EW SMEFT – Exercise sheet 1

Invariance under basis change

A note on notation & conventions. In this sheet we will always use Q_i, C_i to indicate Warsaw basis operators and Wilson coefficients, and $\mathcal{O}_i, f_i$ to indicate HISZ basis operators and their coefficients. We will also work with a $U(3)^5$ flavor symmetry for practicity, so all flavor indices are implicitly contracted and we use $\{m_W, m_Z, G_F\}$ as EW inputs.
A list of Warsaw basis operators relevant to this exercise is provided in the appendix.

1. The following operators are present in the HISZ basis but not in the Warsaw one:

$$\mathcal{O}_B = iB_{\mu\nu} D^\mu \Phi^\dagger D^\nu \Phi \qquad\qquad \mathcal{O}_W = iW^I_{\mu\nu} D^\mu \Phi^\dagger \sigma^I D^\nu \Phi \tag{1}$$

   (a) Convince yourself that the i factor is required for the operators to be Hermitian.

   (b) Translate them to the Warsaw basis:

   Step 1. Start from the operators

$$A_W = (D^\mu W^I_{\mu\nu})(i\Phi^\dagger \overleftrightarrow{D}^{I\nu} \Phi) \qquad\qquad A_B = (\partial^\mu B_{\mu\nu})(i\Phi^\dagger \overleftrightarrow{D}^{\nu} \Phi) \tag{2}$$

   and rewrite them in terms of $\mathcal{O}_W$, $\mathcal{O}_B$ and Warsaw basis operators. Use:

$$i\Phi^\dagger \overleftrightarrow{D}^{\nu} \Phi = i\Phi^\dagger (D^\nu \Phi) - i(D^\nu \Phi^\dagger)\Phi \tag{3}$$

$$i\Phi^\dagger \overleftrightarrow{D}^{I\nu} \Phi = i\Phi^\dagger \sigma^I (D^\nu \Phi) - i(D^\nu \Phi^\dagger)\sigma^I \Phi \tag{4}$$

$$[D_\mu, D_\nu]\Phi = \left[\frac{ig}{2} W^I_{\mu\nu}\sigma^I + \frac{ig'}{2} B_{\mu\nu}\right]\Phi \tag{5}$$

   Step 2. Now write A_W, A_B as a function of Warsaw basis operators only. Use the Equations of motion:

$$D^\mu W^I_{\mu\nu} = \frac{g}{2}(\Phi^\dagger i \overleftrightarrow{D}^I_\nu \Phi) + g\left(\bar{q}\frac{\sigma^I}{2}\gamma_\nu q + \bar{l}\frac{\sigma^I}{2}\gamma_\nu l\right) \tag{6}$$

$$\partial^\mu B^I_{\mu\nu} = \frac{g'}{2}(\Phi^\dagger i \overleftrightarrow{D}_\nu \Phi) + g'\left(\frac{1}{6}\bar{q}\gamma_\nu q + \frac{2}{3}\bar{u}\gamma_\nu u - \frac{1}{3}\bar{d}\gamma_\nu d - \frac{1}{2}\bar{l}\gamma_\nu l - \bar{e}\gamma_\nu e\right) \tag{7}$$

   You will also need the relations [*Bonus*: prove them!]

$$(\Phi^\dagger i \overleftrightarrow{D}_\nu \Phi)^2 = 4Q_{HD} + Q_{H\Box} \tag{8}$$

$$(D_\mu \Phi^\dagger D^\mu \Phi)\Phi^\dagger \Phi = \frac{Q_{H\Box}}{2} - m_h^2 (\Phi^\dagger \Phi)^2 + \lambda Q_H + (Q_{eH} + Q_{uH} + Q_{dH} + \text{h.c.}) \tag{9}$$

   (the term $(\Phi^\dagger \Phi)^2$ appears at $d = 4$ and remains explicitly in the solutions)

Step 3. Put everything together equating the two expressions for A_W, A_B that you got at steps 1 and 2. Solve for $\mathcal{O}_W$ and $\mathcal{O}_B$.

(c) Determine the Wilson coefficients of the Warsaw basis operators as a function of those in the HISZ basis (i.e. $C_i(f_W, f_B)$).

2. Verify the invariance under basis change for the process $e^+e^- \to W^+W^-$. For this exercise and the next we will only look at $\mathcal{O}_B$, as the case of $\mathcal{O}_W$ is very similar.

For this first example it is sufficient to look at the Feynman rules stemming from the operators in the two bases to verify that the basis change derived above leads exactly to the same scattering amplitude.

(a) Consider first the Feynman rules in the Warsaw basis. The relevant ones are: (only operators relevant for the basis conversion are retained, all momenta incoming)

$$
\begin{aligned}
\gamma_\rho W^+_\mu W^-_\nu \quad & ie\,[\eta^{\mu\nu}(p_{W+}-p_{W-})^\rho + \eta^{\mu\rho}(p_\gamma - p_{W+})^\nu - \eta^{\nu\rho}(p_\gamma - p_{W-})^\mu]\left[1-\left(\frac{\bar{C}_{HD}}{4t_\theta^2}+\frac{\bar{C}_{HWB}}{t_\theta}\right)\right]\\
& +ie\left[\eta^{\mu\rho}p_\gamma^\nu - \eta^{\nu\rho}p_\gamma^\mu\right]\frac{\bar{C}_{HWB}}{t_\theta}\\
Z_\rho W^+_\mu W^-_\nu \quad & igc_\theta\,[\eta^{\mu\nu}(p_{W+}-p_{W-})^\rho + \eta^{\mu\rho}(p_Z - p_{W+})^\nu - \eta^{\nu\rho}(p_Z - p_{W-})^\mu]\left[1+\frac{\bar{C}_{HD}}{4}+t_\theta\bar{C}_{HWB}\right]\\
& -ie\,[\eta^{\mu\rho}p_Z^\nu - \eta^{\nu\rho}p_Z^\mu]\,\bar{C}_{HWB}\\
\bar{e}e\gamma_\mu \quad & ie\gamma^\mu\left[1-\left(\frac{\bar{C}_{HD}}{4t_\theta^2}+\frac{\bar{C}_{HWB}}{t_\theta}\right)\right]\\
\bar{e}eZ_\mu \quad & i\frac{g}{c_\theta}\frac{c_{2\theta}}{2}(\gamma^\mu P_L)\left[1+\frac{1}{c_{2\theta}}\left(\bar{C}^{(1)}_{Hl}+\frac{\bar{C}_{HD}}{4}(3-2s_\theta^2)+s_{2\theta}\bar{C}_{HWB}\right)\right]\\
& -i\frac{g}{c_\theta}s_\theta^2(\gamma^\mu P_R)\left[1-\frac{1}{2s_\theta^2}\left(\bar{C}_{He}+\frac{\bar{C}_{HD}}{2}(2-s_\theta^2)+s_{2\theta}\bar{C}_{HWB}\right)\right]
\end{aligned}
$$

Rewrite them replacing the C_i with the functions of f_B determined at point (c) above.

(b) Now consider the rules for the operator $\mathcal{O}_B$:

$$
\begin{aligned}
\gamma_\rho W^+_\mu W^-_\nu \quad & i\frac{g^2 c_\theta}{4}\bar{f}_B(\eta^{\mu\rho}p_\gamma^\nu - \eta^{\nu\rho}p_\gamma^\mu)\\
Z_\rho W^+_\mu W^-_\nu \quad & -i\frac{g^2 s_\theta}{4}\bar{f}_B(\eta^{\mu\rho}p_Z^\nu - \eta^{\nu\rho}p_Z^\mu)
\end{aligned}
$$

Verify that they coincide with those obtained at the previous point, starting from Warsaw basis operators. This directly proves that the two bases give identical predictions at amplitude level.

(c) Convince yourself that the equivalence holds for the γ and Z diagrams individually and also independently for each chirality in initial state.

3. Verify the invariance under basis change in $\bar{u}u \to Zh$.
This case is less trivial! We will actually need to compute the amplitudes.

(a) Compute first the scattering amplitude with one $\mathcal{O}_B$ insertion. The relevant Feynman rules are:

$$Z_\mu Z_\nu h \qquad \frac{ie}{2vc_\theta}\bar{f}_B\left[\eta^{\mu\nu}p_h\cdot(p_{Z1}+p_{Z2})-p_h^\nu p_{Z2}^\mu-p_h^\mu p_{Z1}^\nu\right]$$

$$\gamma_\mu Z_\nu h \qquad -\frac{ig}{2v}\bar{f}_B\left[\eta^{\mu\nu}p_h\cdot p_\gamma-p_\gamma^\nu p_h^\mu\right]$$

⚠ The rules are given for all momenta *incoming*. Change signs appropriately when inserting in the diagrams.

(b) Now consider the FR for the Warsaw basis and convert them to f_B:

$$Z_\mu Z_\nu h \qquad \frac{ig^2 v}{2c_\theta^2}\eta^{\mu\nu}\left[1+\left(\bar{C}_{H\Box}+\frac{\bar{C}_{HD}}{4}\right)\right]+\frac{4is_\theta}{v}\left[p_{Z1}^\nu p_{Z2}^\mu-p_{Z1}\cdot p_{Z2}\eta^{\mu\nu}\right]\left(\bar{C}_{HWB}c_\theta+\bar{C}_{HB}s_\theta\right)$$

$$\gamma_\mu Z_\nu h \qquad -\frac{2i}{v}\left[p_\gamma^\nu p_Z^\mu-p_\gamma\cdot p_Z\eta^{\mu\nu}\right]\left(c_{2\theta}\bar{C}_{HWB}+s_{2\theta}\bar{C}_{HB}\right)$$

$$\begin{aligned}\bar{u}uZ_\mu \qquad & -i\frac{g}{c_\theta}\left(\frac{1}{2}-\frac{2s_\theta^2}{3}\right)(\gamma^\mu P_L)\left[1-\frac{3}{3-4s_\theta^2}\left(\bar{C}_{Hq}^{(1)}+\bar{C}_{HD}\frac{-5+4s_\theta^2}{12}-\frac{2s_{2\theta}}{3}\bar{C}_{HWB}\right)\right]\\ & +i\frac{g}{c_\theta}\frac{2}{3}s_\theta^2(\gamma^\mu P_R)\left[1+\frac{3}{4s_\theta^2}\left(\bar{C}_{Hu}+\bar{C}_{HD}\frac{-2+s_\theta^2}{3}-\frac{2s_{2\theta}}{3}\bar{C}_{HWB}\right)\right]\end{aligned}$$

$$\bar{u}uZ_\mu H \qquad \frac{ig}{vc_\theta}\left[\bar{C}_{Hq}^{(1)}(\gamma^\mu P_L)+\bar{C}_{Hu}(\gamma^\mu P_R)\right]$$

(c) Using the Warsaw basis FR simplified at the previous point, compute the scattering amplitude linear in f_B.

⚠ There are 3 diagrams here: with a Z, with a γ and with a contact term.

(d) Verify that the total amplitudes computed directly with $\mathcal{O}_B$ and through the translation from the Warsaw basis are identical. This should be true independently for each chirality in initial state.

Unlike in the previous example, here the equivalence does not occur diagram by diagram or vertex by vertex, but requires to sum all contributions!

A List of relevant Warsaw basis operators

$$Q_{HW}=W_{\mu\nu}^I W^{I\mu\nu}\Phi^\dagger\Phi \qquad Q_{HD}=(\Phi^\dagger D_\mu\Phi)(D^\mu\Phi^\dagger\Phi)$$

$$Q_{HB}=B_{\mu\nu}B^{\mu\nu}\Phi^\dagger\Phi \qquad Q_{H\Box}=\Box(\Phi^\dagger\Phi)(\Phi^\dagger\Phi) \qquad Q_H=(\Phi^\dagger\Phi)^3$$

$$Q_{HWB}=B_{\mu\nu}W^{I\mu\nu}\Phi^\dagger\sigma^I\Phi$$

$$Q_{Hl}^{(3)}=(i\Phi^\dagger\overleftrightarrow{D}^{I\nu}\Phi)(\bar{l}\gamma_\nu\sigma^I l) \qquad Q_{Hq}^{(3)}=(i\Phi^\dagger\overleftrightarrow{D}^{I\nu}\Phi)(\bar{q}\gamma_\nu\sigma^I q)$$

$$Q_{Hl}^{(1)}=(i\Phi^\dagger\overleftrightarrow{D}^{\nu}\Phi)(\bar{l}\gamma_\nu l) \qquad Q_{Hq}^{(1)}=(i\Phi^\dagger\overleftrightarrow{D}^{\nu}\Phi)(\bar{q}\gamma_\nu q)$$

$$Q_{Hu}=(i\Phi^\dagger\overleftrightarrow{D}^{\nu}\Phi)(\bar{u}\gamma_\nu u) \qquad Q_{Hd}=(i\Phi^\dagger\overleftrightarrow{D}^{\nu}\Phi)(\bar{d}\gamma_\nu d)$$

$$Q_{He}=(i\Phi^\dagger\overleftrightarrow{D}^{\nu}\Phi)(\bar{e}\gamma_\nu e)$$

$$Q_{uH}=(\bar{q}\tilde{\Phi}Y_u^\dagger u)(\Phi^\dagger\Phi) \qquad Q_{dH}=(\bar{q}\Phi Y_d^\dagger d)(\Phi^\dagger\Phi)$$

$$Q_{eH}=(\bar{l}\Phi Y_e^\dagger e)(\Phi^\dagger\Phi)$$