Hyperfine splittings of heavy quarkonium hybrids

J. Soto, S. T. Valls, Phys. rev. D 108, 014025

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6th Workshop on Future Directions in Spectroscopy Analysis (FDSA2025)

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EXCELENCIA MARÍA DE MAEZTU 2020-2023

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

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Outline

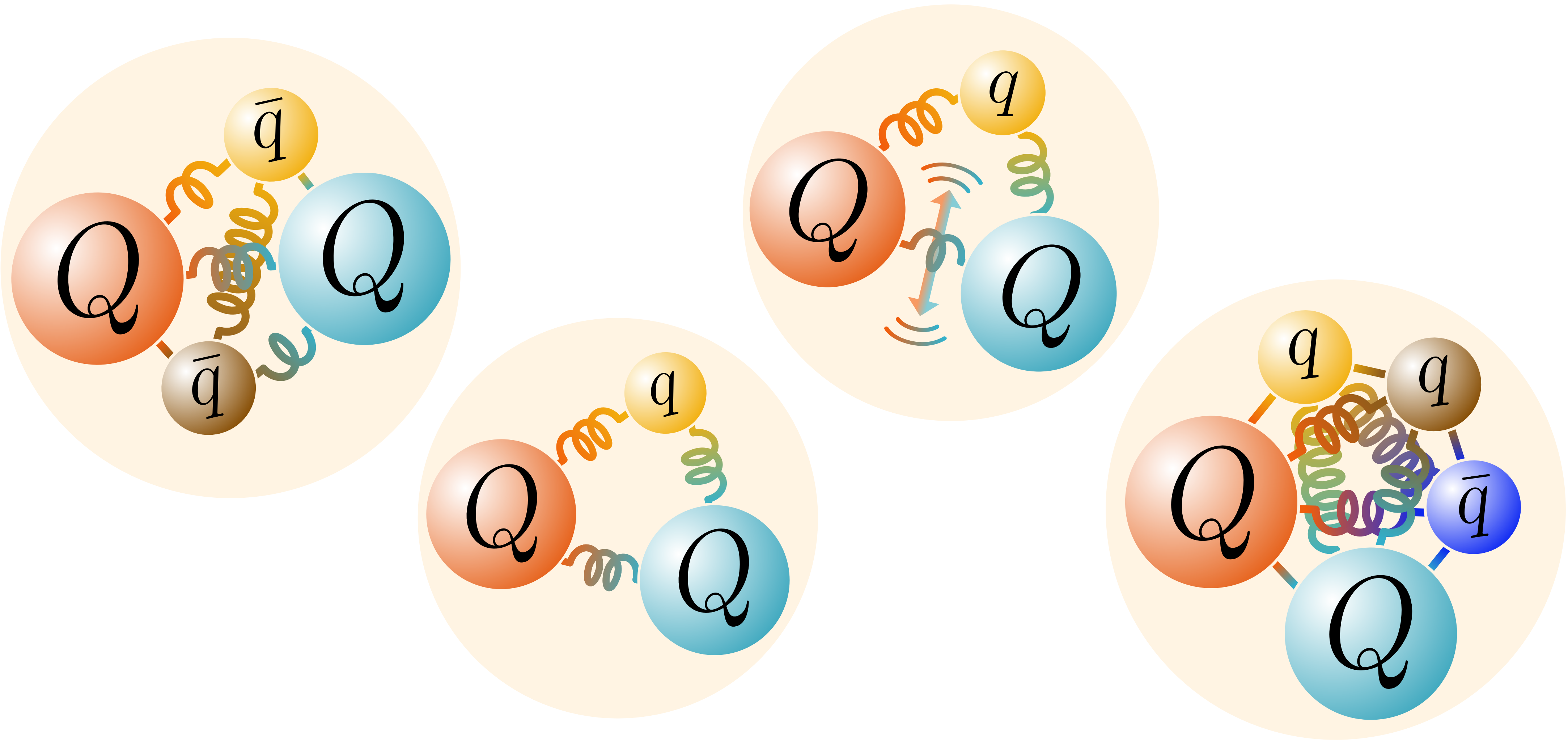
- What are heavy quarkonium hybrids?
- The Born-Oppenheimer Approximation for QCD
- Hyperfine splitting for quarkonium hybrids
- Results and Comparison
- Conclusion

What are heavy hybrids?

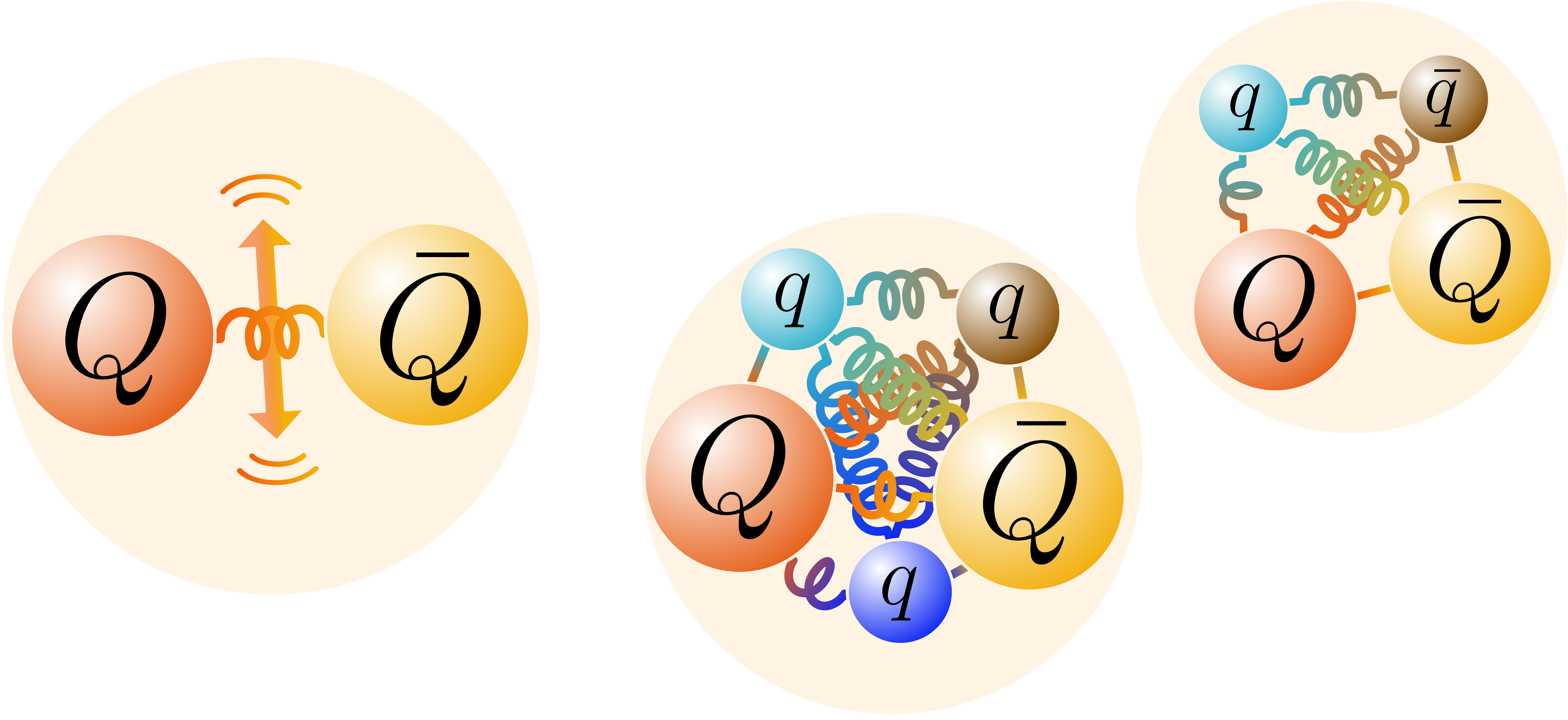
Exotic heavy hadrons

Using as heavy quarks $Q = c, b$ and as light quarks $q = u, d, s$ the hadrons with two heavy quarks can be

QQ + light quarks and gluons



$Q\bar{Q}$ + light quarks and gluons



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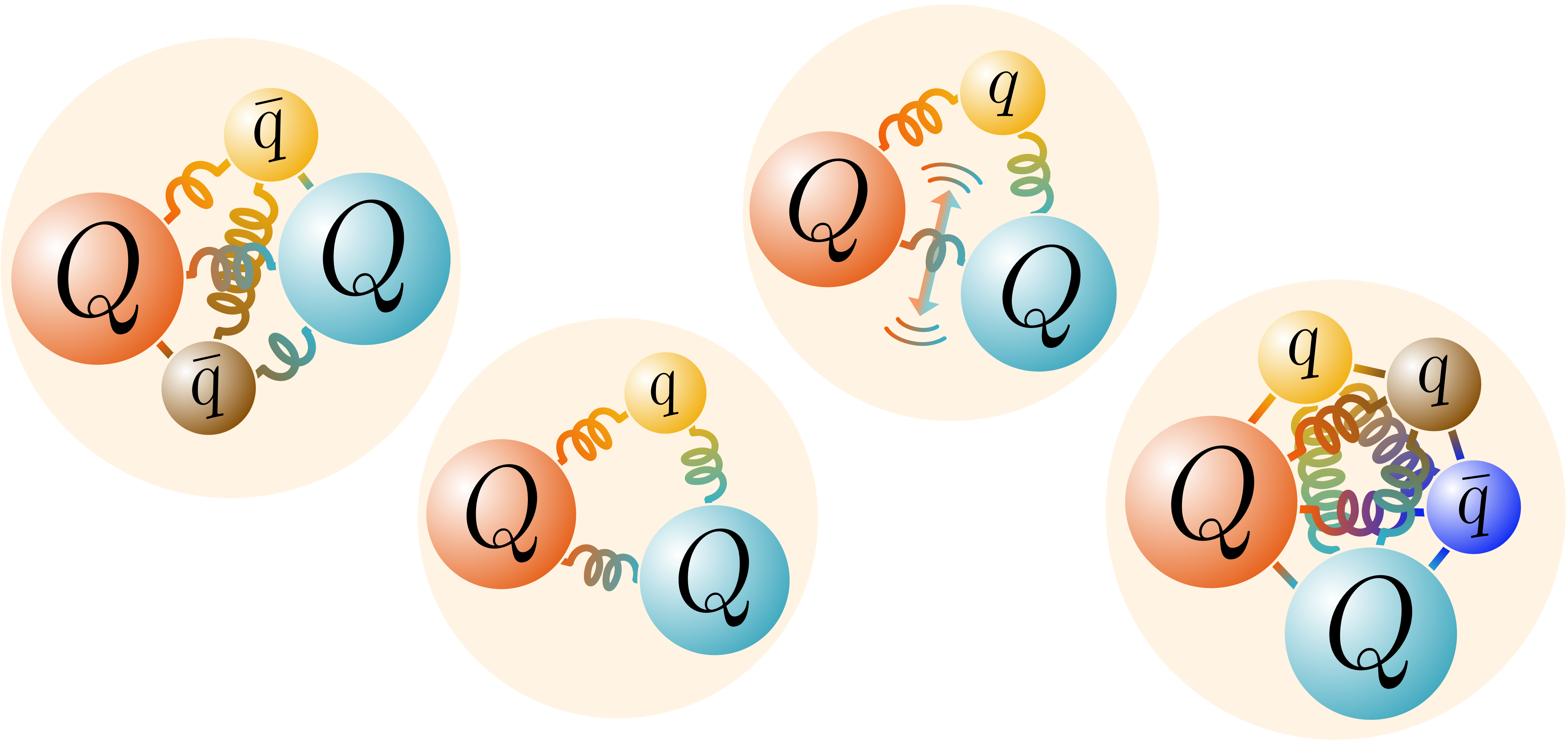

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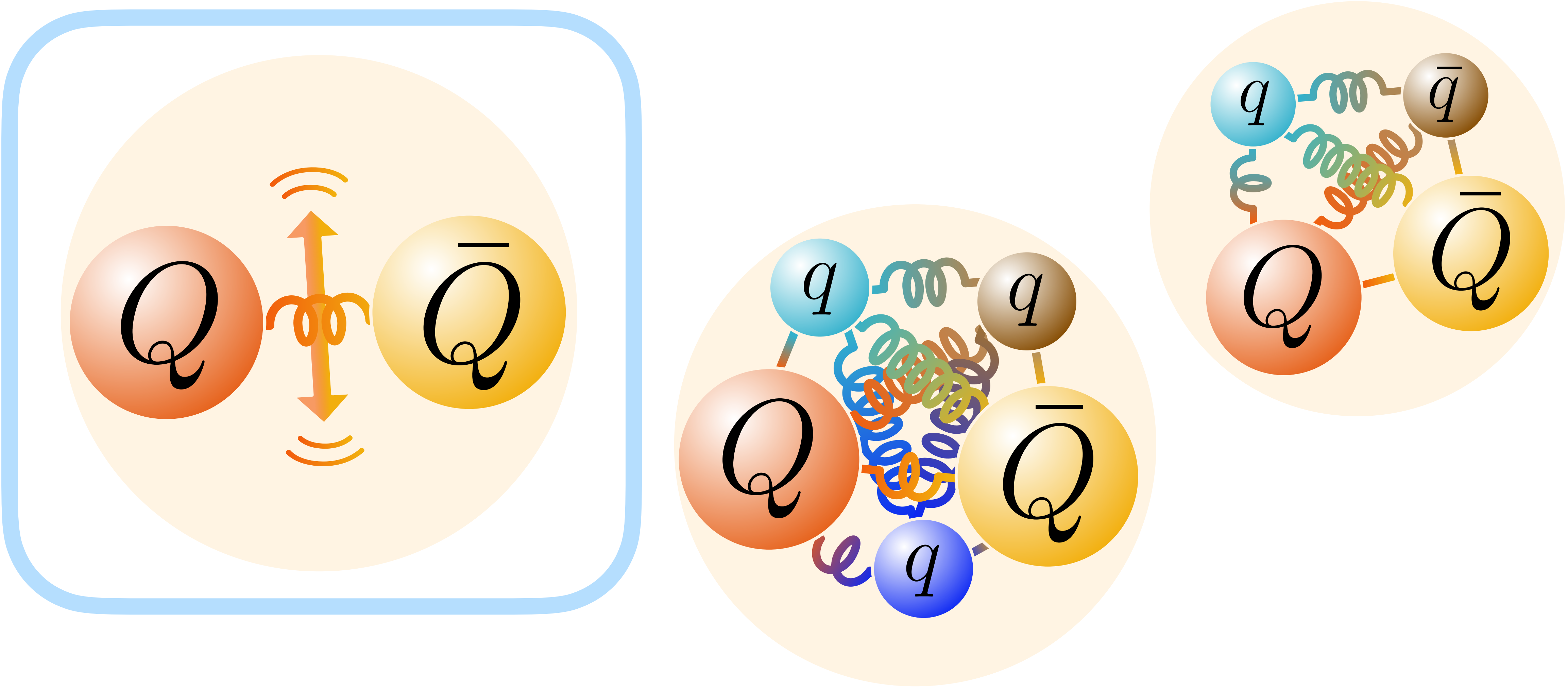


What are heavy hybrids?

Scales in heavy hadrons

- Heavy quarks: $Q = c, b \rightarrow m_Q \gg \Lambda_{QCD}$

In the hadron rest frame the heavy quark moves slowly compared to the LDOF

- We have a **non-relativistic system** → multiscale problem

$$m_Q \gg p \gtrsim \Lambda_{QCD} \gg E$$

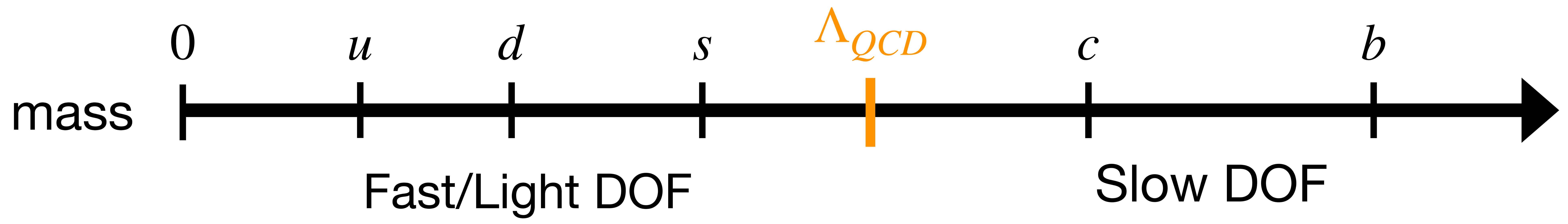
- We use an **Effective Field Theory** → Born-Oppenheimer EFT

[Berwein, Brambilla, Krein, Mohapatra, Scirpa, Vairo,…]

The B-O Approx for QCD

Philosophy of the B-O Approximation

$m_Q \gg p \gtrsim \Lambda_{QCD} \gg E$



Two-step procedure

- Calculate the QCD potential models using data from lattice QCD
- Solve Schrödinger equation for $Q\bar{Q}$

[Juge, Kuti, Morningstar]

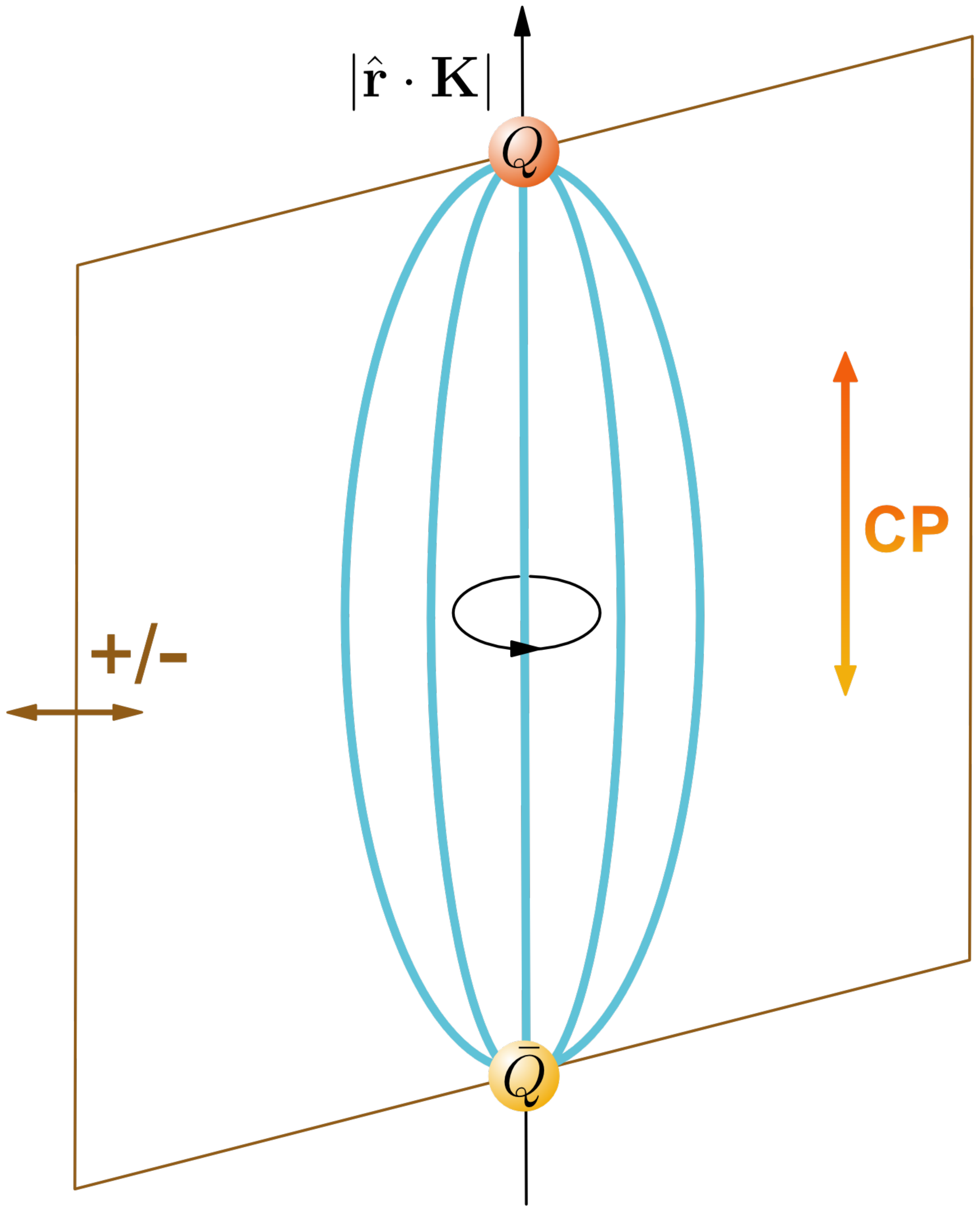
BOEFT

- At LO: $m_Q \rightarrow \infty$ (static quarks)
- At NLO: $\mathcal{O}(1/m_Q)$ expansion

[Berwein, Brambilla, Krein, Mohapatra, Vairo,…]
[Oncala, Soto, Tarrús Castellà,…]

The B-O Approx for QCD

B-O Symmteries and Quantum numbers

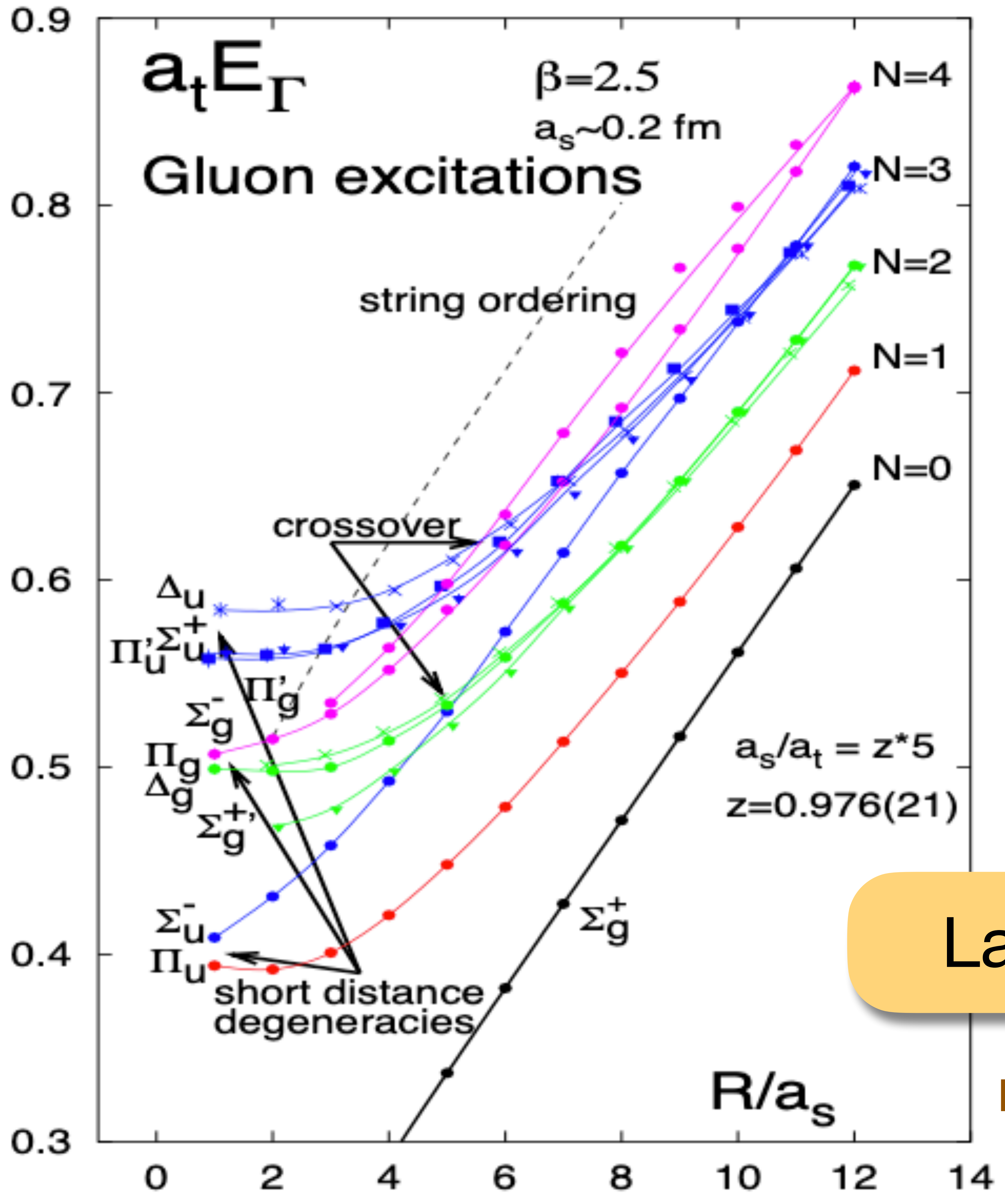


- Irreps of the $D_{\infty h}$ group → Λ_η^σ
- Total angual momentum $\mathbf{K}$ of LDOF with eigenvalue κ
- $|\mathbf{K} \cdot \hat{\mathbf{r}}|$ with eigenvalue λ and $\Lambda \equiv |\lambda|$ $\Lambda = 0,1,\ldots = \Sigma, \Pi, \ldots$
- CP eigenvalue $\eta = +1, -1 = g, u$
- Sign under reflection $\sigma = +, -$

The B-O Approx for QCD

B-O Hybrid Quantum numbers

LDOF: κ^{PC}	Λ_η^σ
1^{+-}	Σ_u^-, Π_u



Lattice results

Figure: [Juge, Kuti, Morningstar]

The B-O Approx for QCD

B-O Hybrid Potentials

- **Model the potentials** with correct behaviour at short and long distances [Oncala, Soto]

$$V_{\Sigma_u^-}(r) = \frac{\sigma_p}{r} + \kappa_s r + E_s^{\bar{Q}Q}$$

$$V_{\Pi_u}(r) = \frac{\sigma_s}{r}\left(\frac{1 + b_1 r + b_2 r^2}{1 + a_1 r + a_2 r^2}\right) + \kappa_p r + E_p^{\bar{Q}Q}$$

- **Fit the parameters** with lattice QCD results

Two-step procedure

- Calculate the QCD potential models using data from lattice QCD
- Solve Schrödinger equation for $Q\bar{Q}$

[Juge, Kuti, Morningstar]

The B-O Approx for QCD

Hybrid spectrum with BOEFT

- Lagrangian density:

$$\mathcal{L} = \mathrm{tr}\left(H^{i\dagger}\left(\delta_{ij} i\partial_0 - h_{Hij}\right) H_j\right)$$

$$h_{Hij} = \left(-\frac{\nabla^2}{m_Q} + V_{\Sigma_u^-}(r)\right)\delta_{ij} + \left(\delta_{ij} - \hat{r}_i \hat{r}_j\right)\left[V_{\Pi_u}(r) - V_{\Sigma_u^-}(r)\right]$$

Two-step procedure

- Calculate the QCD potential models using data from lattice QCD
- Solve Schrödinger equation for $Q\bar{Q}$

[Juge, Kuti, Morningstar]

- **Solve** the eigenvalue problem to find the **degenerate** energy spectrum

The B-O Approx for QCD

Hybrid spectrum with BOEFT

- Spectrum for chamonium hybrid multiplets

$\kappa^{PC} = 1^{+-}$	NL_J	$M_{c\bar{c}g}$	$J^{PC}(S_{\bar{Q}Q} = 0)$	$J^{PC}(S_{\bar{Q}Q} = 1)$	Λ_η^σ
H_1	$1(s/d)_1$	4011	1^{--}	$(0,1,2)^{-+}$	Σ_u^-, Π_u
H_2	$1p_1$	4145	1^{++}	$(0,1,2)^{+-}$	Π_u
H_3	$1p_0$	4486	0^{++}	1^{+-}	Σ_u^-
H_4	$1(p/f)_2$	4231	2^{++}	$(1,2,3)^{+-}$	Σ_u^-, Π_u

[Oncala, Soto]

- **Spin effects** enter at $\mathcal{O}(1/m_Q)$ in hybrids

Hyperfine splitting for hybrids

Spin dependent potentials

$$\left[V^{(1)}_{\kappa^p}(\mathbf{r})\right]^{n'm';nm} = -2V_{hf}(r)\left(\delta^{n'm}\delta^{nm'} - \delta^{n'm'}\delta^{nm}\right) + 2V_{hf2}(r)\left(\hat{r}^i\hat{r}^j - \frac{\delta^{ij}}{3}\right)\times$$
$$\times\left(\delta^{jm}\delta^{n'i}\delta^{nm'} + \delta^{ni}\delta^{jm'}\delta^{n'm} - \delta^{jm'}\delta^{n'i}\delta^{nm} - \delta^{ni}\delta^{jm}\delta^{n'm'}\right)$$

Two-step procedure

- Calculate the QCD potential models using data from lattice QCD
- Solve Schrödinger equation for $Q\bar{Q}$

[Juge, Kuti, Morningstar]

We need to find the **hyperfine potentials**

$$V_{hf}\,,\,V_{hf2}$$

Hyperfine splitting for hybrids

Spin dependent potentials

- **Short** distance behaviour

$$V_{hf}(r)/m_Q = A + \mathcal{O}(r^2)$$
$$V_{hf2}(r)/m_Q = Br^2 + \mathcal{O}(r^4)$$

- **Long** distance behaviour

$$V_{hf}(r) = \frac{1}{6}V^{sa}_{1+11}(r) - \frac{1}{3}V^{sb}_{1+10}(r)$$
$$V_{hf2}(r) = \frac{1}{2}\left(V^{sa}_{1+11}(r) + V^{sb}_{1+10}(r)\right)$$

$$\begin{cases} \dfrac{V^{sa}_{1+11}}{m_Q} = -\dfrac{2c_F\pi^2 g\Lambda'''}{m_Q\kappa r^3} \equiv V^{sa}_{ld}(r) \\ \dfrac{V^{sb}_{1+10}}{m_Q} = \mp\dfrac{2c_F\pi^2 g\Lambda'}{m_Q\sqrt{\pi\kappa}r^2} \equiv V^{sb}_{ld}(r) \end{cases}$$

Hyperfine splitting for hybrids

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[Soto, Tarrús]

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Hyperfine splitting for hybrids

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Lattice results of the spectrum of $c\bar{c}g$

- **Long** distance behaviour

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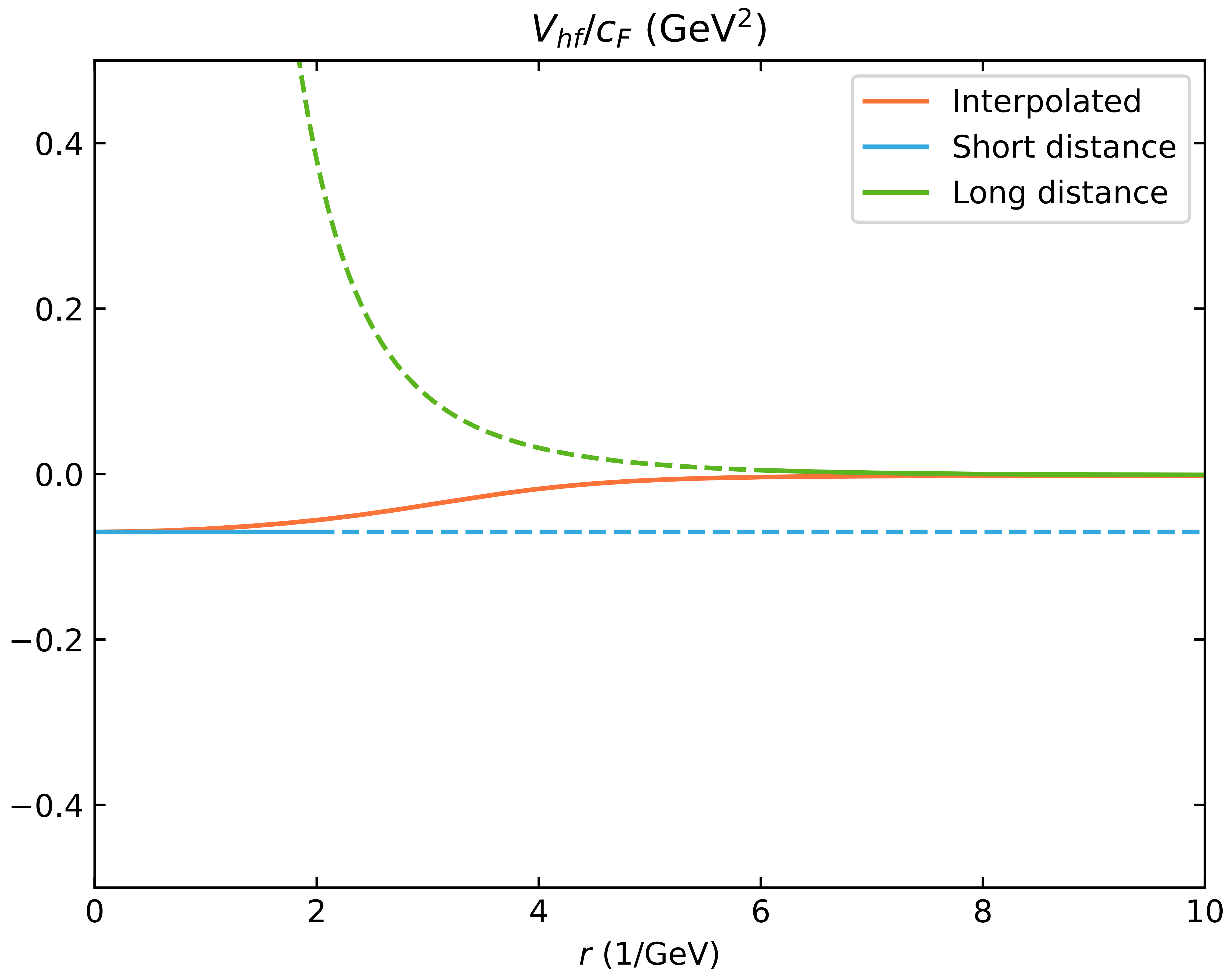
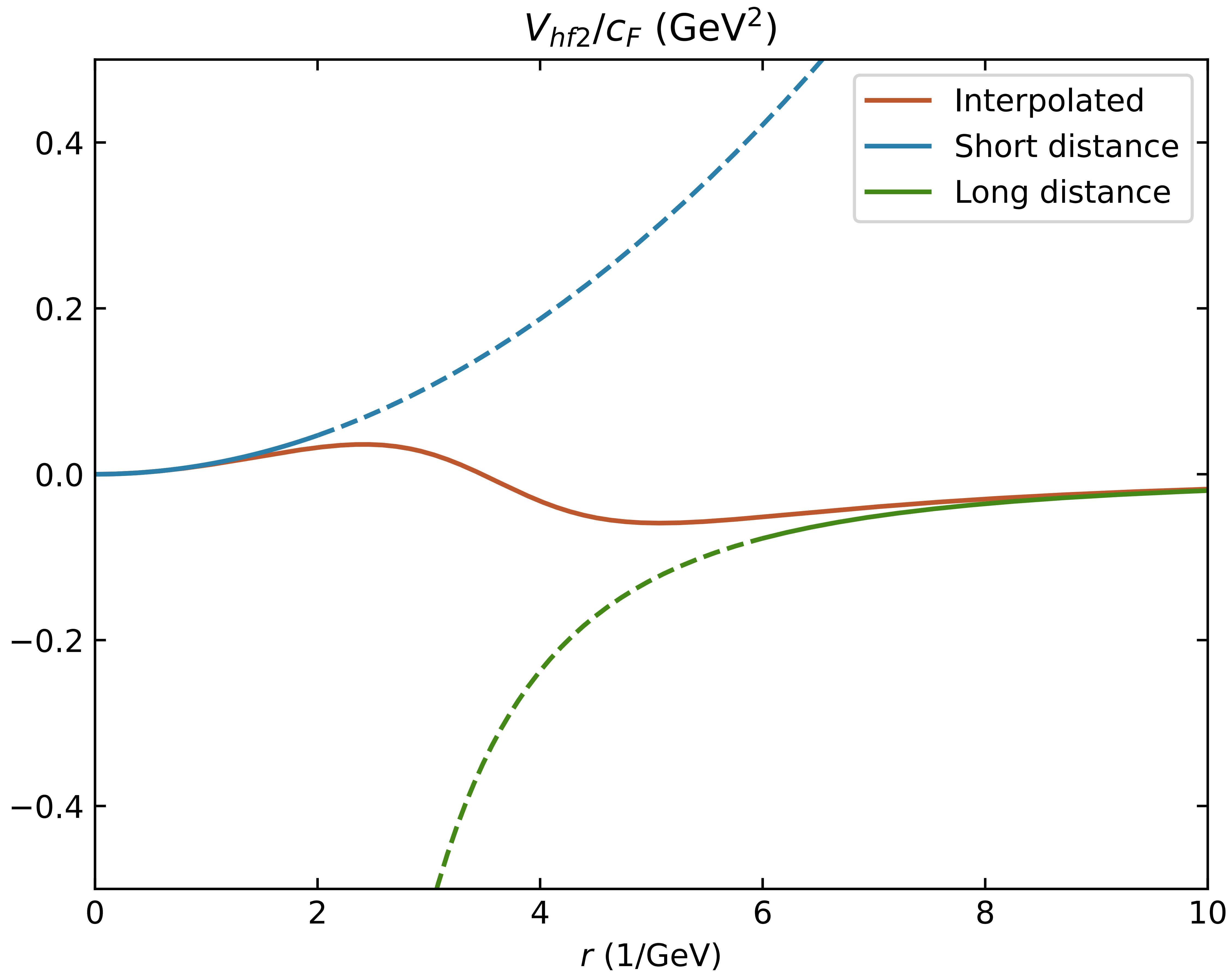
[Soto, Tarrús]

Hyperfine splitting for hybrids

Hybrid potential interpolation

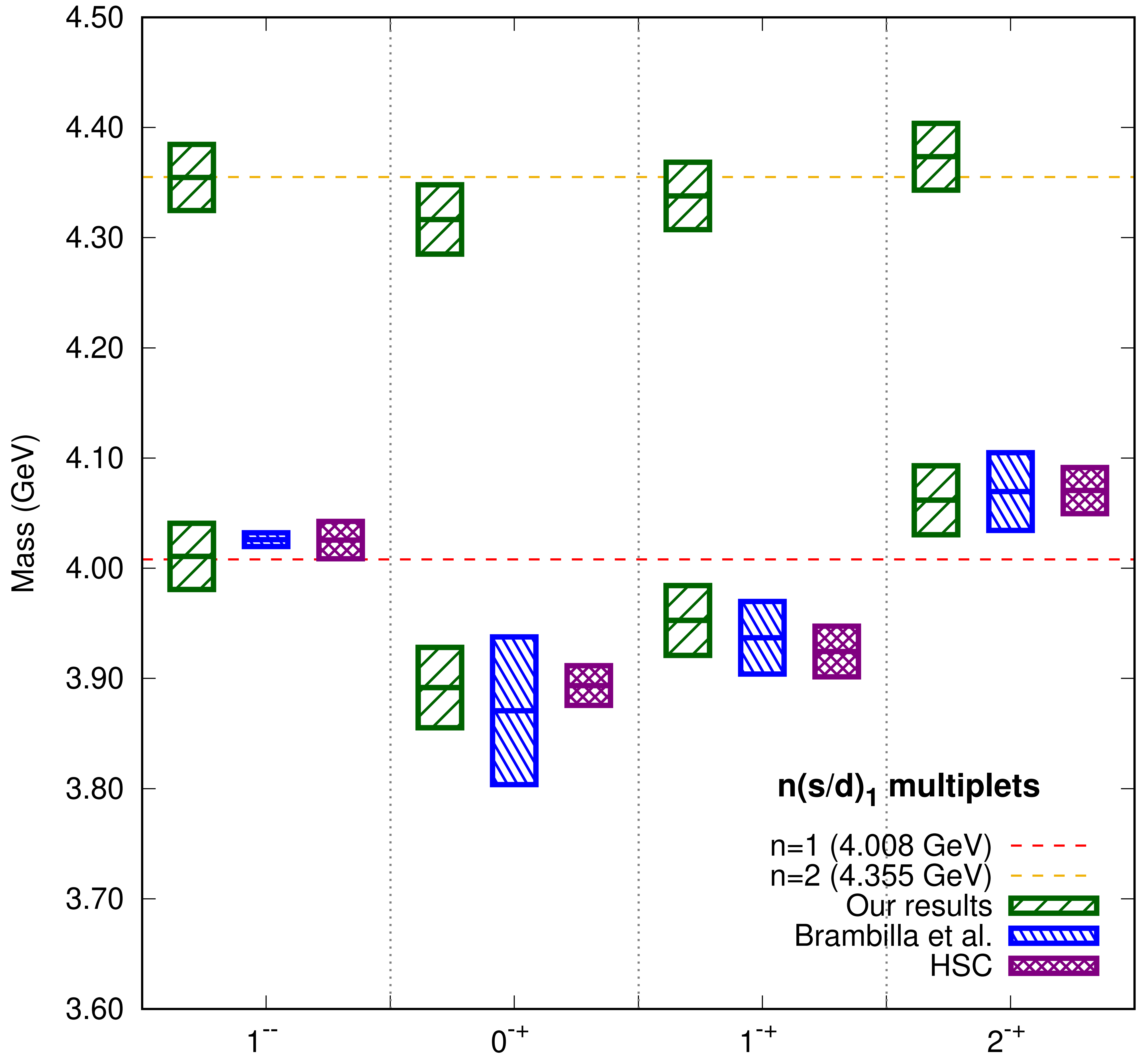


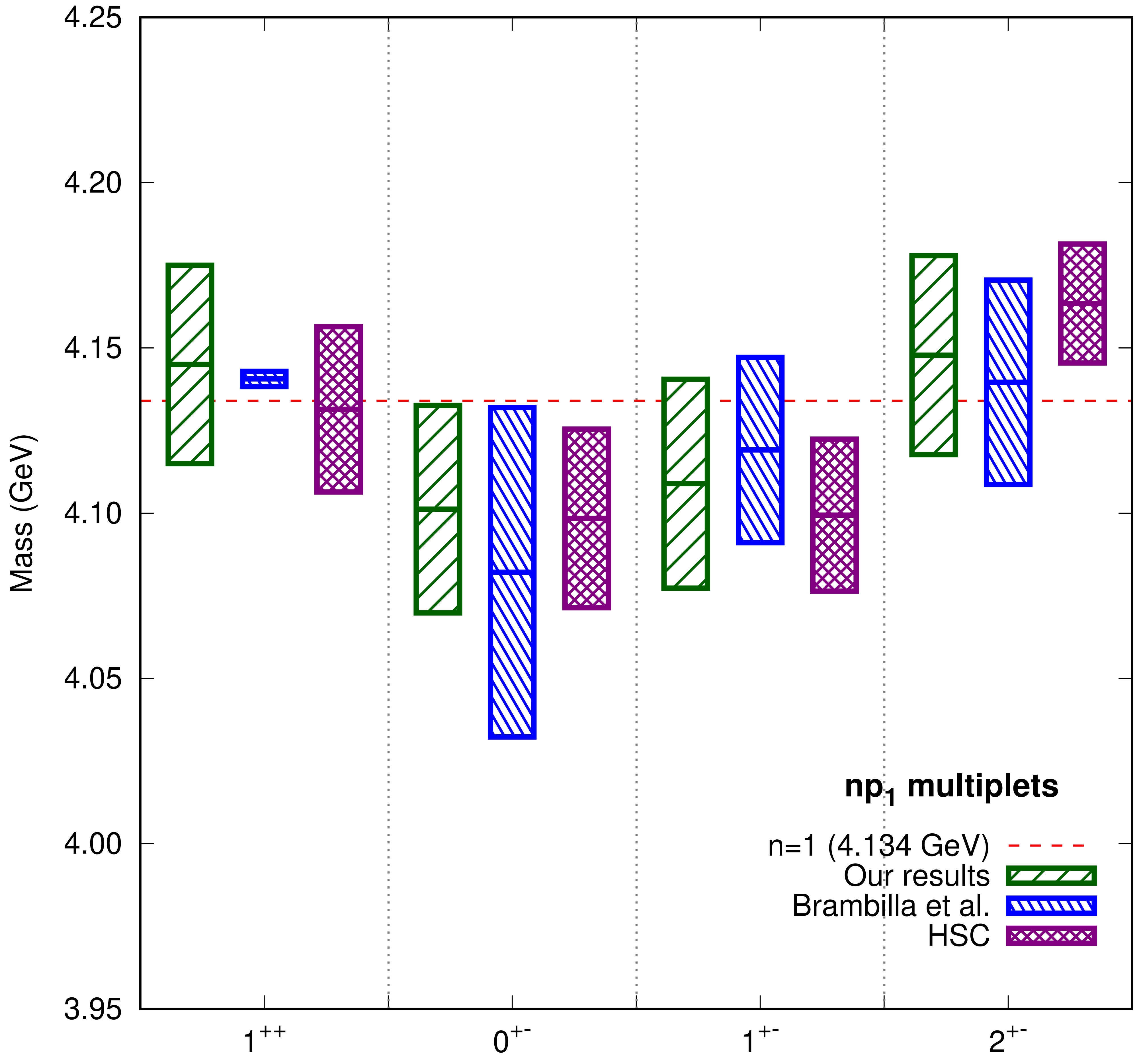


Results

Hyperfine splitting of H_1, H_2 with $m_c = 1.477$GeV

[Brambilla, Lai, Segovia, Tarrús, Vairo]
[Cheung, O'Hara, Moir, Peardon …]





Results

Hyperfine splitting of H_3, H_4 with $m_c = 1.477$GeV

[Brambilla, Lai, Segovia, Tarrús, Vairo]
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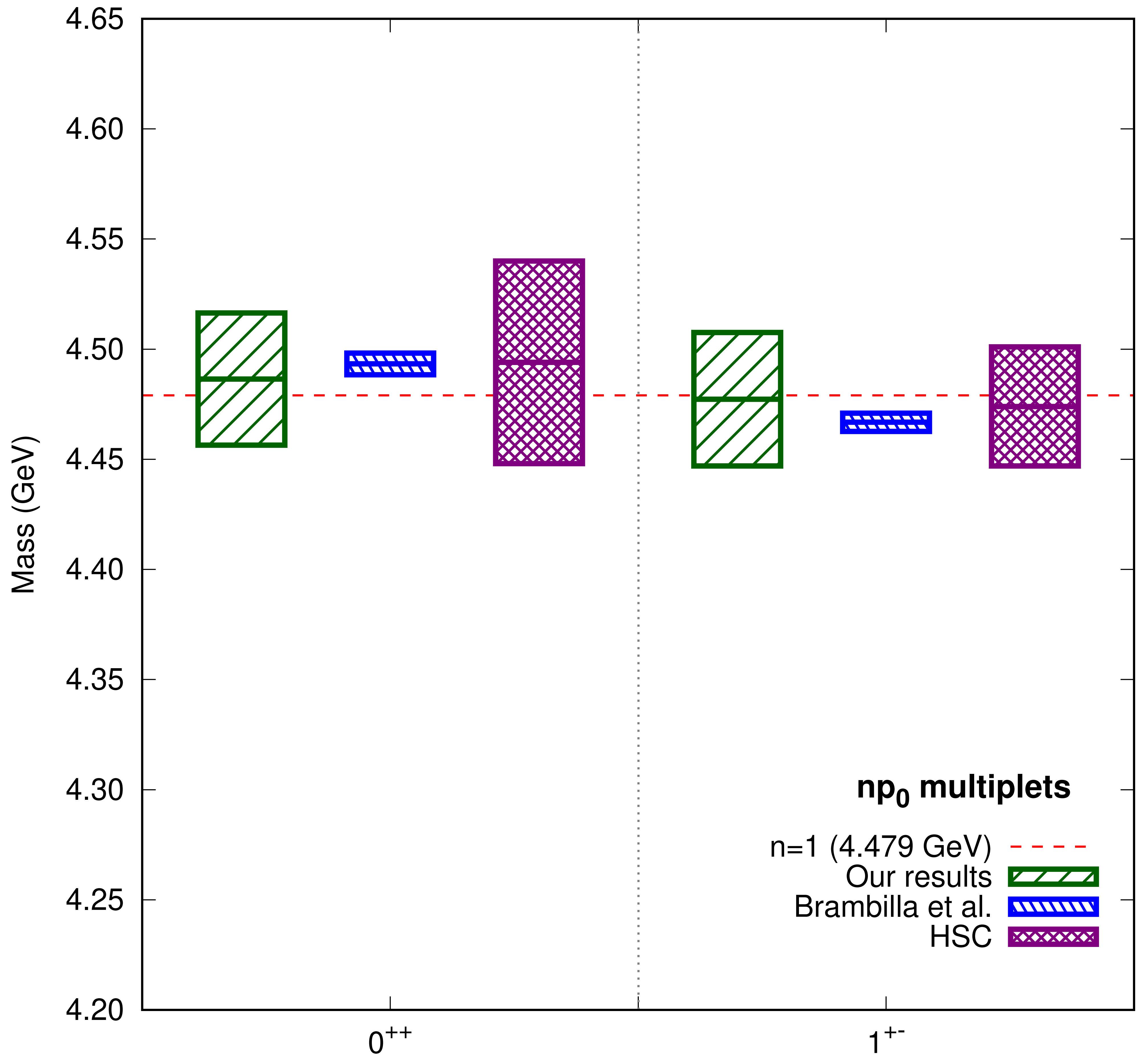

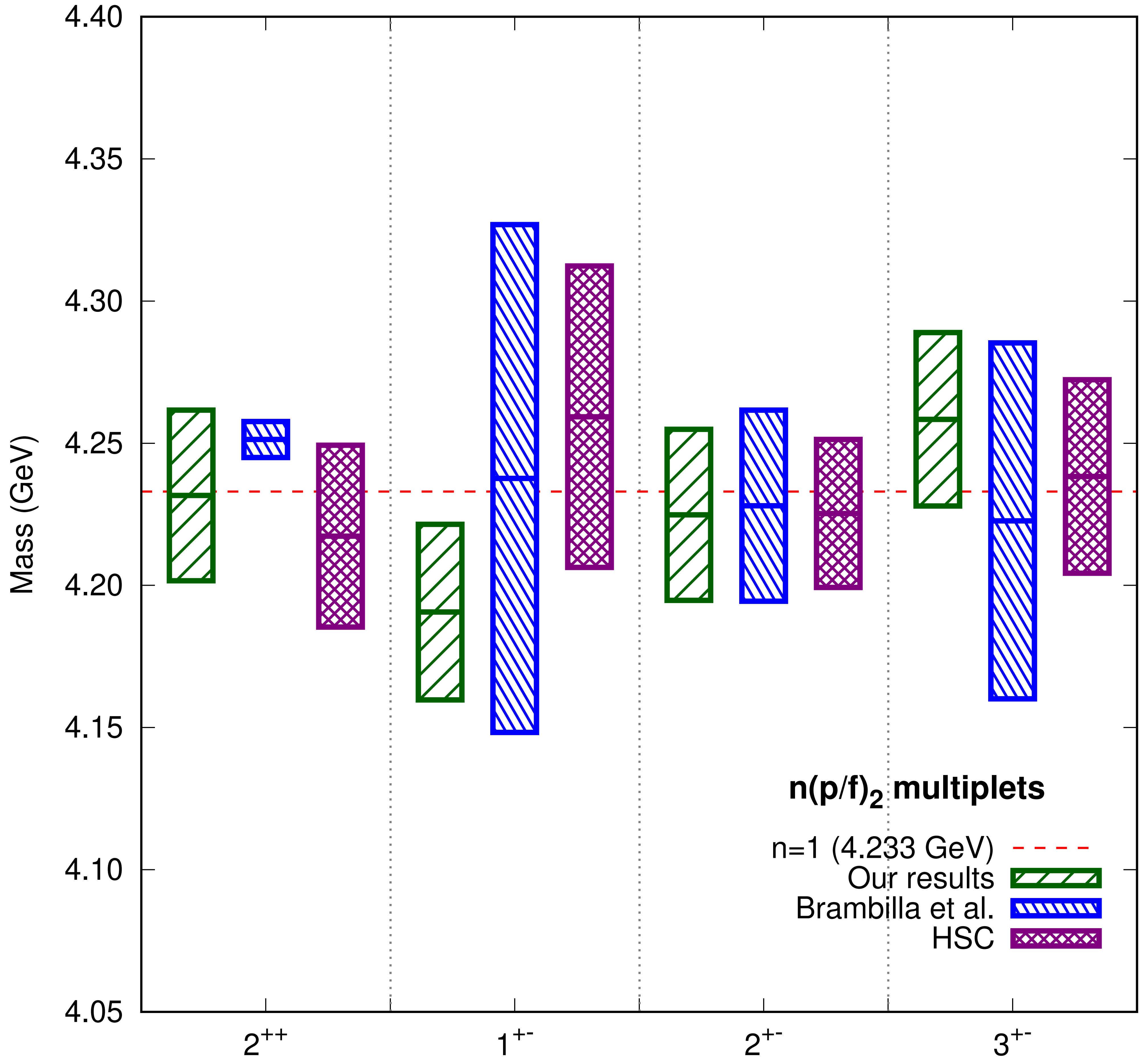

Results

Hyperfine splitting of H_1, H_2 with $m_b = 4.863$GeV

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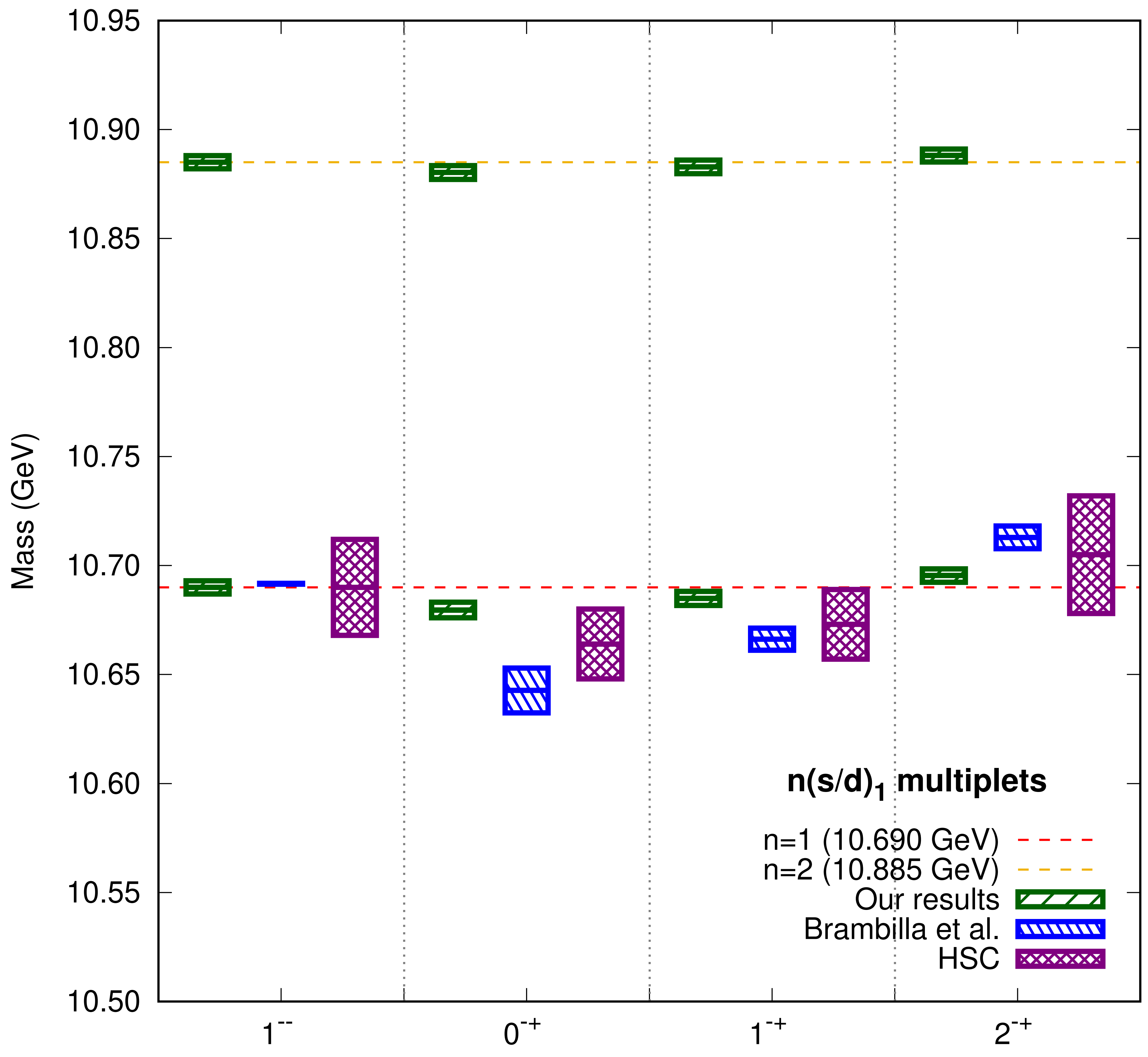

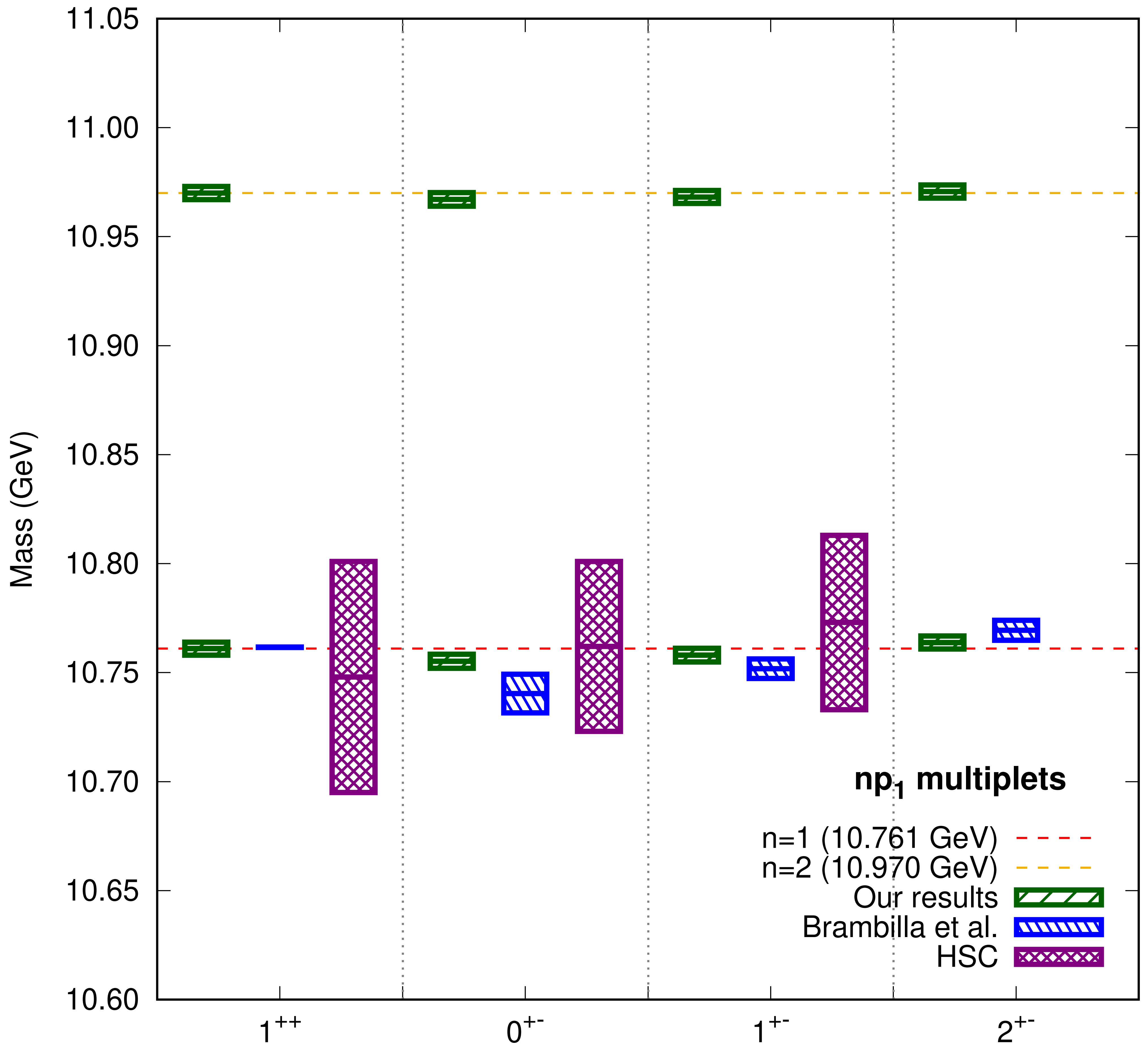

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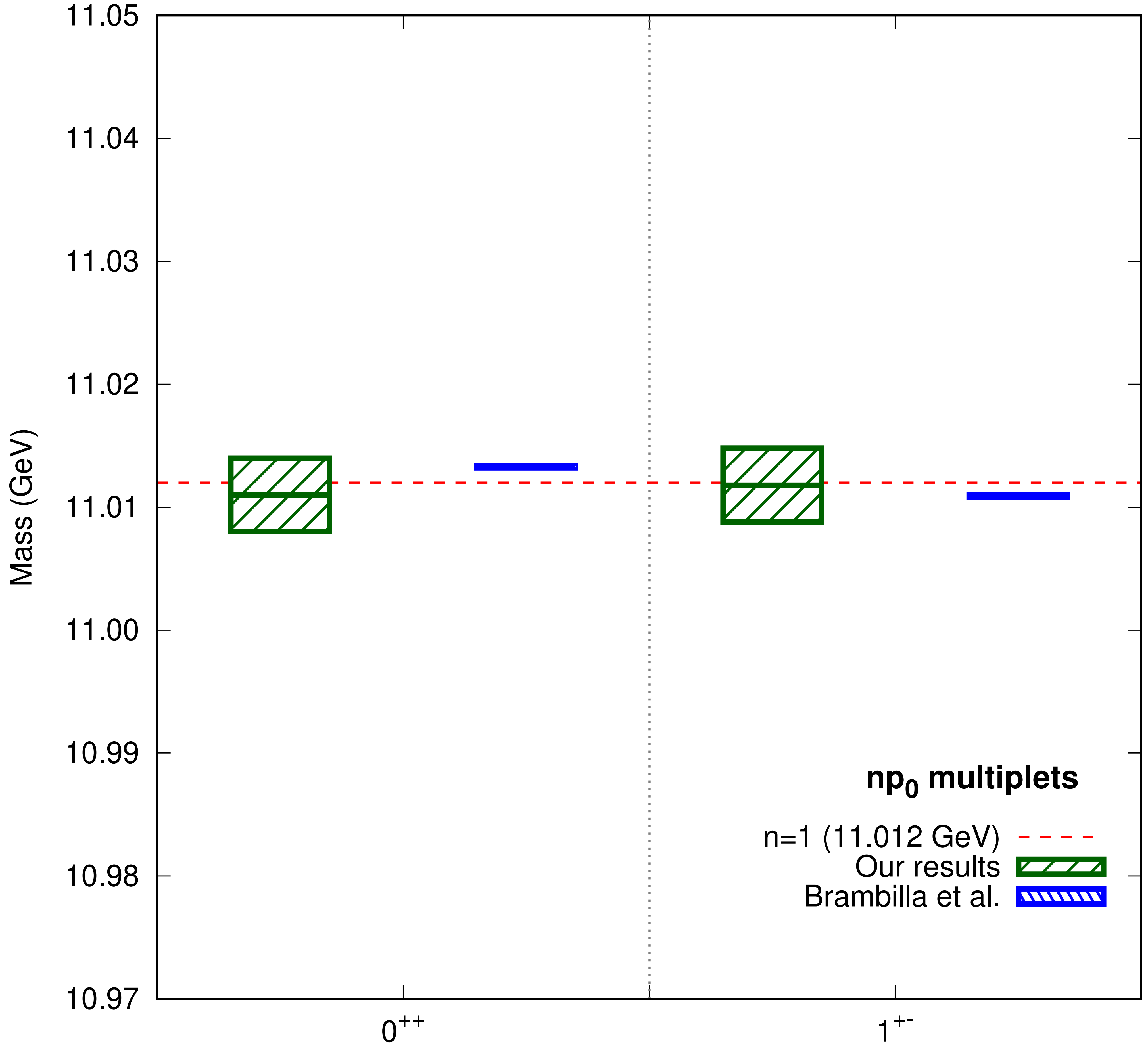
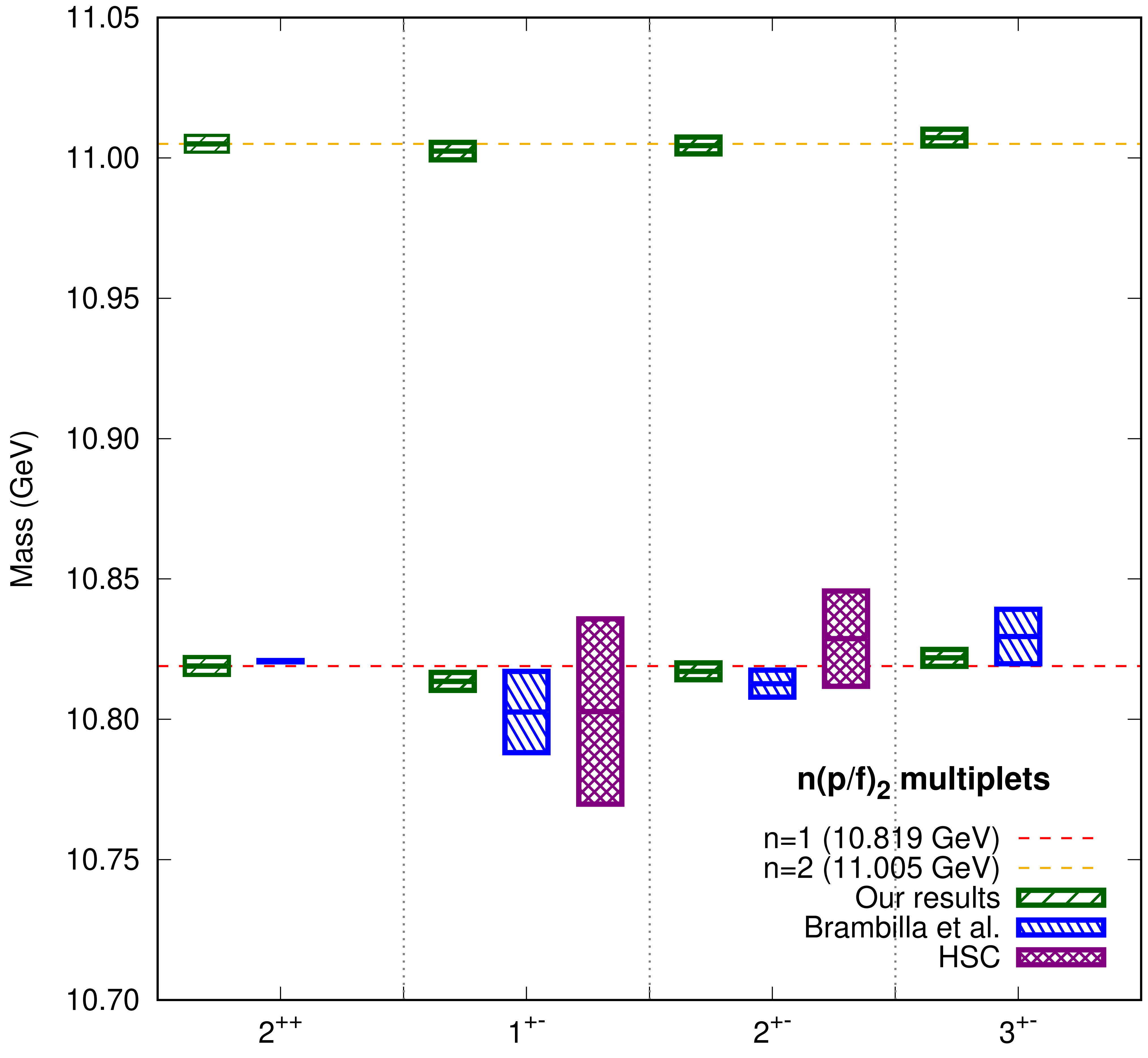
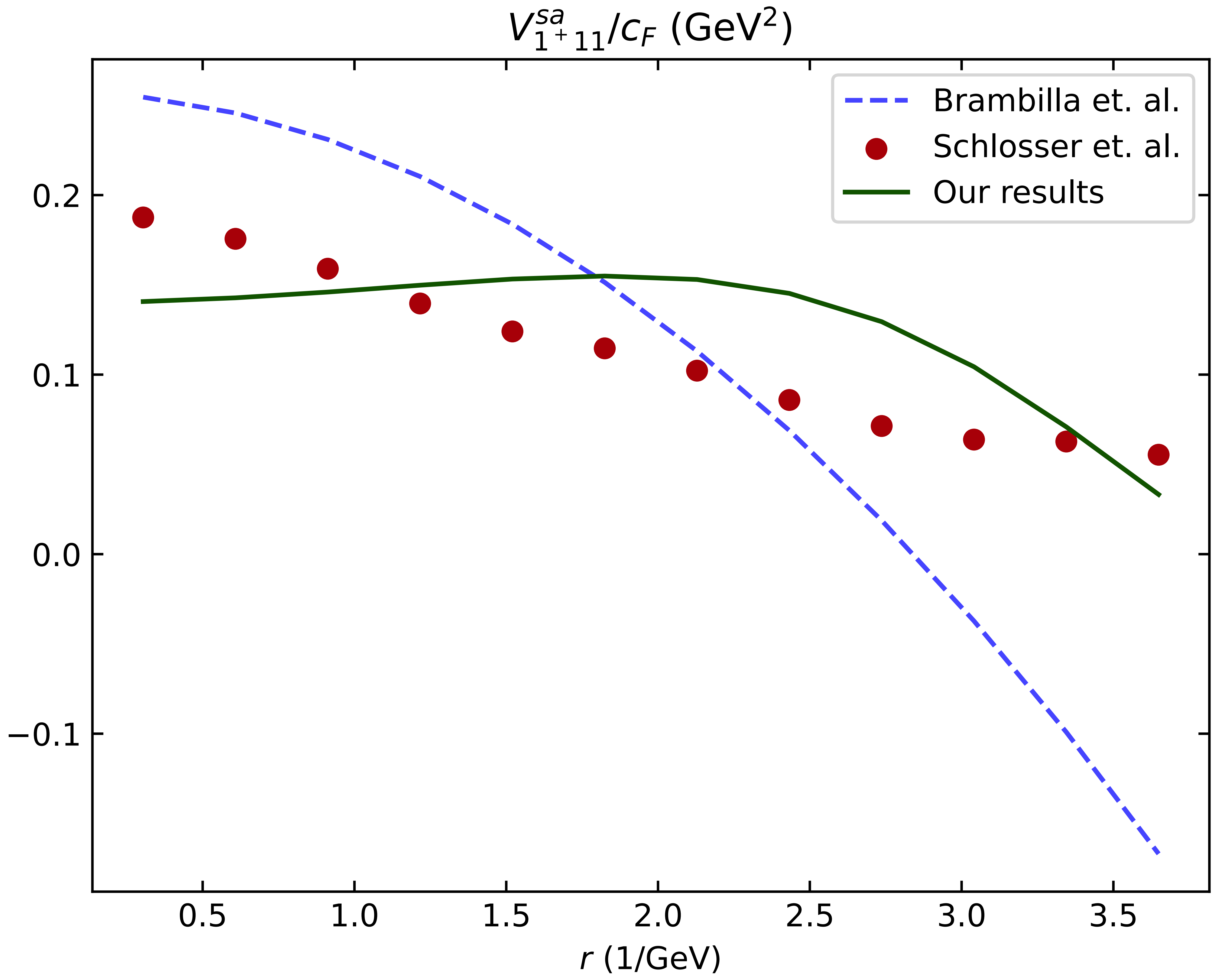


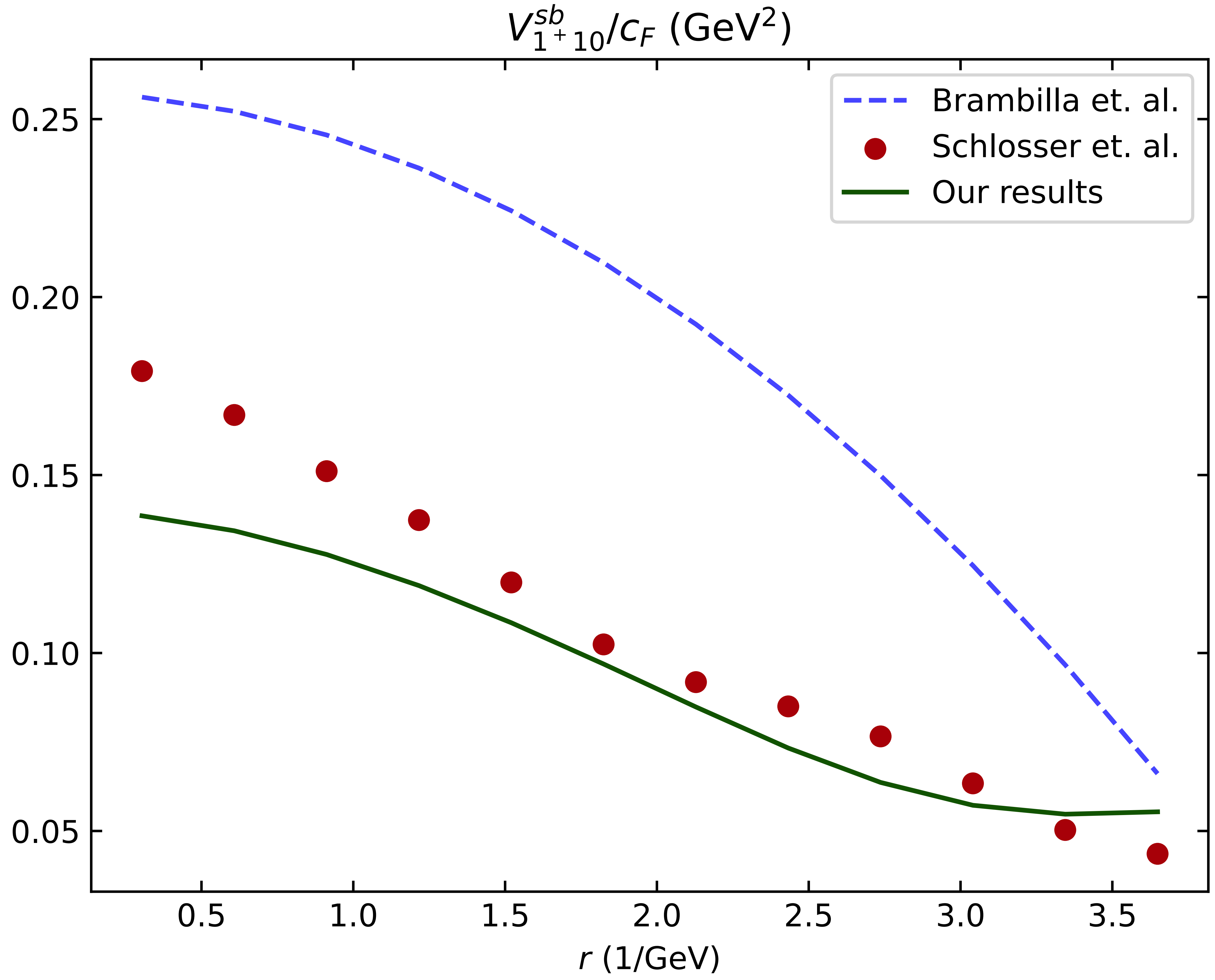


Comparison

Further analysis of the potentials

[Brambilla, Lai, Segovia, Tarrús]
[Schlosser, Wagner]





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Conclusions

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Questions?

The B-O Approx for QCD

Coupled channel Schrödinger equation

- Equations for $J \neq 0$:

$$\left[-\frac{1}{m_Q}\frac{\partial^2}{\partial r^2} + \begin{pmatrix} \frac{(J+1)J}{m_Q r^2} & 0 \\ 0 & \frac{(J+1)(J+2)}{m_Q r^2} \end{pmatrix} + V_{\Sigma_u^-}(r) + V_q(r) \begin{pmatrix} \frac{J+1}{2J+1} & \frac{\sqrt{(J+1)J}}{2J+1} \\ \frac{\sqrt{(J+1)J}}{2J+1} & \frac{J}{2J+1} \end{pmatrix}\right] \begin{pmatrix} P_J^-(r) \\ P_J^+(r) \end{pmatrix} = E \begin{pmatrix} P_J^-(r) \\ P_J^+(r) \end{pmatrix}$$

$$\left(-\frac{1}{m_Q}\frac{\partial^2}{\partial r^2} + \frac{J(J+1)}{m_Q r^2} + V_{\Pi_u}(r)\right) P_J^0(r) = E P_J^0(r)$$

- Equations for $J = 0$:

$$\left(-\frac{1}{m_Q}\frac{\partial^2}{\partial r^2} + \frac{2}{m_Q r^2} + V_{\Pi_u}(r)\right) P_0^+(r) = E P_0^+(r)$$

Hyperfine splitting for hybrids

Long distance potential

- By solving the Wilson Loop [Soto, Tarrús]

$$\frac{V^{sa}_{1+11}(r)}{m_Q} = -\lim_{T\to\infty}\frac{g\,c_F}{m_Q T}\int_{-\frac{T}{2}}^{\frac{T}{2}} dt\frac{\langle B^*(\mathbf{0},\frac{T}{2})B^3(\frac{\mathbf{r}}{2},t)B(\mathbf{0},-\frac{T}{2}) - B(\mathbf{0},\frac{T}{2})B^3(\frac{\mathbf{r}}{2},t)B^*(\mathbf{0},-\frac{T}{2})\rangle_\Box}{\langle B(\mathbf{0},\frac{T}{2})B^*(\mathbf{0},-\frac{T}{2}) + B^*(\mathbf{0},\frac{T}{2})B(\mathbf{0},-\frac{T}{2})\rangle_\Box}$$

$$\frac{V^{sb}_{1+10}(r)}{m_Q} = \lim_{T\to\infty}\frac{g\,c_F}{4m_Q}\frac{V_{\Pi_u} - V_{\Sigma_u^-}}{\sin\left((V_{\Pi_u} - V_{\Sigma_u^-})\frac{T}{2}\right)}\int_{-\frac{T}{2}}^{\frac{T}{2}} dt\frac{\langle\left(B^*(\mathbf{0},\frac{T}{2})B(\frac{\mathbf{r}}{2},t) - B(\mathbf{0},\frac{T}{2})B^*(\frac{\mathbf{r}}{2},t)\right)B^3(\mathbf{0},-\frac{T}{2})\rangle_\Box}{\langle B(\mathbf{0},\frac{T}{2})B^*(\mathbf{0},-\frac{T}{2}) + B^*(\mathbf{0},\frac{T}{2})B(\mathbf{0},-\frac{T}{2})\rangle_\Box^{1/2}\langle B^3(\mathbf{0},\frac{T}{2})B^3(\mathbf{0},-\frac{T}{2})\rangle_\Box^{1/2}}$$

- With the chromomagnetic field

$$B(\mathbf{r},t) = B^1(\mathbf{r},t) + iB^2(\mathbf{r},t)$$
$$B^*(\mathbf{r},t) = B^1(\mathbf{r},t) - iB^2(\mathbf{r},t)$$

Hyperfine splitting for hybrids

Interpolated potentials

- Interpolation between the short and long distances

$$\frac{V_{hf}(r)}{m_Q} = \frac{A + \left(\frac{r}{r_0}\right)^2 \left(\frac{1}{6} V_{ld}^{sa}(r_0) - \frac{r}{3r_0} V_{ld}^{sb}(r_0)\right)}{1 + \left(\frac{r}{r_0}\right)^5}$$

$$\frac{V_{hf2}(r)}{m_Q} = \frac{Br^2 - \left(\frac{r}{r_0}\right)^5 \left(\frac{r_0}{2r} V_{ld}^{sa}(r_0) + \frac{1}{2} V_{ld}^{sb}(r_0)\right)}{1 + \left(\frac{r}{r_0}\right)^7}$$

- With $r_0 \simeq 3.96\,\text{GeV}^{-1} \sim 1/\Lambda_{QCD}$

Hyperfine splitting for hybrids

Parameter results

$$A = c_F k_A / m_Q, \quad k_A \sim \Lambda^2_{\rm QCD}$$
$$B = c_F k_A / m_Q, \quad k_B \sim \Lambda^4_{\rm QCD}$$

$$A = -0.070 \pm 0.010 \text{GeV}$$
$$B = 0.0117 \pm 0.0003 \text{GeV}^3$$

$$c_F(m_c) \equiv c_F(\nu = 1\,\text{GeV}, m_c) = 1.12155$$
$$c_F(m_b) \equiv c_F(\nu = 1\,\text{GeV}, m_b) = 0.87897$$

$$A' = A\frac{c_F(m_b)m_c}{c_F(m_c)m_b} \qquad B' = B\frac{c_F(m_b)m_c}{c_F(m_c)m_b}$$

$$g\Lambda' \sim -59\,\text{MeV}$$
$$g\Lambda''' \sim \pm 230\,\text{MeV}$$

$$A' = -0.017 \pm 0.002 \text{GeV}$$
$$B' = 0.0028 \pm 0.0001 \text{GeV}^3$$

$$\kappa \simeq 0.187 \text{GeV}^2$$

$$\chi^2/\text{dof} = 0.582$$