EW SMEFT – Exercise sheet 1 – Solutions

Invariance under basis change

1. (a)

$$\mathcal{O}_B^\dagger = (-i)B_{\mu\nu}(D^\mu\Phi)(D^\nu\Phi^\dagger) = iB_{\mu\nu}(D^\nu\Phi^\dagger)(D^\mu\Phi) = \mathcal{O}_B \tag{1}$$

and the same for $\mathcal{O}_W$. If the i were not there, there would be an overall minus sign to the right hand side.

(b) **Step 1.**

$$\begin{aligned}
A_W &= -W^I_{\mu\nu}D^\mu\left(\Phi^\dagger i\overleftrightarrow{D}^{I\nu}\Phi\right) && (2)\\
&= -2iW^I_{\mu\nu}D^\mu\Phi^\dagger\sigma^I D^\nu\Phi + \frac{g}{2}W^I_{\mu\nu}W^{I\mu\nu}\Phi^\dagger\Phi + \frac{g'}{2}W^I_{\mu\nu}B^{\mu\nu}\Phi^\dagger\sigma^I\Phi && (3)\\
&= -2\mathcal{O}_W + \frac{g}{2}Q_{HW} + \frac{g}{2}Q_{HWB}\,, && (4)\\
A_B &= -B_{\mu\nu}D^\mu\left(i\Phi^\dagger\overleftrightarrow{D}^\nu\Phi\right) && (5)\\
&= -2iB_{\mu\nu}D^\mu\Phi^\dagger D^\nu\Phi + \frac{g'}{2}B_{\mu\nu}B^{\mu\nu}\Phi^\dagger\Phi + \frac{g}{2}W^I_{\mu\nu}B^{\mu\nu}\Phi^\dagger\sigma^I\Phi && (6)\\
&= -2\mathcal{O}_B + \frac{g'}{2}Q_{HB} + \frac{g}{2}Q_{HWB}. && (7)
\end{aligned}$$

Step 2.

$$\begin{aligned}
A_W &= g\left(2\Phi^\dagger\Phi\left(D_\mu\Phi^\dagger D^\mu\Phi\right) + \frac{Q_{H\Box}}{2} + \frac{1}{2}\left(Q^{(3)}_{Hl} + Q^{(3)}_{Hq}\right)\right), && (8)\\
&= g\left[\frac{3}{2}Q_{H\Box} - 2m_h^2(\Phi^\dagger\Phi)^2 + 2\lambda Q_H + 2\left(Q_{eH} + Q_{uH} + Q_{dH} + \text{h.c.}\right) + \frac{1}{2}\left(Q^{(3)}_{Hl} + Q^{(3)}_{Hq}\right)\right] && (9)\\
A_B &= g'\left(\frac{1}{6}Q^{(1)}_{Hq} + \frac{2}{3}Q_{Hu} - \frac{1}{3}Q_{Hd} - \frac{1}{2}Q^{(1)}_{Hl} - Q_{He} + \frac{Q_{H\Box}}{2} + 2Q_{HD}\right). && (10)
\end{aligned}$$

Step 3.

$$\mathcal{O}_W = \frac{1}{2}\left[\frac{g}{2}Q_{HW} + \frac{g'}{2}Q_{HWB} - \frac{3g}{2}Q_{H\Box} + 2gm^2\left(\Phi^\dagger\Phi\right)^2 - 2g\lambda Q_H \right.$$
$$\left. - g\left(Q_{eH} + Q_{uH} + Q_{dH} + \text{h.c.}\right) - g\left(\frac{1}{2}Q_{Hl}^{(3)} + \frac{1}{2}Q_{Hq}^{(3)}\right)\right], \qquad (11)$$

$$\mathcal{O}_B = \frac{1}{2}\left[\frac{g'}{2}Q_{HB} + \frac{g}{2}Q_{HWB} - \frac{g'}{2}Q_{H\Box} - 2g'Q_{HD} \right.$$
$$\left. - g'\left(\frac{1}{6}Q_{Hq}^{(1)} + \frac{2}{3}Q_{Hu} - \frac{1}{3}Q_{Hd} - \frac{1}{2}Q_{Hl}^{(1)} - Q_{He}\right)\right]. \qquad (12)$$

(c) Conversion of the Wilson coefficients:

$$\begin{aligned} \mathcal{O} &= RQ \\ f\mathcal{O} &= CQ = CR^{-1}\mathcal{O} \qquad\qquad \to f = CR^{-1}, \quad C = fR \end{aligned} \qquad (13)$$

where R is the basis rotation matrix between operators. Since we do not have the complete HISZ basis, we can only convert C into f and not vice versa.
In order to translate from Warsaw to $\mathcal{O}_W$ one has to map simultaneously

$$\begin{array}{llll} C_{HW} \to \frac{g}{4}f_W & C_{HWB} \to \frac{g'}{4}f_W & C_{H\Box} \to -\frac{3g}{4}f_W & C_H \to -g\lambda f_W \\ C_{Hq}^{(3)} \to -\frac{g}{4}f_W & C_{Hl}^{(3)} \to \frac{g}{4}f_W & & \\ C_{uH} \to -\frac{g}{2}f_W & C_{dH} \to -\frac{g}{2}f_W & C_{eH} \to -\frac{g}{2}f_W & \end{array} \qquad (14)$$

(and add the quartic contribution to the potential).
In order to translate from Warsaw to $\mathcal{O}_B$ one has to map simultaneously

$$\begin{array}{lllll} C_{HB} \to \frac{g'}{4}f_B & C_{HWB} \to \frac{g}{4}f_B & C_{H\Box} \to -\frac{g'}{4}f_B & C_{HD} \to -g'f_B & \\ C_{Hq}^{(1)} \to -\frac{g'}{12}f_B & C_{Hu} \to -\frac{g'}{3}f_B & C_{Hd} \to \frac{g'}{6}f_B & C_{Hl}^{(1)} \to \frac{g'}{4}f_B & C_{He} \to \frac{g'}{2}f_B \end{array} \qquad (15)$$

2. (a) The shifts appearing in the FR map to

$$\frac{\bar{C}_{HD}}{4t_\theta^2} + \frac{\bar{C}_{HWB}}{t_\theta} \to 0 \qquad (16)$$

$$\bar{C}_{HWB} \to \frac{g}{4}\bar{f}_B \qquad (17)$$

$$\bar{C}_{Hl}^{(1)} + \frac{\bar{C}_{HD}}{4}(3 - 2s_\theta^2) + s_{2\theta}\bar{C}_{HWB} \to 0 \qquad (18)$$

$$\bar{C}_{He} + \frac{\bar{C}_{HD}}{2}(2 - s_\theta^2) + s_{2\theta}\bar{C}_{HWB} \to 0 \qquad (19)$$

so, of all contributions, only the corrections to $WW\gamma$ and WWZ survive and they become

$$\gamma_\rho W^+_\mu W^-_\nu \qquad ie\left[\eta^{\mu\rho} p^\nu_\gamma - \eta^{\nu\rho} p^\mu_\gamma\right] \frac{g}{4t_\theta} \bar{f}_B$$

$$Z_\rho W^+_\mu W^-_\nu \qquad -ie\left[\eta^{\mu\rho} p^\nu_Z - \eta^{\nu\rho} p^\mu_Z\right] \frac{g}{4} \bar{f}_B$$

(b) the rules for the operator $\mathcal{O}_B$ are indeed identical to those derived at the previous point.

(c) The equivalence holds for the γ and Z diagrams individually because it happens separately for the $WW\gamma$ and WWZ vertices. It also holds independently for each chirality in initial state, because the fermionic current is unaffected by the operators in any case.

3. (a) There are two diagrams, one with a Z and one with a γ in s channel. Let's compute them separately. We will use $q = p_Z + k$ for the s channel momentum, p_Z for the momentum of the outgoing Z and k for the momentum of the outgoing Higgs. We also shorten

$$J^\rho_{uuZ,SM} = \bar{u}\gamma^\rho(g^u_L P_L + g^u_R P_R)v \qquad g^u_L = \frac{1}{2} - \frac{2}{3}s^2_\theta \qquad g^u_R = -\frac{2}{3}s^2_\theta \tag{20}$$

$$J^\rho_{uu,\gamma} = \frac{2}{3}\bar{u}\gamma^\rho v \tag{21}$$

$$\begin{aligned}
A^{\mathcal{O}_B}_Z &= -\frac{ig}{c_\theta} J^\rho_{uuZ,SM} \frac{-i}{q^2 - m^2_Z}\left(\eta^{\mu\rho} - \frac{q^\rho q^\mu}{m^2_Z}\right) \frac{ie}{vc_\theta} \bar{f}_B \left[-\eta^{\mu\nu}\frac{(q-p_Z)^2}{2} - q^\nu p^\mu_Z + \frac{1}{2}q^\mu q^\nu\right] \varepsilon^*_\nu(p_Z) \\
&= -\frac{ige}{vc^2_\theta} \bar{f}_B \frac{J^\rho_{uuZ,SM}}{q^2 - m^2_Z}\left[-\eta^{\rho\nu}\frac{k^2}{2} - q^\nu p^\rho_Z + \frac{1}{2}q^\nu q^\rho + \frac{q^\rho q^\nu}{m^2_Z}\frac{(q-p_Z)^2}{2} + \frac{q^\rho q^\nu}{m^2_Z} q\cdot p_Z - \frac{1}{2}q^\rho q^\nu \frac{q^2}{m^2_Z}\right] \varepsilon^*_\nu(p_Z) \\
&= -\frac{ige}{vc^2_\theta} \bar{f}_B \frac{J^\rho_{uuZ,SM}}{q^2 - m^2_Z}\left[-\frac{m^2_h}{2}\eta^{\rho\nu} - q^\nu p^\rho_Z + \frac{q^\rho q^\nu}{m^2_Z}\left(\frac{1}{2}m^2_Z + \frac{1}{2}q^2 + \frac{1}{2}m^2_Z - q\cdot p_Z + q\cdot p_Z - \frac{1}{2}q^2\right)\right] \varepsilon^*_\nu(p_Z) \\
&= -\frac{ige}{vc^2_\theta} \bar{f}_B \frac{J^\rho_{uuZ,SM}}{q^2 - m^2_Z}\left[-\frac{m^2_h}{2}\eta^{\rho\nu} + k^\rho q^\nu\right] \varepsilon^*_\nu(p_Z)
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
A^{\mathcal{O}_B}_\gamma &= -ieJ^\rho_{uu\gamma,SM} \frac{-i\eta^{\mu\rho}}{q^2} \frac{-ig}{2v} \bar{f}_B \left[-\eta^{\mu\nu} q\cdot k + q^\nu k^\mu\right] \varepsilon^*_\nu(p_Z) \\
&= i\frac{eg}{2v} \bar{f}_B \frac{J^\rho_{uu\gamma,SM}}{q^2}\left[-\eta^{\rho\nu}\frac{q^2 + m^2_h - m^2_Z}{2} + q^\nu k^\rho\right] \varepsilon^*_\nu(p_Z)
\end{aligned} \tag{23}$$

(b) The converted FR are

$$Z_\mu Z_\nu h \qquad \frac{ig^2 v}{2c^2_\theta}\eta^{\mu\nu}\left[1 - \frac{g'}{2}\bar{f}_B\right] + \frac{4is_\theta}{v}\left[p^\nu_{Z1} p^\mu_{Z2} - p_{Z1}\cdot p_{Z2}\eta^{\mu\nu}\right] \frac{g}{4c_\theta}\bar{f}_B$$

$$\gamma_\mu Z_\nu h \qquad -\frac{2i}{v}\left[p^\nu_\gamma p^\mu_Z - p_\gamma \cdot p_Z \eta^{\mu\nu}\right] \frac{g}{4}\bar{f}_B$$

$$\bar{u}uZ_\mu \qquad -i\frac{g}{c_\theta}\left(\frac{1}{2} - \frac{2s^2_\theta}{3}\right)(\gamma^\mu P_L) + i\frac{g}{c_\theta}\frac{2}{3}s^2_\theta(\gamma^\mu P_R)$$

$$\bar{u}uZ_\mu H \qquad -\frac{igs_\theta}{3vc^2_\theta}\bar{f}_B\left[\frac{1}{4}(\gamma^\mu P_L) + (\gamma^\mu P_R)\right]$$

(c)

$$
\begin{aligned}
A_Z^{\text{Warsaw}\to\text{f}_\text{B}} &= -\frac{ig}{c_\theta} J^\rho_{uuZ,SM} \frac{-i}{q^2-m_Z^2}\left(\eta^{\mu\rho} - \frac{q^\rho q^\mu}{m_Z^2}\right)\frac{ie}{vc_\theta}\bar{f}_B\left[-(m_Z^2 - p_Z\cdot q)\eta^{\mu\nu} - q^\nu p_Z^\mu\right]\varepsilon^*_\nu(p_Z) \\
&= -\frac{ige}{vc_\theta^2}\bar{f}_B\frac{J^\rho_{uuZ,SM}}{q^2-m_Z^2}\left[\eta^{\rho\nu}p_Z\cdot k - q^\nu p_Z^\rho - \frac{q^\rho q^\nu}{m_Z^2}(p_Z\cdot k) + \frac{q^\rho q^\nu}{m_Z^2}q\cdot p_Z\right]\varepsilon^*_\nu(p_Z) \\
&= -\frac{ige}{vc_\theta^2}\bar{f}_B\frac{J^\rho_{uuZ,SM}}{q^2-m_Z^2}\left[\eta^{\rho\nu}p_Z\cdot k - q^\nu p_Z^\rho + q^\rho q^\nu\right]\varepsilon^*_\nu(p_Z) \\
&= -\frac{ige}{vc_\theta^2}\bar{f}_B\frac{J^\rho_{uuZ,SM}}{q^2-m_Z^2}\left[\eta^{\rho\nu}\frac{q^2-m_Z^2-m_h^2}{2} + k^\rho q^\nu\right]\varepsilon^*_\nu(p_Z) \\
&= -\frac{ige}{vc_\theta^2}\bar{f}_B\left[\bar{u}\gamma^\rho(g_L^u P_L + g_R^u P_R)v\right]\left[\frac{1}{2}\eta^{\rho\nu}\right]\varepsilon^*_\nu(p_Z) + A_Z^{\mathcal{O}_B} \qquad (24)
\end{aligned}
$$

Note that in the last line, in subtracting $A_Z^{\mathcal{O}_B}$, we remained with a factor $(q^2 - m_Z^2)$ in the numerator that canceled out with the propagator dependence in the denominator. For the photon we have something similar:

$$
\begin{aligned}
A_\gamma^{\text{Warsaw}\to\text{f}_\text{B}} &= -ieJ^\rho_{uu\gamma,SM}\frac{-i\eta^{\mu\rho}}{q^2}\frac{-ig}{2v}\bar{f}_B\left[\eta^{\mu\nu}q\cdot p_Z - q^\nu p_Z^\mu\right]\varepsilon^*_\nu(p_Z) \\
&= i\frac{eg}{2v}\bar{f}_B\frac{J^\rho_{uu\gamma,SM}}{q^2}\left[\eta^{\nu\rho}\frac{q^2+m_Z^2-m_h^2}{2} - q^\nu p_Z^\rho\right]\varepsilon^*_\nu(p_Z) \qquad (25) \\
&= i\frac{eg}{2v}\bar{f}_B J^\rho_{uu\gamma,SM}\left[\eta^{\nu\rho} - \frac{q^\nu q^\rho}{q^2}\right] + A_\gamma^{\mathcal{O}_B} \qquad (26)
\end{aligned}
$$

In the last row, the term in $q^\nu q^\rho$ can be removed because $J^\rho_{uu\gamma}q_\rho = 0$.
Now the contact terms: for these we need to split LH and RH up quarks:

$$
A_{uuZH,L}^{\text{Warsaw}\to\text{f}_\text{B}} = -\frac{ieg}{12vc_\theta^2}\bar{f}_B(\bar{u}\gamma^\nu P_L v)\varepsilon^*_\nu(p_Z) \qquad (27)
$$

$$
A_{uuZH,R}^{\text{Warsaw}\to\text{f}_\text{B}} = -\frac{ieg}{3vc_\theta^2}\bar{f}_B(\bar{u}\gamma^\nu P_R v)\varepsilon^*_\nu(p_Z) \qquad (28)
$$

(d) Let's put everything together, for LH and RH up quarks separately:

$$
\begin{aligned}
&A_{Z,L}^{\text{Warsaw}\to\text{f}_\text{B}} + A_{\gamma,L}^{\text{Warsaw}\to\text{f}_\text{B}} + A_{uuZH,L}^{\text{Warsaw}\to\text{f}_\text{B}} \qquad (29) \\
&= A_{Z,L}^{\mathcal{O}_B} + A_{\gamma,L}^{\mathcal{O}_B} + \frac{ieg}{vc_\theta^2}\bar{f}_B\varepsilon^*_\nu(p_Z)(\bar{u}\gamma^\nu P_L v)\left[-g_L^u\frac{1}{2} + \frac{c_\theta^2}{2}\frac{2}{3} - \frac{1}{12}\right] \qquad (30) \\
&= A_{Z,L}^{\mathcal{O}_B} + A_{\gamma,L}^{\mathcal{O}_B} \qquad (31)
\end{aligned}
$$

and

$$
\begin{aligned}
&A_{Z,R}^{\text{Warsaw}\to\text{f}_\text{B}} + A_{\gamma,R}^{\text{Warsaw}\to\text{f}_\text{B}} + A_{uuZH,R}^{\text{Warsaw}\to\text{f}_\text{B}} \qquad (32) \\
&= A_{Z,R}^{\mathcal{O}_B} + A_{\gamma,R}^{\mathcal{O}_B} + \frac{ieg}{vc_\theta^2}\bar{f}_B\varepsilon^*_\nu(p_Z)(\bar{u}\gamma^\nu P_R v)\left[-g_R^u\frac{1}{2} + \frac{c_\theta^2}{2}\frac{2}{3} - \frac{1}{3}\right] \qquad (33) \\
&= A_{Z,L}^{\mathcal{O}_B} + A_{\gamma,L}^{\mathcal{O}_B} \qquad (34)
\end{aligned}
$$

as

$$-\frac{g_L^u}{2}+\frac{c_\theta^2}{3}-\frac{1}{12}=-\frac{1}{4}+\frac{1}{3}s_\theta^2+\frac{c_\theta^2}{3}-\frac{1}{12}=0 \tag{35}$$

$$-\frac{g_R^u}{2}+\frac{c_\theta^2}{3}-\frac{1}{3}=\frac{1}{3}s_\theta^2+\frac{c_\theta^2}{3}-\frac{1}{3}=0 \tag{36}$$

It's interesting to note that really all diagrams are needed in order for the contributions to cancel.

The messages of these exercises are:

- Naive operator classifications into "bosonic operators" vs "fermionic operators" or "operators affecting TGC" etc are basis dependent statements.
- When computing a SMEFT observable, in order to have a physical, basis independent result, we need to retain contributions from *all* operators in the basis. In the example above, if we had neglected one or more Warsaw basis operators, we wouldn't have been able to reconstruct properly the effect of $\mathcal{O}_B$.