{\mathrm{i}}}\Bigg(& n^i n^j \int_0^L \mathrm{d}\lambda \left(h_{ij}^{(1)}\right)' \\
& + n^i \left(V_{\mathrm{D},i}^{(1)}\left(\tau_{\mathrm{Df}}\right) - V_{\mathrm{E},i}^{(1)}\left(\tau_{\mathrm{Ei}}\right)\right) \\
& + \Phi_{\mathrm{f}}^{(1)} - \Phi_{\mathrm{i}}^{(1)} - \int_0^L \mathrm{d}\lambda \left(\Phi^{(1)} + \Psi^{(1)}\right)' \\
& + \left(\frac{a'}{a} X_{\mathrm{D}}^{(1)0}\right)\left(\tau_{\mathrm{Df}}\right) - \left(\frac{a'}{a} X_{\mathrm{E}}^{(1)0}\right)\left(\tau_{\mathrm{Ei}}\right)\Bigg).
\end{aligned}$$

$$1+z_{\cos}=\frac{a(x_{\mathrm{f}})}{a(x_{\mathrm{i}})}$$



HELLINGS-DOWNS CURVE

$$z_{\mathrm{GW}}^{(1)} = \frac{1}{2}\frac{a_{\mathrm{f}}}{a_{\mathrm{i}}} n^i n^j \int_0^L \, \mathrm{d}\lambda \left(h_{ij}^{(1)} \right)'$$

HELLINGS-DOWNS CURVE

$$z_{\mathrm{GW}}^{(1)} = \frac{1}{2}\frac{a_{\mathrm{f}}}{a_{\mathrm{i}}} n^i n^j \int_0^L \, \mathrm{d}\lambda \left(h_{ij}^{(1)} \right)'$$

$$h_{ij}^{(1)} = h(t - x^3) e_{ij}^{+/\times}$$

$$z_{\mathrm{GW}}^{(1)} = \frac{1}{2} n^i n^j e_{ij}^{+/\times} \left(h(t - L\, n^3) - h(t) \right) .$$

HELLINGS-DOWNS CURVE

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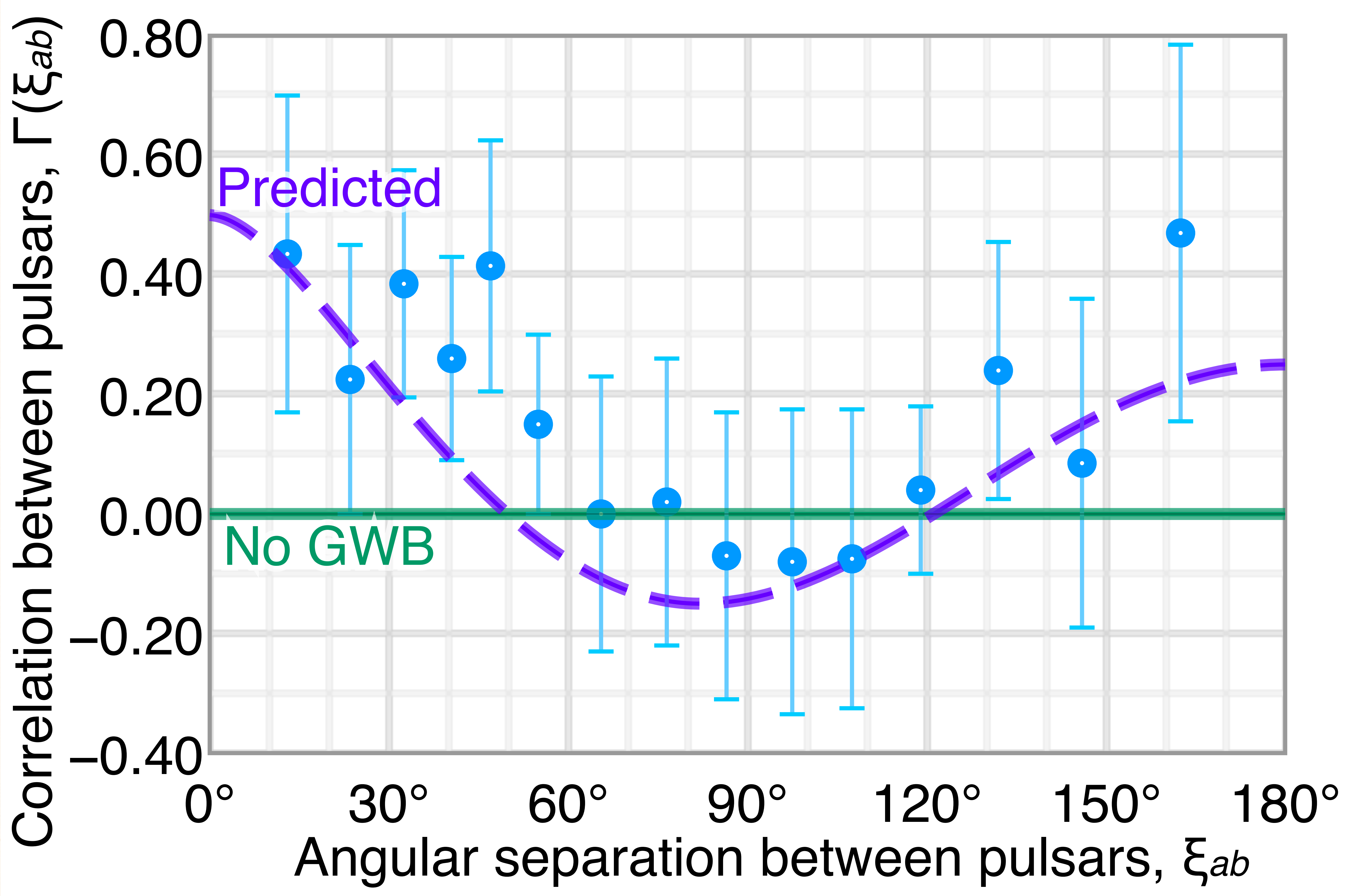
$$z_{\mathrm{GW}}^{(1)} = \frac{1}{2} n^i n^j e_{ij}^{+/\times} \left(h(t - L\,n^3) - h(t)\right).$$

$$\left\langle z_a\, z_b \right\rangle = \Gamma_{ab} \left\langle h^2 \right\rangle$$

$$\Gamma_{ab} = \frac{1}{4} \int (n_a^i n_a^j e_{ij}^{+/\times}(\Omega))\,(n_b^i n_b^j e_{ij}^{+/\times}(\Omega)) \mathrm{d}\Omega$$



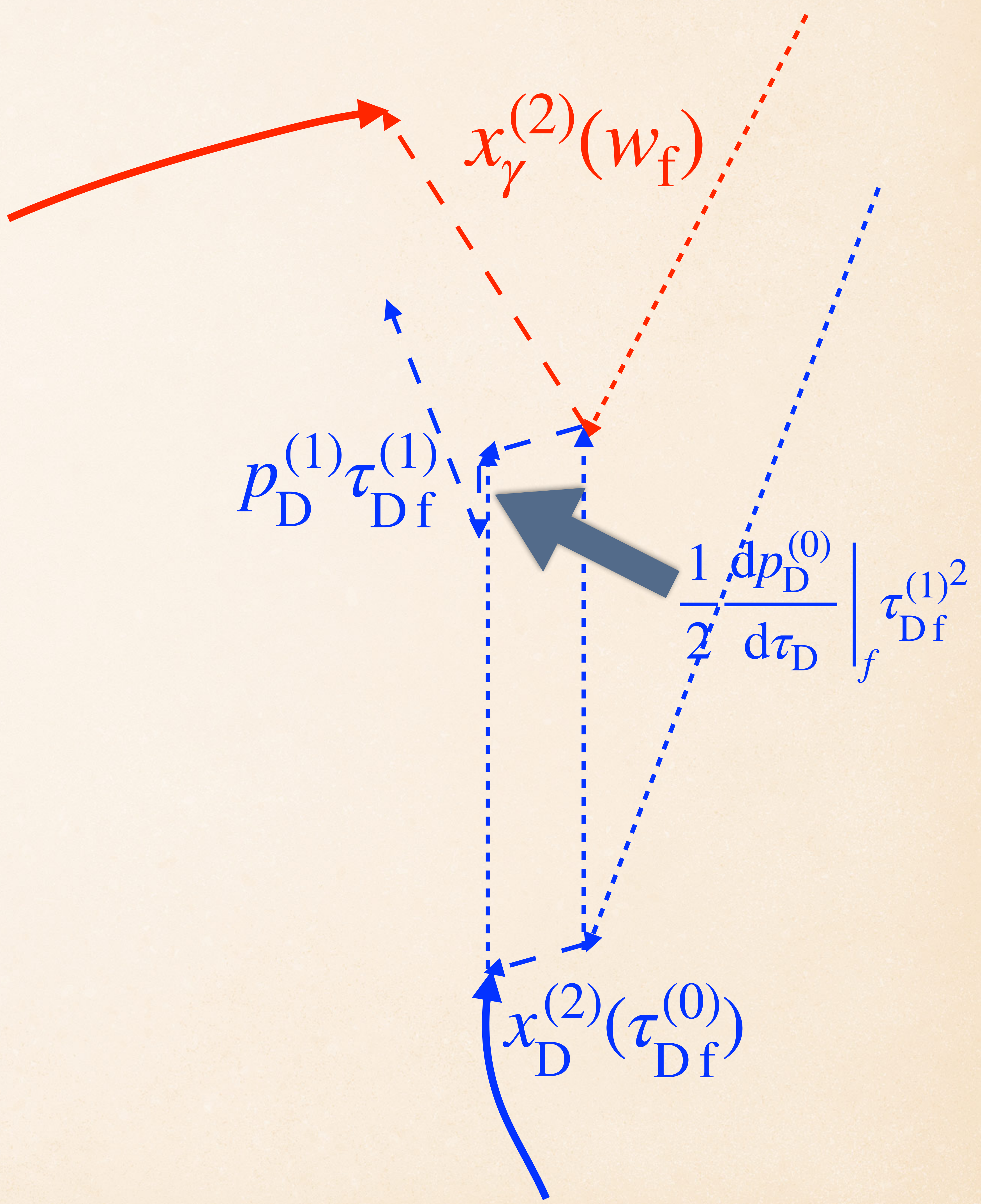
Credit: NANOGrav

SECOND-ORDER

GW EFFECT (II)

$\tau_{\mathrm{D}} = \tau_{\mathrm{D}}^{(0)} + \tau_{\mathrm{D}}^{(1)}$

		1-st	2-nd
$x_{\mathrm{D}}^{(0)}$	$x_{\mathrm{D}}^{(0)}(\tau_{\mathrm{D\,f}}^{(0)})$	$+\, p_{\mathrm{D\,f}}^{(0)}\tau_{\mathrm{D\,f}}^{(1)}$	$+\, \frac{1}{2}\frac{\mathrm{d}p_{\mathrm{D}}^{(0)}}{\mathrm{d}\tau_{\mathrm{D}}}\Big\vert_{f}\tau_{\mathrm{D\,f}}^{(1)2}$
$x_{\mathrm{D}}^{(1)}$		$x_{\mathrm{D}}^{(1)}(\tau_{\mathrm{D\,f}}^{(0)})$	$+\, p_{\mathrm{D}}^{(1)}\tau_{\mathrm{D\,f}}^{(1)}$
$x_{\mathrm{D}}^{(2)}$			$x_{\mathrm{D}}^{(2)}(\tau_{\mathrm{D\,f}}^{(0)})$
			$\neq$
	$x_{\gamma}^{(0)}(w_{\mathrm{f}})$	$x_{\gamma}^{(1)}(w_{\mathrm{f}})$	$x_{\gamma}^{(2)}(w_{\mathrm{f}})$



GW EFFECT (II)

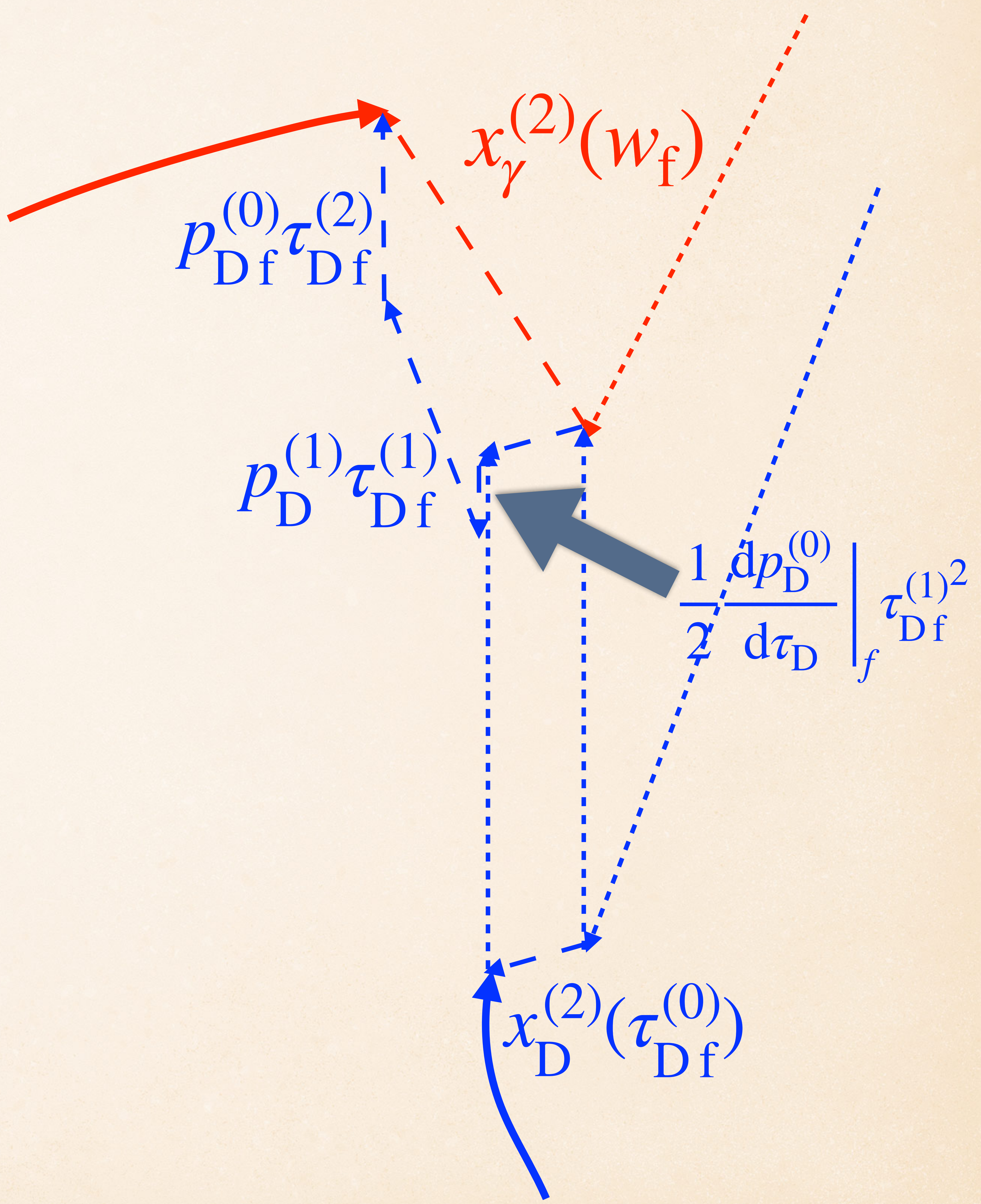
$$\tau_{\mathrm{D}} = \tau_{\mathrm{D}}^{(0)} + \tau_{\mathrm{D}}^{(1)}$$

	1-st	2-nd
$x_{\mathrm{D}}^{(0)}$	$x_{\mathrm{D}}^{(0)}(\tau_{\mathrm{D\,f}}^{(0)}) + p_{\mathrm{D\,f}}^{(0)}\tau_{\mathrm{D\,f}}^{(1)}$	$+ \dfrac{1}{2}\dfrac{\mathrm{d}p_{\mathrm{D}}^{(0)}}{\mathrm{d}\tau_{\mathrm{D}}}\Big\vert_f \tau_{\mathrm{D\,f}}^{(1)2}$
$x_{\mathrm{D}}^{(1)}$		$x_{\mathrm{D}}^{(1)}(\tau_{\mathrm{D\,f}}^{(0)}) + p_{\mathrm{D}}^{(1)}\tau_{\mathrm{D\,f}}^{(1)}$
$x_{\mathrm{D}}^{(2)}$		$x_{\mathrm{D}}^{(2)}(\tau_{\mathrm{D\,f}}^{(0)})$

$$\neq$$

$$x_\gamma^{(0)}(w_{\mathrm{f}}) \qquad x_\gamma^{(1)}(w_{\mathrm{f}}) \qquad x_\gamma^{(2)}(w_{\mathrm{f}})$$

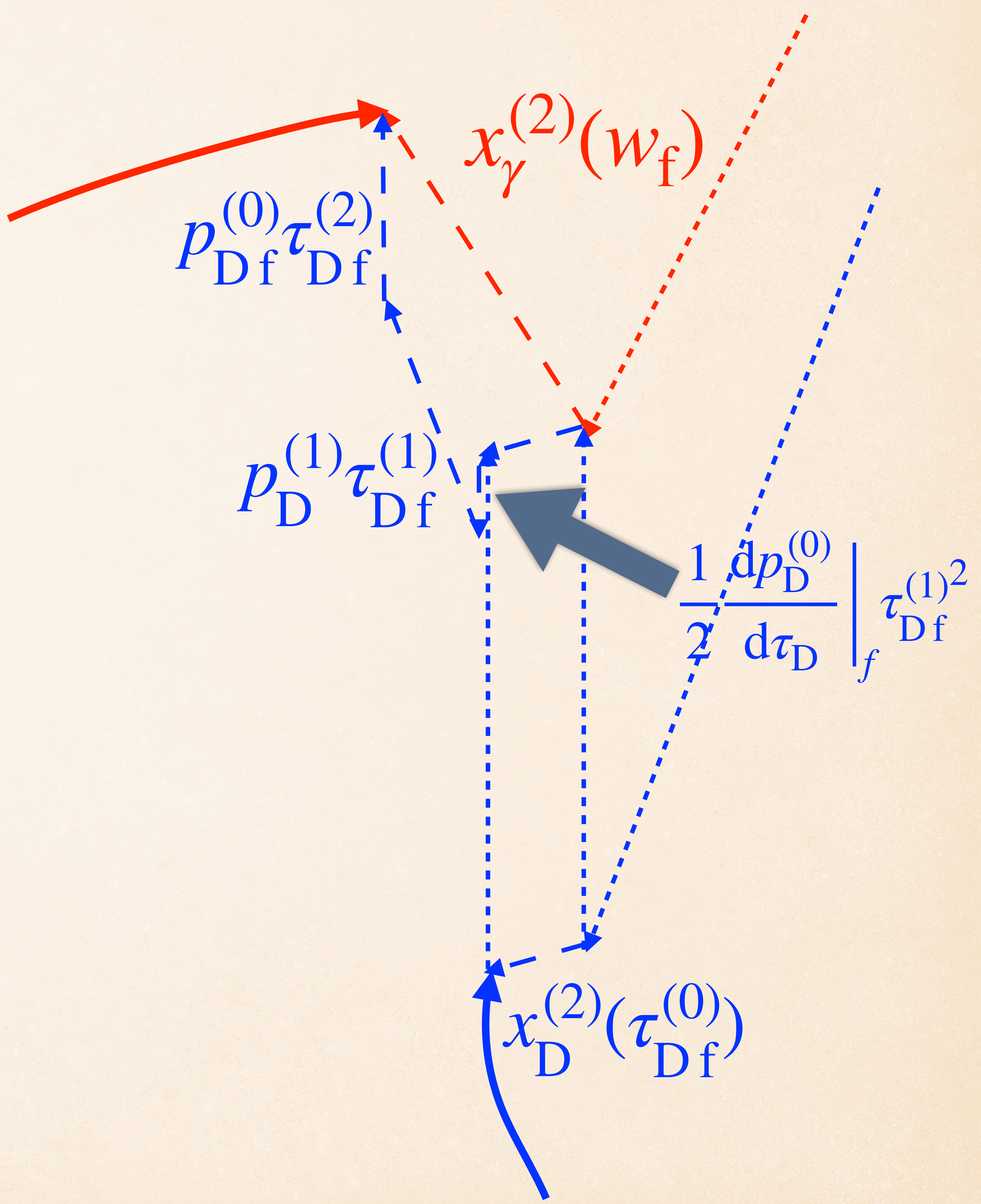
$$\tau_{\mathrm{D\,f}}^{(2)} = \frac{1}{p_{\mathrm{D\,f}}^{(0)}}\left(x_\gamma^{(2)}(w_{\mathrm{f}}) - x_{\mathrm{D}}^{(2)}(\tau_{\mathrm{D\,f}}^{(0)}) - p_{\mathrm{D\,f}}^{(0)}\tau_{\mathrm{D\,f}}^{(1)} - \frac{1}{2}\frac{\mathrm{d}p_{\mathrm{D}}^{(0)}}{\mathrm{d}\tau_{\mathrm{D}}}\Big\vert_f \tau_{\mathrm{D\,f}}^{(1)2}\right)$$



GW EFFECT (II)

$$\left(x_\gamma^{(1)0} - n^i x_\gamma^{(1)i}\right)\Big|_0^{w_\mathrm{f}} = -\frac{1}{2}\int_0^{w_f} \mathrm{d}w \left(g_{//}^{(2)} + 2k_\gamma^{(1)\mu} g_{\mu\nu}^{(1)} k_\gamma^{(0)\nu}\right.$$

$$\left. + a^2 k_\gamma^{(0)\mu}\left(g_{\mu\nu}^{(1)}/a^2\right)_{,\sigma} x_\gamma^{(1)\sigma} k_\gamma^{(0)\nu} + k_\gamma^{(1)\mu} g_{\mu\nu}^{(0)} k_\gamma^{(1)\nu}\right)$$



GW EFFECT (II)

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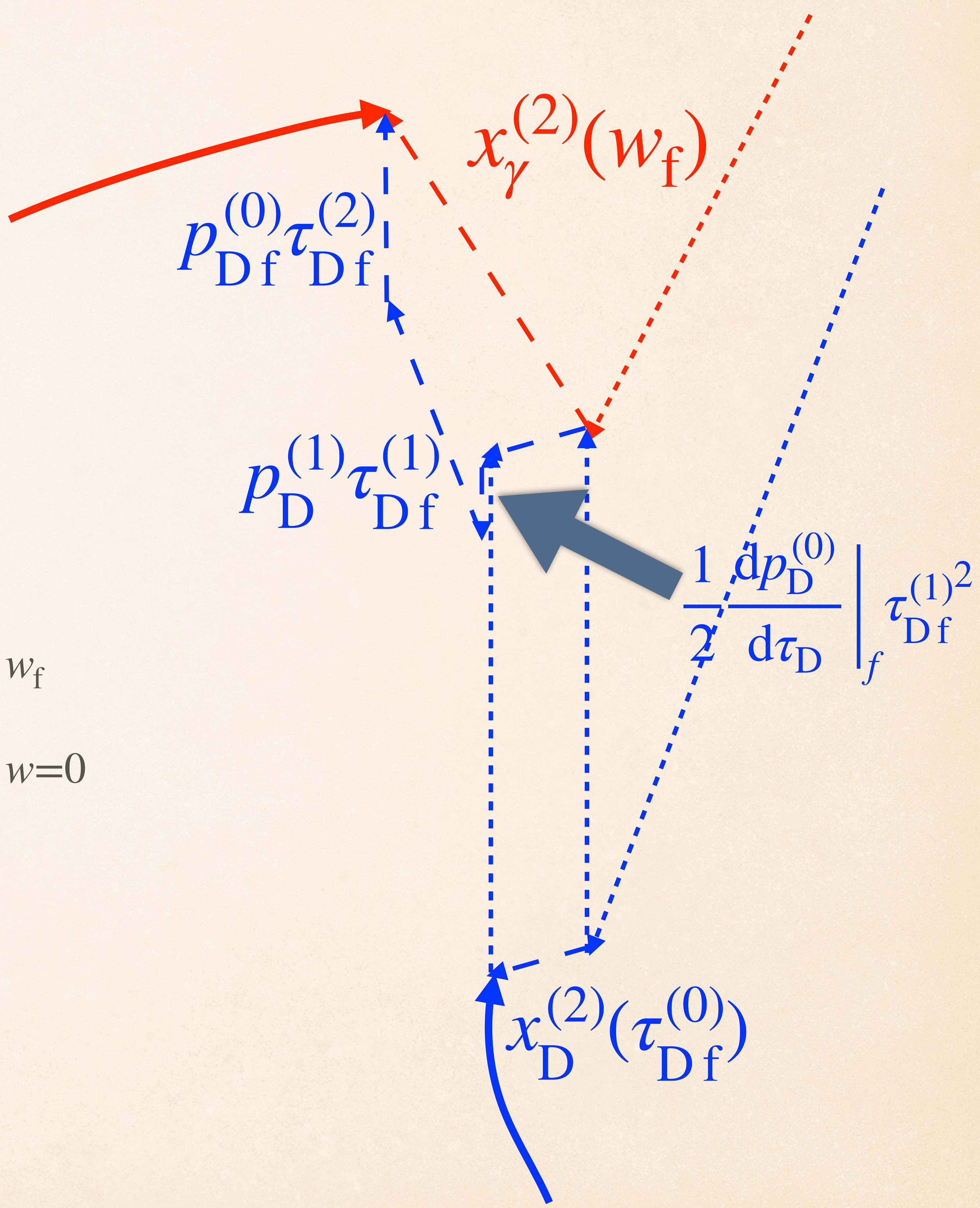
$$\supset n^i n^j \left[\frac{1}{2}\int_0^L \mathrm{d}\lambda\, h_{ij}^{N(2)} + E_{,i}^{(1)'}\left(x_{\gamma\mathrm{f}}^{(0)}\right)\Delta X_\gamma^{(1)j} + \frac{a_\mathrm{f}^4}{2w_\mathrm{f}^2} L \Delta X_\gamma^{(1)i}\Delta X_\gamma^{(1)j}\right]$$

$$h_{ij}^{N(2)} \equiv h_{ij}^{(2)} + \Lambda_{ij}{}^{lk}\left(\sigma_l^{(1)}\sigma_k^{(1)} + E_{,lm}^{(1)}E_{,mk}^{(1)}\right) \qquad \Delta X_\gamma^{(1)j} \equiv X_\gamma^{(1)j}\Big|_{w=0}^{w_\mathrm{f}}$$



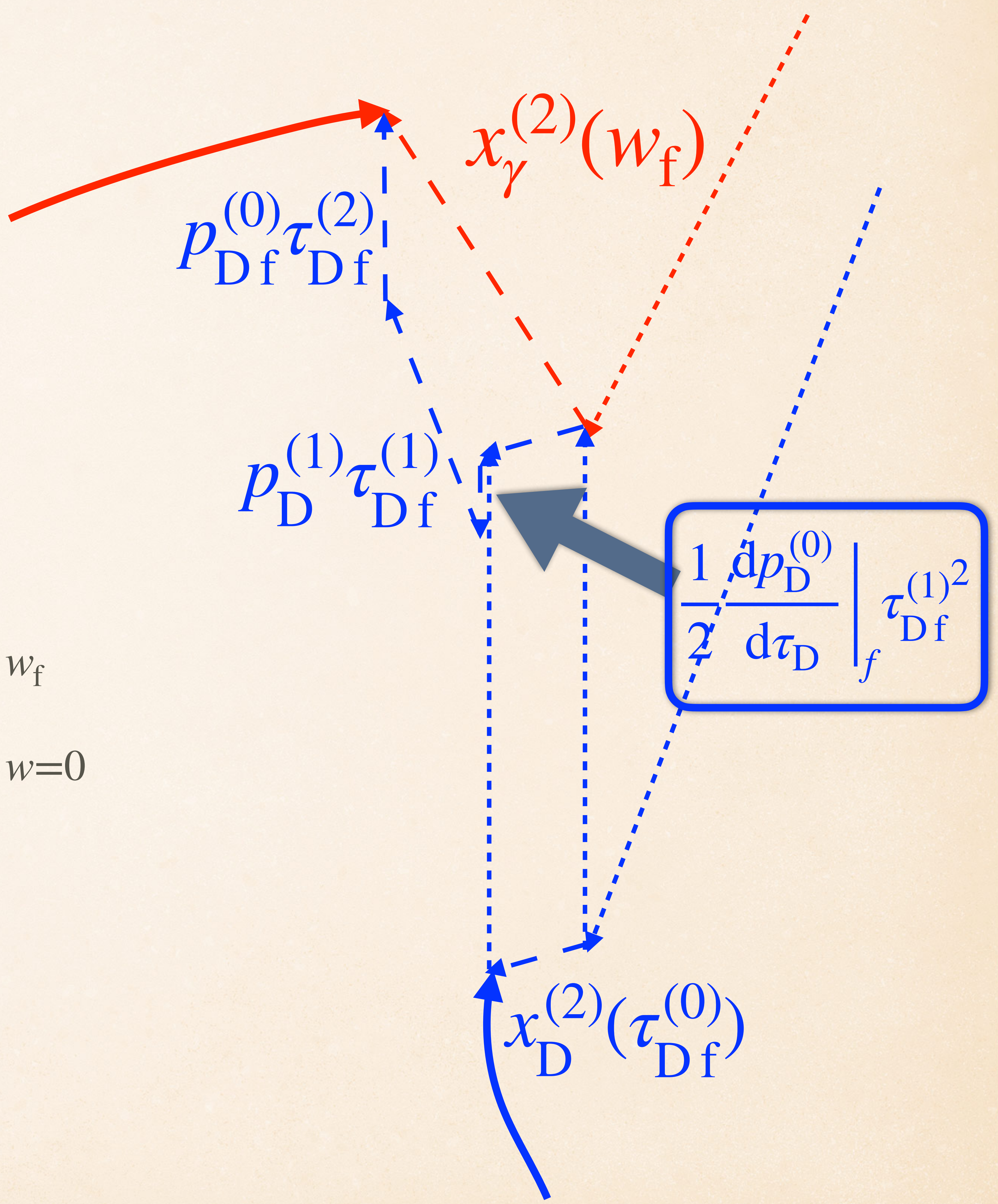
GW EFFECT (II)

$$\left(x_\gamma^{(1)0} - n^i x_\gamma^{(1)i}\right)\Big|_0^{w_\mathrm{f}} = -\frac{1}{2}\int_0^{w_f} \mathrm{d}w \left(g_{//}^{(2)} + 2k_\gamma^{(1)\mu} g_{\mu\nu}^{(1)} k_\gamma^{(0)\nu}\right.$$

$$\left. + a^2 k_\gamma^{(0)\mu}\left(g_{\mu\nu}^{(1)}/a^2\right)_{,\sigma} x_\gamma^{(1)\sigma} k_\gamma^{(0)\nu} + k_\gamma^{(1)\mu} g_{\mu\nu}^{(0)} k_\gamma^{(1)\nu}\right)$$

$$\supset n^i n^j \left[\frac{1}{2}\int_0^L \mathrm{d}\lambda\, h_{ij}^{N(2)} + E_{,i}^{(1)'}\left(x_{\gamma\mathrm{f}}^{(0)}\right)\Delta X_\gamma^{(1)j} + \frac{a_\mathrm{f}^4}{2w_\mathrm{f}^2} L \Delta X_\gamma^{(1)i}\Delta X_\gamma^{(1)j}\right]$$

$$h_{ij}^{N(2)} \equiv h_{ij}^{(2)} + \Lambda_{ij}{}^{lk}\left(\sigma_l^{(1)}\sigma_k^{(1)} + E_{,lm}^{(1)}E_{,mk}^{(1)}\right) \qquad \Delta X_\gamma^{(1)j} \equiv X_\gamma^{(1)j}\Big|_{w=0}^{w_\mathrm{f}}$$



GW EFFECT (II)

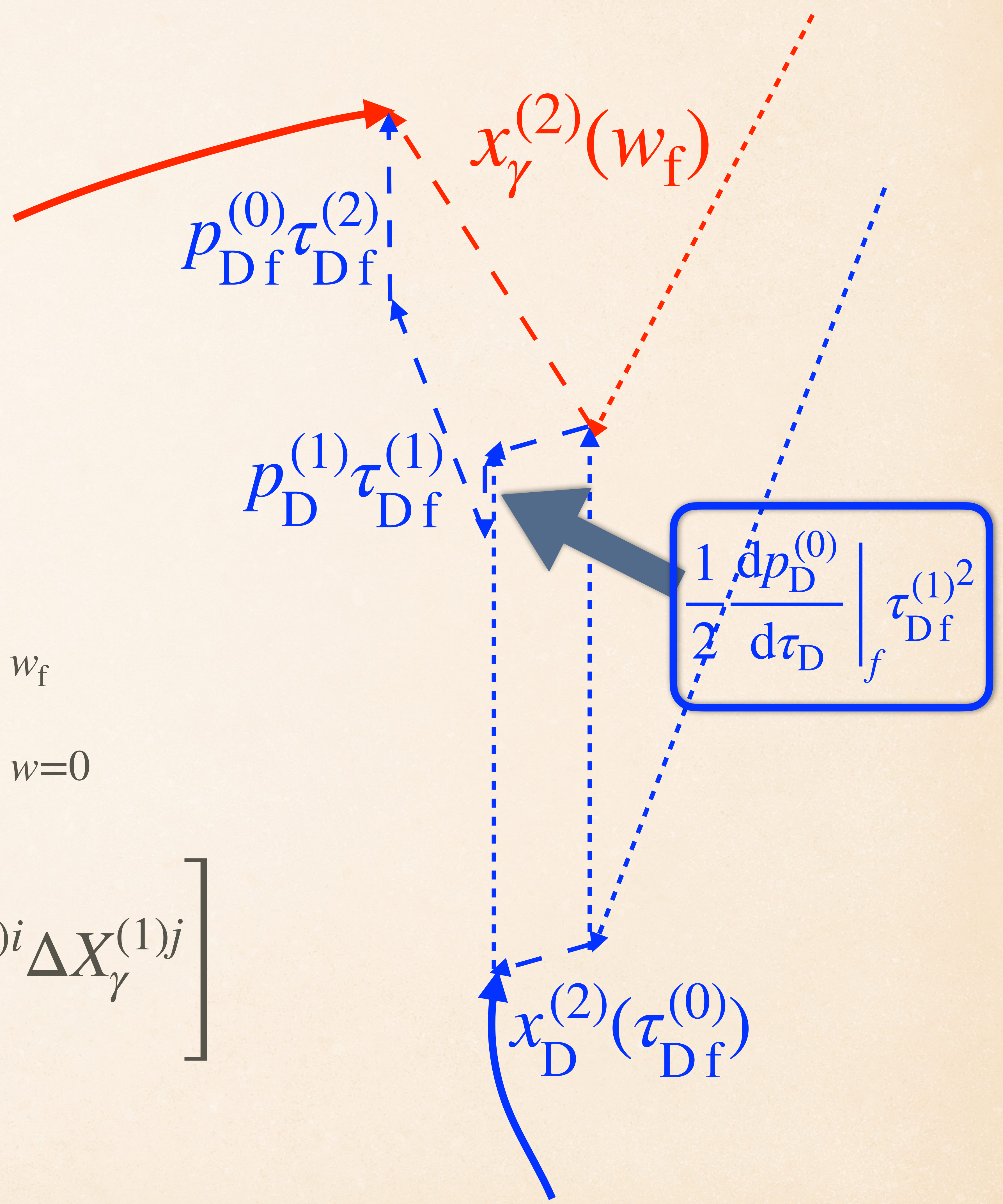
$$\left(x_\gamma^{(1)0} - n^i x_\gamma^{(1)i}\right)\Big|_0^{w_{\rm f}} = -\frac{1}{2}\int_0^{w_f} \mathrm{d}w \left(g_{//}^{(2)} + 2k_\gamma^{(1)\mu} g^{(1)}_{\mu\nu} k_\gamma^{(0)\nu} \right.$$

$$\left. + a^2 k_\gamma^{(0)\mu}\left(g^{(1)}_{\mu\nu}/a^2\right)_{,\sigma} x_\gamma^{(1)\sigma}k_\gamma^{(0)\nu} + k_\gamma^{(1)\mu} g^{(0)}_{\mu\nu} k_\gamma^{(1)\nu}\right)$$

$$\supset n^i n^j \left[\frac{1}{2}\int_0^L \mathrm{d}\lambda\, h_{ij}^{N(2)} + E^{(1)\prime}_{,i}\left(x^{(0)}_{\gamma \rm f}\right)\Delta X_\gamma^{(1)j} + \frac{a_{\rm f}^4}{2w_{\rm f}^2} L \Delta X_\gamma^{(1)i}\Delta X_\gamma^{(1)j}\right]$$

$$h_{ij}^{N(2)} \equiv h_{ij}^{(2)} + \Lambda_{ij}{}^{lk}\left(\sigma_l^{(1)}\sigma_k^{(1)} + E^{(1)}_{,lm}E^{(1)}_{,mk}\right) \qquad \Delta X_\gamma^{(1)j}\equiv X_\gamma^{(1)j}\Big|_{w=0}^{w_{\rm f}}$$

$$\tau^{(2)}_{\rm Df}\supset a_{\rm f} n^i n^j\left[\int_0^L \mathrm{d}\lambda\, h^{N(2)}_{ij} + V^{(1)}_{{\rm Df},i}\Delta X^{(1)}_\gamma + \left(\frac{a_{\rm f}'}{2a_{\rm f}^2} - \frac{a_{\rm f}^4}{2w_{\rm f}^2}L\right)\Delta X^{(1)i}_\gamma \Delta X^{(1)j}_\gamma\right]$$



GW EFFECT (II)

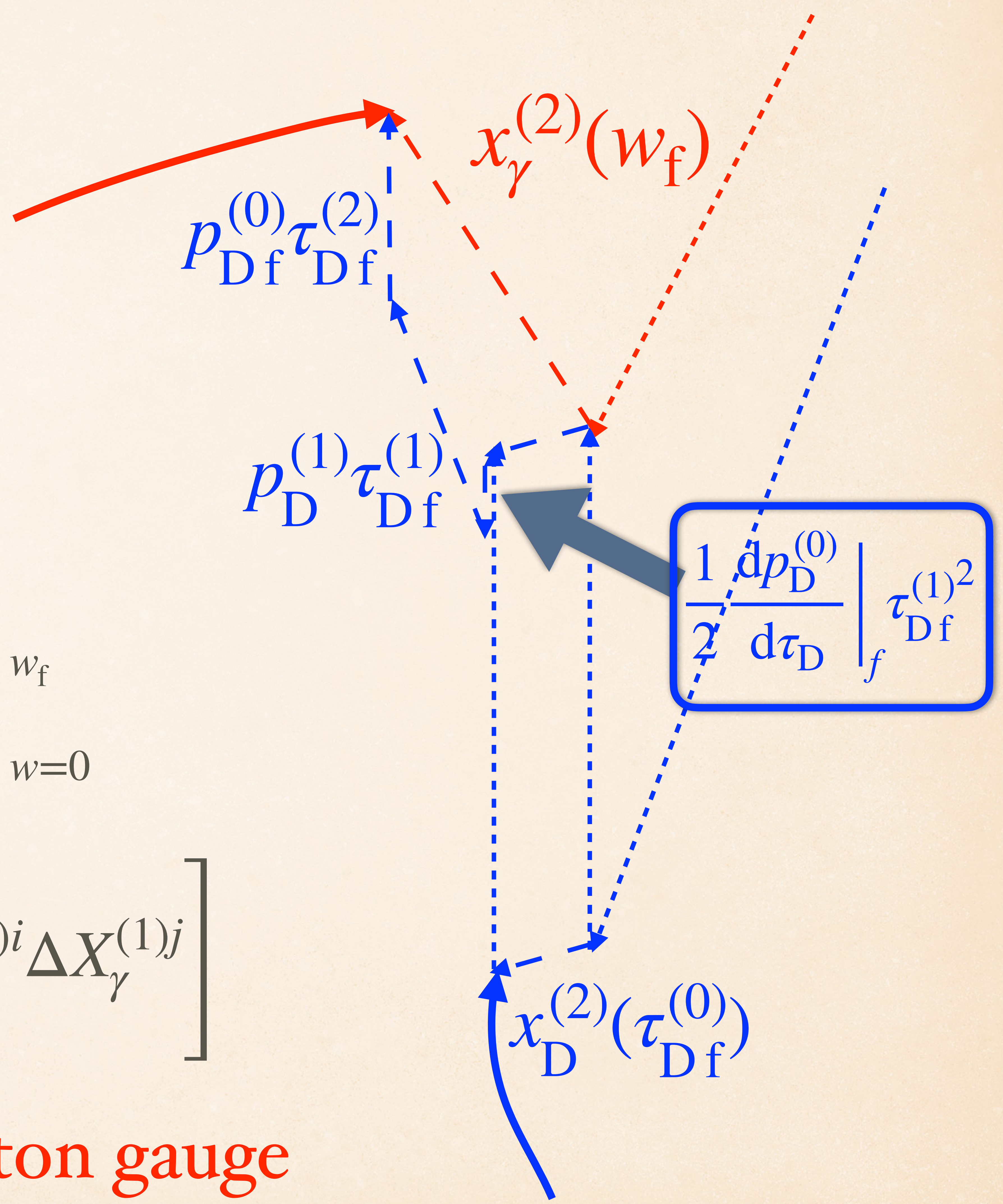
$$\left(x_\gamma^{(1)0} - n^i x_\gamma^{(1)i}\right)\Big|_0^{w_\mathrm{f}} = -\frac{1}{2}\int_0^{w_f} \mathrm{d}w \left(g_{//}^{(2)} + 2k_\gamma^{(1)\mu} g_{\mu\nu}^{(1)} k_\gamma^{(0)\nu}\right.$$

$$\left. + a^2 k_\gamma^{(0)\mu}\left(g^{(1)}_{\mu\nu}/a^2\right)_{,\sigma} x_\gamma^{(1)\sigma} k_\gamma^{(0)\nu} + k_\gamma^{(1)\mu} g^{(0)}_{\mu\nu} k_\gamma^{(1)\nu}\right)$$

$$\supset n^i n^j \left[\frac{1}{2}\int_0^L \mathrm{d}\lambda\, h_{ij}^{N(2)} + E_{,i}^{(1)'}\left(x^{(0)}_{\gamma\mathrm{f}}\right)\Delta X_\gamma^{(1)j} + \frac{a_\mathrm{f}^4}{2w_\mathrm{f}^2} L \Delta X_\gamma^{(1)i}\Delta X_\gamma^{(1)j}\right]$$

$$h^{N(2)}_{ij} \equiv h^{(2)}_{ij} + \Lambda_{ij}{}^{lk}\left(\sigma_l^{(1)}\sigma_k^{(1)} + E^{(1)}_{,lm}E^{(1)}_{,mk}\right) \qquad \Delta X_\gamma^{(1)j} \equiv X_\gamma^{(1)j}\Big|_{w=0}^{w_\mathrm{f}}$$

$$\tau^{(2)}_{\mathrm{Df}} \supset a_\mathrm{f} n^i n^j \left[\int_0^L \mathrm{d}\lambda\, h_{ij}^{N(2)} + V^{(1)}_{\mathrm{Df},i}\Delta X_\gamma^{(1)} + \left(\frac{a_\mathrm{f}'}{2a_\mathrm{f}^2} - \frac{a_\mathrm{f}^4}{2w_\mathrm{f}^2}L\right)\Delta X_\gamma^{(1)i}\Delta X_\gamma^{(1)j}\right]$$



The TT components of the spatial metric in the Newton gauge

SUMMARY

- The physical effect of GWs is to perturb the spacetime separation between events.
- The GW effect is inherently **quadrupolar**, and the Hellings–Downs curve is a direct consequence of this quadrupole nature.
- Up to the second order, the GW effect is captured by the TT components in the **Newton gauge**.

THANK YOU FOR ATTENTION

IGW IN A GENERAL UNIVERSE

For a universe dominated by a perfect fluid with EoS w

$$\Phi(k) = \begin{cases} \text{Const.} & c_s k \gg \mathscr{H} \\ a^{-2} & c_s k \ll \mathscr{H} \end{cases}$$

$$k = aH \propto a^{-\frac{1+3w}{2}}$$

$$h_k = \begin{cases} \text{Const.} & k \ll \mathscr{H} \\ a^{-1} & k \gg \mathscr{H} \end{cases}$$