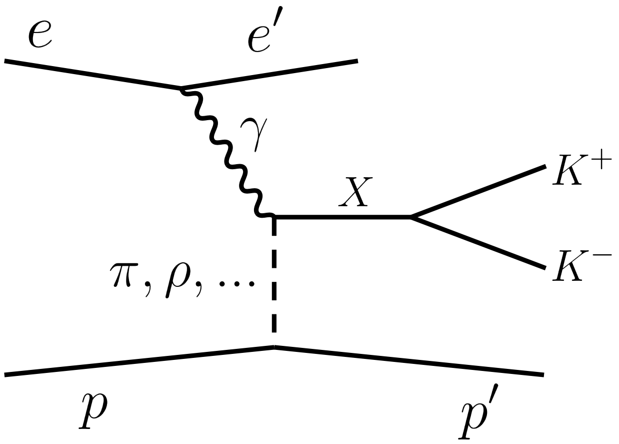
Moments of Angular Distribution of K^+K^- with CLAS12

Charlie Velasquez

Reaction of Interest

The reaction of interest is $ep \rightarrow epX \rightarrow e'pK^+K^-$ and is shown below.



Resonances

Resonances	Mass (MeV)	Width (MeV)	Spin
$\phi(1020)$	1019.5	4.3	1
$a_2(1320)$	1318.2	107.8	2
$f_0(1500)$	1522	108	0
$f'_2(1525)$	1517.3	84.4	2
$a_2(1700)$	1706	380	2

Reaction of Interest

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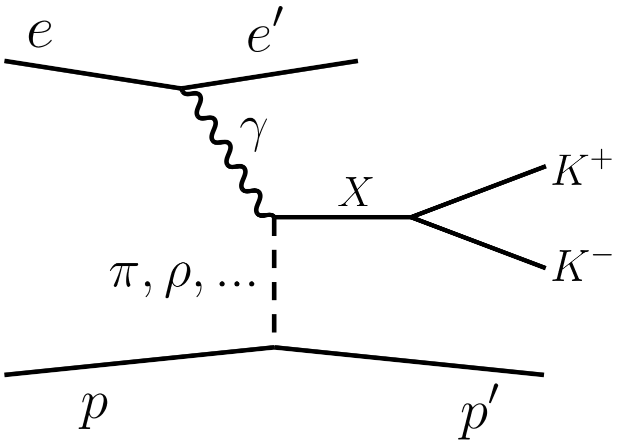
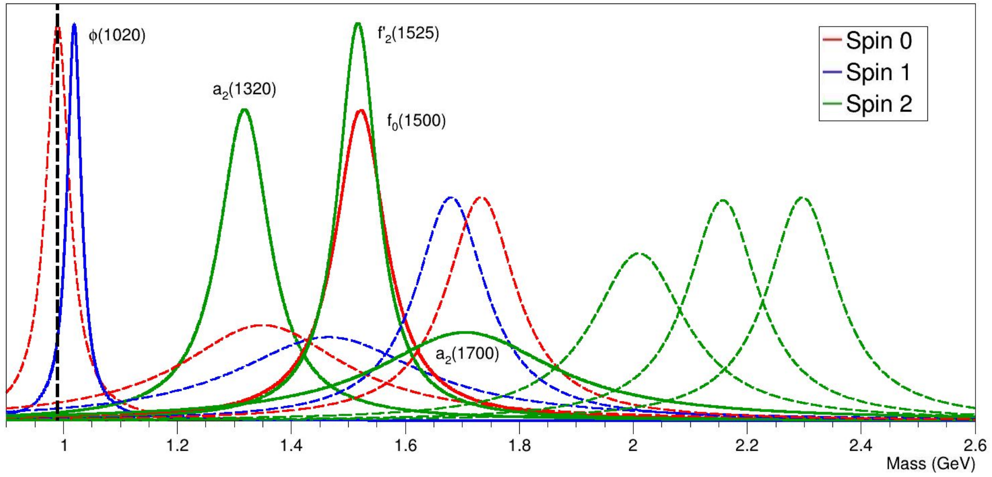
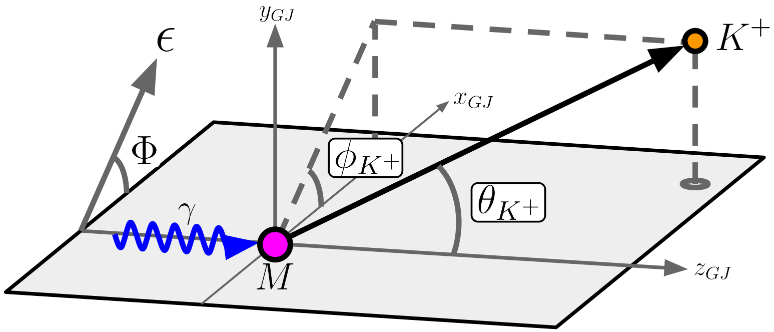


Resonances



Reaction of Interest

The reaction of interest is ep→epX→e'pK$^+$K$^-$ and is shown below.



$$I(\Omega,\Phi) = \frac{\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}m_{\eta\pi^0}\,\mathrm{d}\Omega\,\mathrm{d}\Phi}$$

$$= \sum_{\lambda} A_\lambda(\Omega)\rho(\Phi)A^*_\lambda(\Omega)$$

$$A_\lambda = \sum_{lm} T^l_{\lambda m} Y^m_l$$

Moments of Angular Distribution

$$I(\Omega, \Phi) = \frac{\mathrm{d}\sigma}{\mathrm{d}t\mathrm{d}m_{\eta\pi^0}\mathrm{d}\Omega\mathrm{d}\Phi} = I^0(\Omega) + \boldsymbol{I}(\Omega) \cdot \boldsymbol{P}_\gamma(\Phi)$$

$$I^0(\Omega) = \sum_{L,M\geqslant 0} \left(\frac{2L+1}{4\pi}\right) \tau(M) \underbrace{H^0(LM)}_{\text{moments}} \underbrace{d^L_{M0}(\theta) \cos M\phi}_{Y_l^m(\Omega)}$$

Moments of Angular Distribution

UNIVERSITY of York

$$I^0(\Omega) = \sum_{L,M\geqslant 0} \left(\frac{2L+1}{4\pi}\right) \tau(M) H^0(LM) d^L_{M0}(\theta) \cos M\phi$$

For L = 0, 1 and 2

$$I^0 = \frac{1}{4\pi} H^0(00) + \frac{3}{4\pi} H^0(10) \cos(\theta) - \frac{3\sqrt{(2)}}{4\pi} H^0(11) \sin(\theta)\cos(\phi) + \frac{5}{8\pi} H^0(20) (3\cos^2(\theta) - 1) - \frac{5\sqrt{(6)}}{16\pi} H^0(21) \sin(2\theta) cos(\phi) + \frac{5\sqrt{(6)}}{16\pi} H^0(22) \sin^2(\theta)\cos(2\phi)$$

Moments of Angular Distribution

The relation between moments can be written in terms of the partial wave amplitudes T^l, for example,

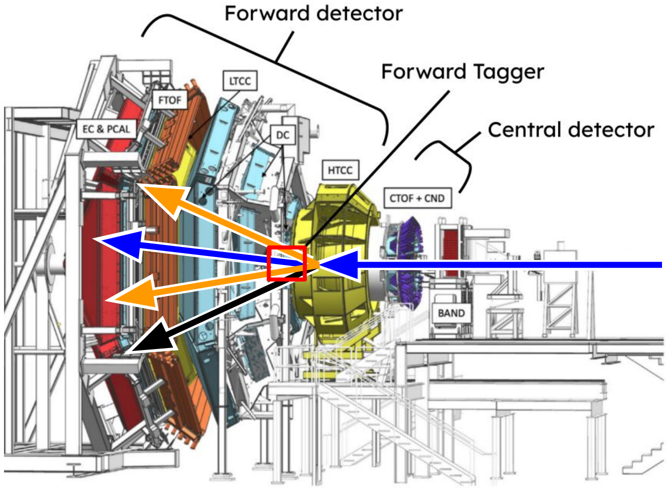
$$H^0(00) = H^1(00) + 2\left[|P_1^{(+)}|^2 + |D_1^{(+)}|^2 + |D_2^{(+)}|^2\right]$$

$$H^1(00) = 2\left[|S_0^{(+)}|^2 + |P_0^{(+)}|^2 + |D_0^{(+)}|^2\right]$$

$$H^0(10) = H^1(10) + \frac{4}{\sqrt{5}}\mathrm{Re}(P_1^{(+)} D_1^{(+)*})$$

CLAS12 at Jefferson Lab

- 10.2 GeV electron beam
- Stationary liquid hydrogen target
- Electron detected in range 2.5-5°
- Hadrons detected in range 5-125°
- Measures energy and time-of-flight of particles for identification



Event Selection

An event is a group of particles produced in an electron-proton interaction.

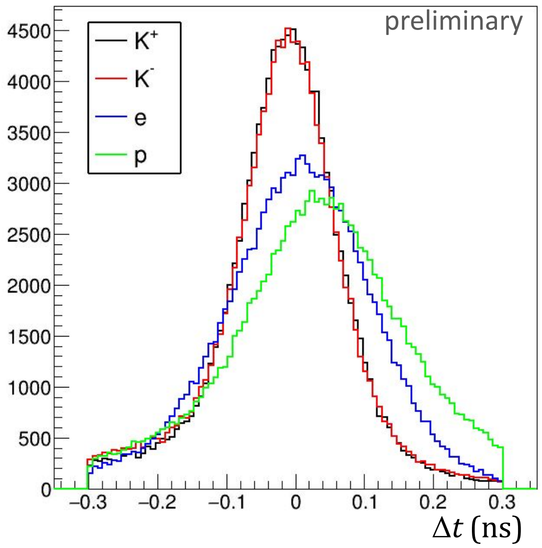
Event are chosen based on the following criteria:

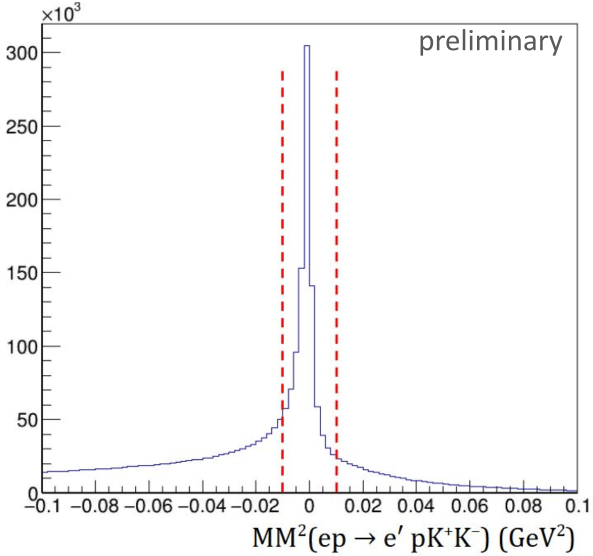
- One electron, one proton and two kaons
- Time-of-flight difference less than 0.3 ns
- Electron energy less than 5 GeV
- Kaons detected in forward part of CLAS12
- Missing mass of the event near zero

Event Selection

$$\Delta t = TOF - \frac{d}{c}\frac{p}{\sqrt{p^2 + m^2}}$$

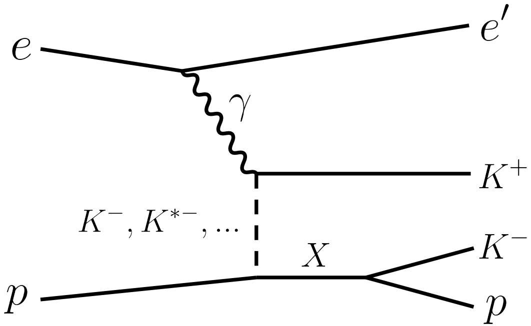
$$K^{-}X) = |P_b + P_t|^2 - |P_{e'} + P_p + P_{K^+} + P_{K^-}|^2$$

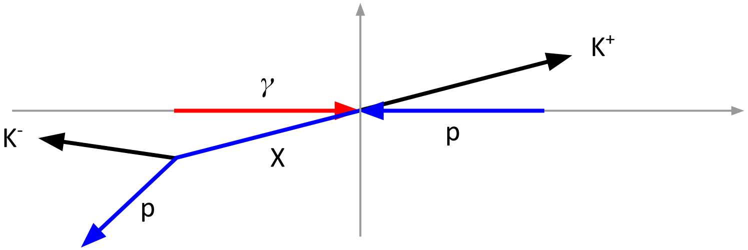


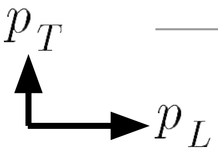


Baryon Resonances

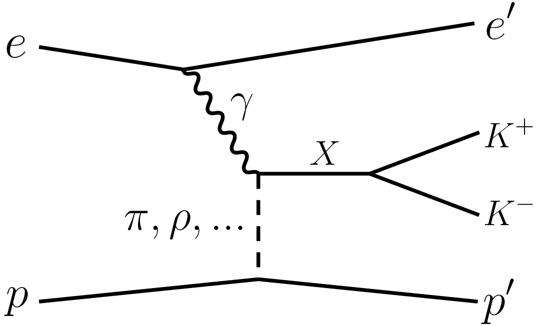
Baryon Resonance

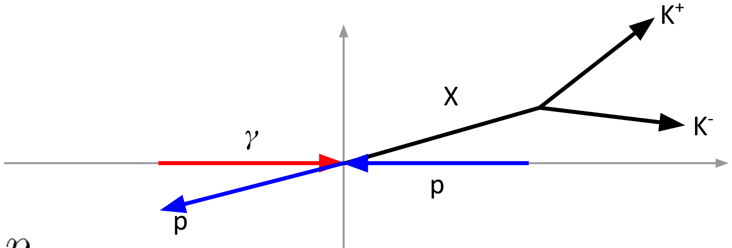






Meson Resonance





Longitudinal Phase Space Plot

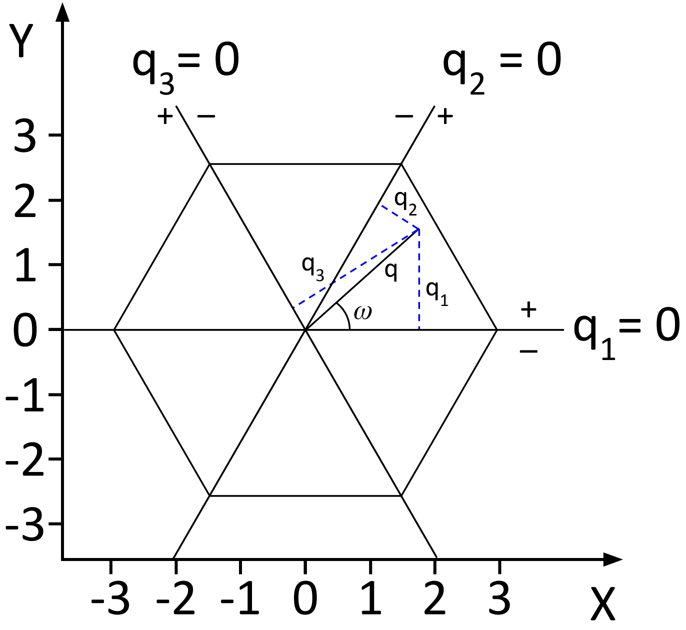
The longitudinal momenta are,

$$q_1 = \sqrt{\frac{2}{3}} q \sin(\omega), \quad q_2 = \sqrt{\frac{2}{3}} q \sin\left(\omega + \frac{2}{3}\pi\right), \quad q_3 = \sqrt{\frac{2}{3}} q \sin\left(\omega + \frac{4}{3}\pi\right),$$

The longitudinal phase space (LPS) polar coordinates, q and ω, and Cartesian coordinates, X and Y, are,

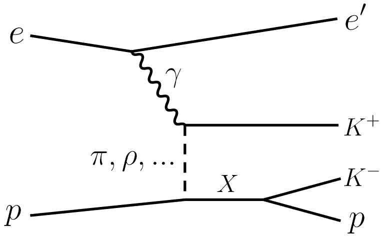
$$q = (q_1^2 + q_2^2 + q_3^2)^{\frac{1}{2}}, \qquad \omega = tan^{-1}\left(\frac{-\sqrt{3}q_1}{2q_2 + q_1}\right)$$

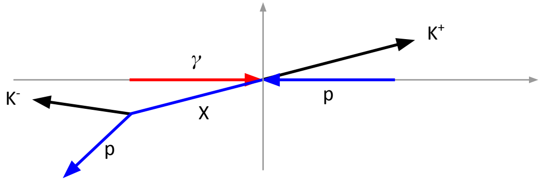
$$X = q\cos(\omega), \qquad Y = q\sin(\omega).$$

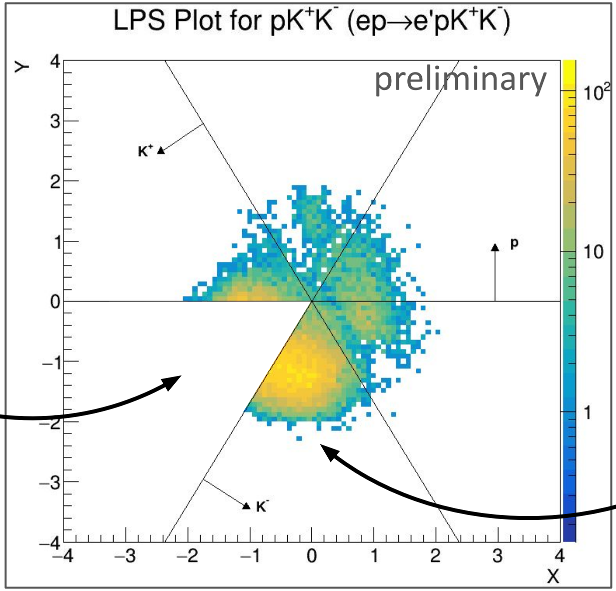


Baryon Resonances

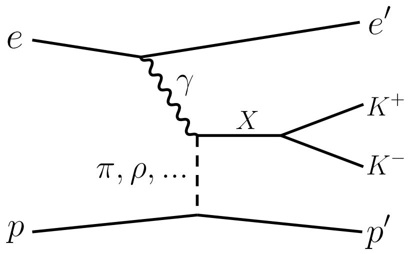
Baryon resonances

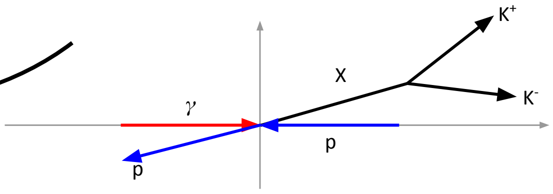




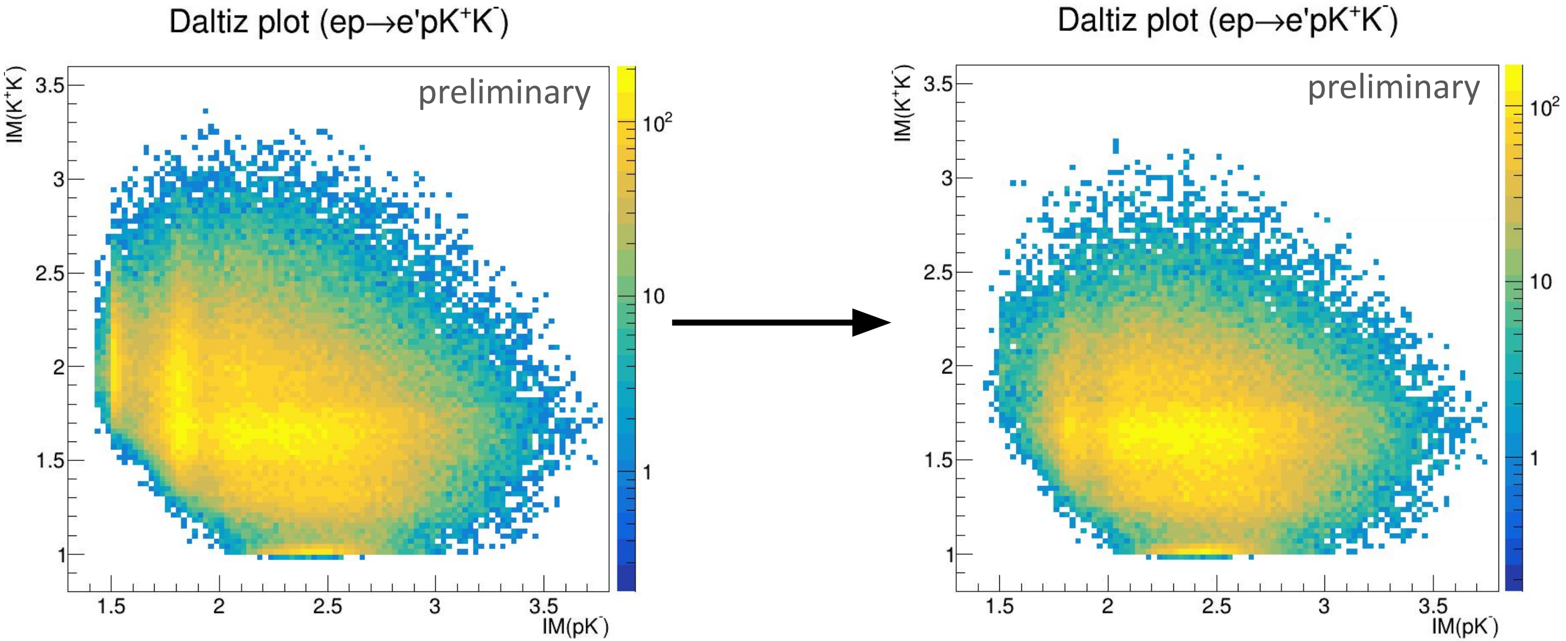


Meson resonances



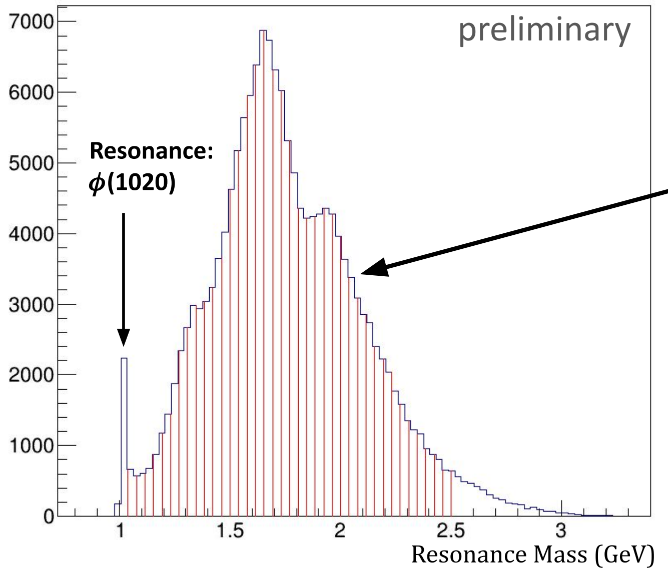


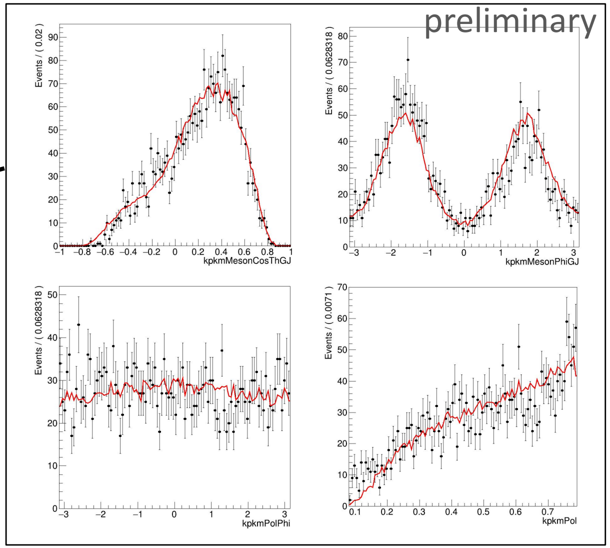
Baryon Resonances



Fitting Angular Distributions

Intensity function is fitted to angular distributions in each bin using brufit which employs an MCMC fitting algorithm.

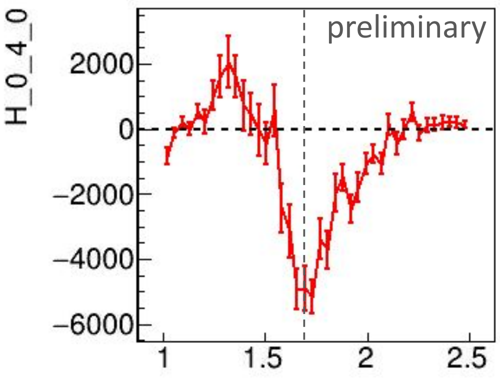




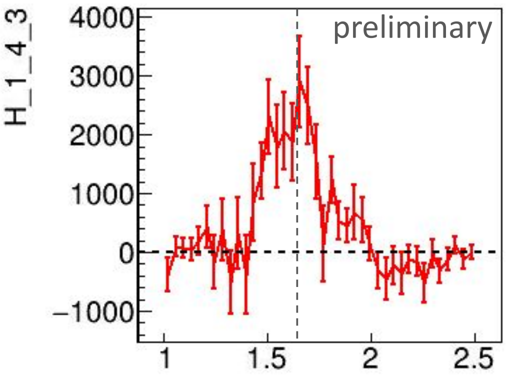
Projections of intensity function (red) on data (black)

Extracted Moments

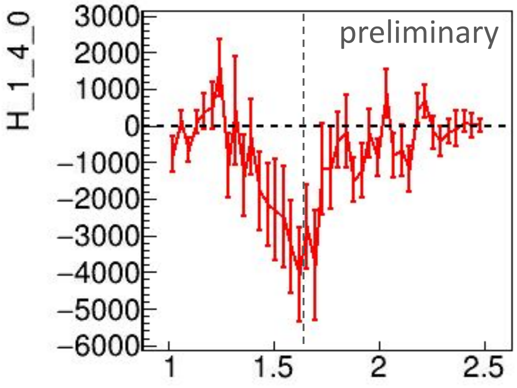
Structures seen in moments with dominant D wave contributions, however conclusions cannot be drawn yet.



$$H_0(4,0) = \frac{2}{21}\sum_{\epsilon}\Big(|D^{\epsilon}_{+2}|^2 - 4|D^{\epsilon}_{+1}|^2 + 6|D^{\epsilon}_{0}|^2 - 4|D^{\epsilon}_{-1}|^2 + |D^{\epsilon}_{-2}|^2\Big),$$



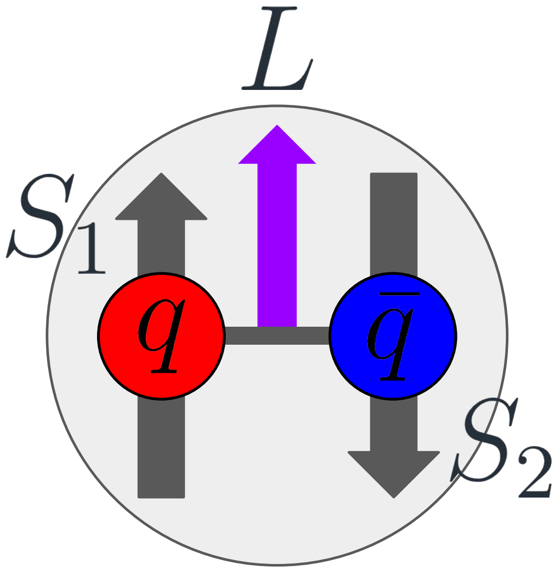
$$H_1(4,3) = \frac{2\sqrt{35}}{21}\sum_{\epsilon}\epsilon\Big(\mathrm{Re}[D^{\epsilon}_{+2}(D^{\epsilon}_{+1})^*] - \mathrm{Re}[D^{\epsilon}_{-1}(D^{\epsilon}_{-2})^*]\Big)$$



$$H_1(4,0) = \frac{4}{21}\sum_{\epsilon}\epsilon\Big(+\mathrm{Re}[D^{\epsilon}_{+2}(D^{\epsilon}_{-2})^*] + 4\mathrm{Re}[D^{\epsilon}_{+1}(D^{\epsilon}_{-1})^*] + 3|D^{\epsilon}_{0}|^2\Big)$$

Thank you for listening

The Quark Model



Meson Spin $J = L + S$

		J^{PC}	$I=1$	$I=0\ (n\bar{n})$	$I=0\ s\bar{s}$	Strange
$L=0$	$S=0$	0^{-+}	π	η	η'	K
	$S=1$	1^{--}	ρ	ω	ϕ	K^*
$L=1$	$S=0$	1^{+-}	b_1	h	h'	K_1
	$S=1$	0^{++}	a_0	f_0	f_0'	K_0^*
		1^{++}	a_1	f_1	f_1'	K_1
		2^{++}	a_2	f_2	f_2'	K_2^*
$L=2$	$S=0$	2^{-+}	π_2	η_2	η_2'	K_2
	$S=1$	1^{--}	ρ	ω	ϕ	K_1^*
		2^{--}	ρ_2	ω_2	ϕ_2	K_2
		3^{--}	ρ_3	ω_3	ϕ_3	K_3^*
⋮	⋮	⋮	⋮	⋮	⋮	⋮

Resonances

Moments of Angular Distribution

$$I(\Omega,\Phi) = \frac{\mathrm{d}\sigma}{\mathrm{d}t\mathrm{d}m_{\eta\pi^0}\mathrm{d}\Omega\mathrm{d}\Phi}$$

$$= \kappa \sum_{\substack{\lambda,\lambda' \\ \lambda_1,\lambda_2}} A_{\lambda;\lambda_1\lambda_2}(\Omega)\rho^{\gamma}_{\lambda\lambda'}(\Phi)A^*_{\lambda';\lambda_1\lambda_2}(\Omega)$$

$$= I^0(\Omega) + \boldsymbol{I}(\Omega)\cdot\boldsymbol{P}_\gamma(\Phi)$$

$$A_{\lambda;\lambda_1\lambda_2}(\Omega) = \sum_{\ell m} T^{\ell}_{\lambda m;\lambda_1\lambda_2} Y^m_\ell(\Omega)$$

$$I^0(\Omega) = \frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2} A_{\lambda;\lambda_1\lambda_2}(\Omega)A^*_{\lambda;\lambda_1\lambda_2}(\Omega),$$

$$I^1(\Omega) = \frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2} A_{-\lambda;\lambda_1\lambda_2}(\Omega)A^*_{\lambda;\lambda_1\lambda_2}(\Omega),$$

$$I^2(\Omega) = i\frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2} \lambda A_{-\lambda;\lambda_1\lambda_2}(\Omega)A^*_{\lambda;\lambda_1\lambda_2}(\Omega).$$

Moments of Angular Distribution

$$\boldsymbol{H}(LM) = -\sum_{\substack{\ell\ell' \\ mm'}} \left(\frac{2\ell'+1}{2\ell+1}\right)^{1/2} C^{\ell 0}_{\ell' 0 L 0} C^{\ell m}_{\ell' m' L M}\ \boldsymbol{\rho}^{\ell\ell'}_{mm'}$$

$$\rho^{\alpha,\ell\ell'}_{mm'} = \frac{\kappa}{2} \sum_{\lambda,\lambda_1,\lambda_2} T^{\ell}_{\lambda m;\lambda_1\lambda_2} \sigma^{\alpha}_{\lambda\lambda'} T^{\ell' *}_{\lambda' m';\lambda_1\lambda_2}$$

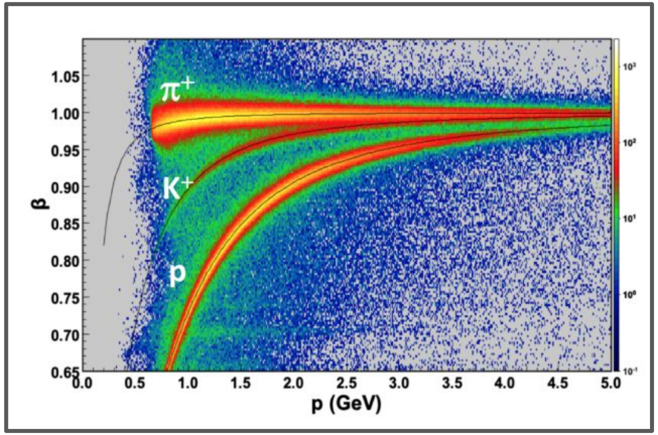
For example:

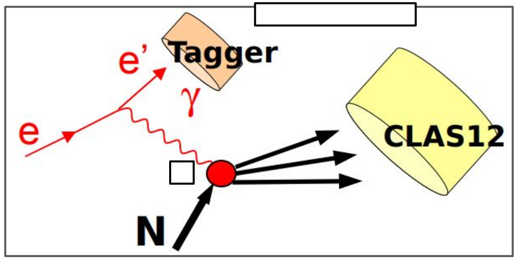
$$H^1(00) = 2\left[|S_0^{(+)}|^2 + |P_0^{(+)}|^2 + |D_0^{(+)}|^2\right]$$

$$H^0(LM) = \frac{P_\gamma}{2} \int_\circ I(\Omega,\Phi)\, d^L_{M0}(\theta) \cos M\phi,$$

$$H^1(LM) = \int_\circ I(\Omega,\Phi)\, d^L_{M0}(\theta) \cos M\phi\ \cos 2\Phi,$$

CLAS12 at Jefferson Lab





Forward time-of-flight average timing resolution: 85-155 ps

Parameter	Current	Resolution
$\Delta p/p$ (%)	0 nA	0.52
	60 nA	0.67
	120 nA	0.86
$\Delta\phi$ (mrad)	0 nA	3.3
	60 nA	3.8
	120 nA	4.4
$\Delta\theta$ (mrad)	0 nA	0.66
	60 nA	0.85
	120 nA	0.85
Δv_z (mm)	0 nA	3.5
	60 nA	4.6
	120 nA	5.6

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Event Selection

$$\beta_1 = \frac{v}{c} = \frac{d}{ct} \qquad \beta_2 = \frac{p}{E} = \frac{p}{\sqrt{p^2+m^2}} \qquad t = \frac{d}{c\beta} = \frac{d}{c}\frac{\sqrt{p^2+m^2}}{p}$$

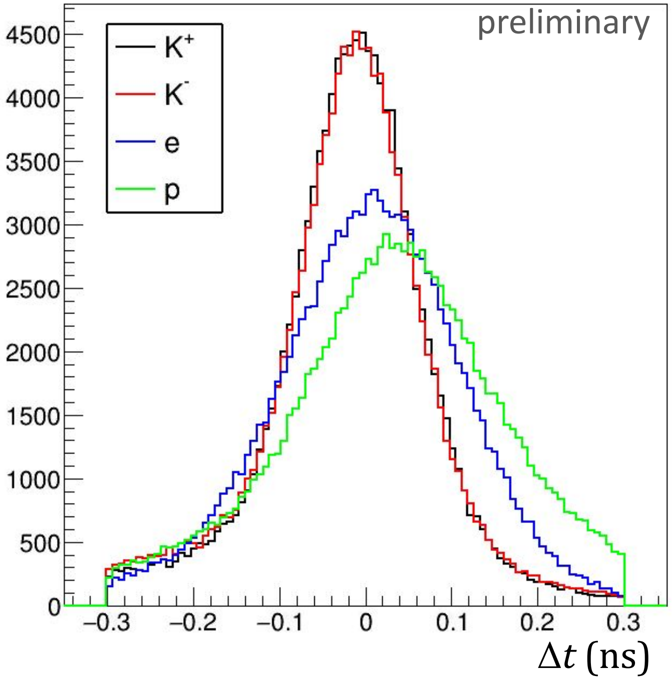
$$\begin{aligned} MM^2(ep \to e' p K^+ K^- X) &= |P_b + P_t|^2 - |P_{e'} + P_p + P_{K^+} + P_{K^-}|^2 \\ &= (E_b + m_t - E_{e'} - E_p - E_{K^+} - E_{K^-})^2 \\ &\quad - |\bar{p}_b + \bar{p}'_e - \bar{p}_p - \bar{p}_{K^+} - \bar{p}_{K^-}|^2 \end{aligned}$$

Event Selection

$$\Delta t = TOF - \frac{d}{c}\frac{p}{\sqrt{p^2 + m^2}}$$

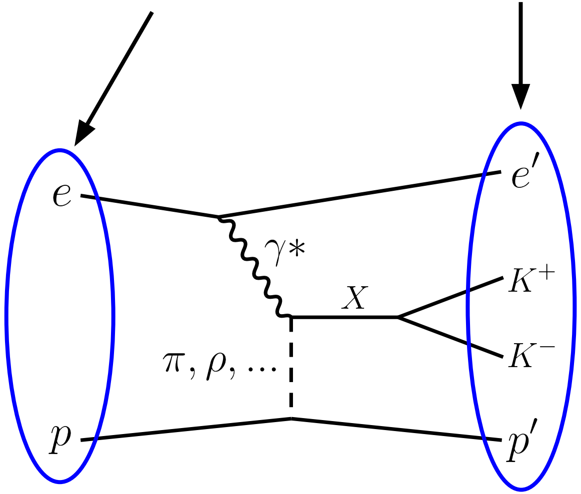
Time-of-flight measured by forward time-of-flight detector

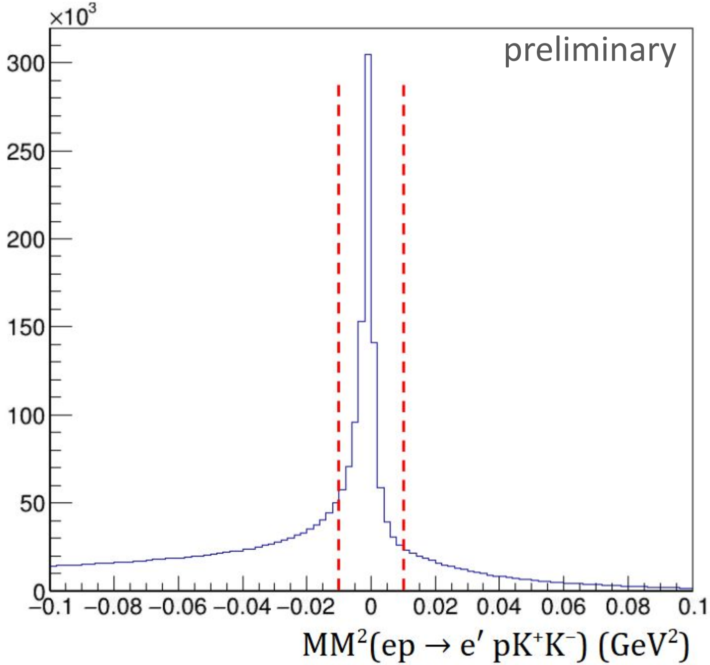
Time-of-flight calculated using distance travelled *d*, momentum *p* (from drift chambers) and assumed mass *m*



Event Selection

$$MM^2 = |P_b + P_t|^2 - |P_{e'} + P_p + P_{K^+} + P_{K^-}|^2$$





Fitting

Fitting uses a Monte-Carlo Markov Chain (MCMC) algorithm:

- Random steps (chain property) in parameter space generated only using information of previous step (Markov property) to find global minimum/maximum

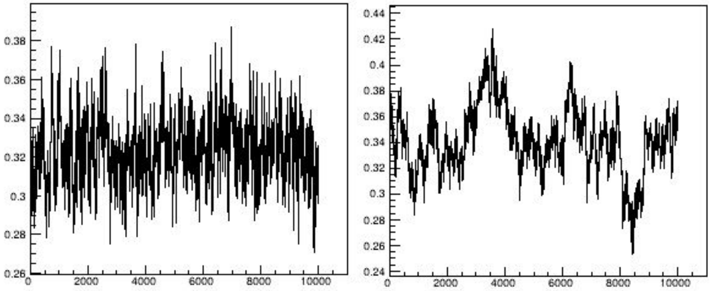
$$-\ln L^{ext}_{acc}(p) \;\propto\; -\sum_{i}^{N} \ln f(\tau_i : p)\,\eta(\tau_i) + A(p)$$

Monte-carlo normalisation

$$A(p) \;\simeq\; \sum_{j}^{M} f(\tau_j : p)$$

https://indico.jlab.org/event/829/contributions/14273/attachments/10776/16321/PolarisedTwoPion.pdf

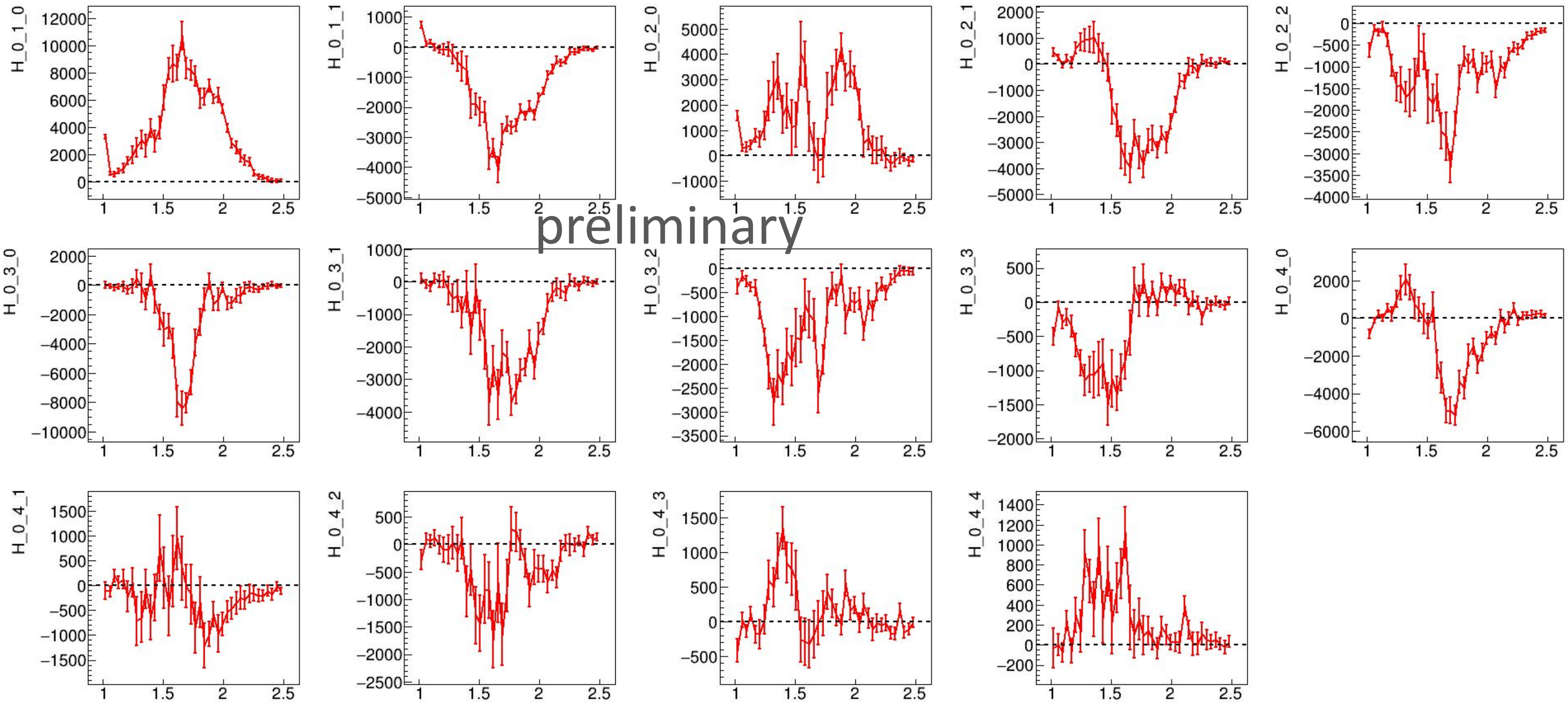
Example: MCMC chain for same parameter, different data sets



Small error bars

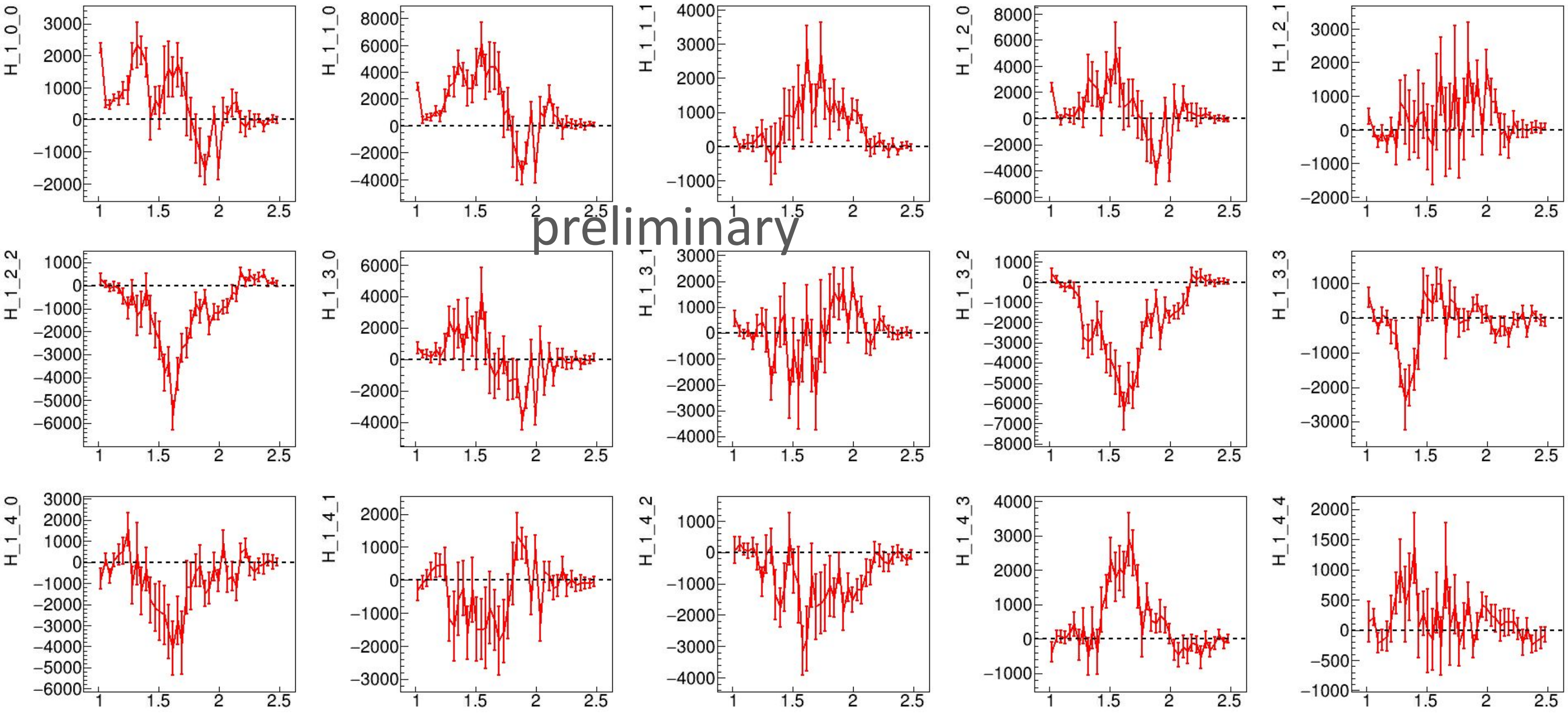
Large error bars

H0 Moments



H_0_1_0
H_0_1_1
H_0_2_0
H_0_2_1
H_0_2_2
preliminary
H_0_3_0
H_0_3_1
H_0_3_2
H_0_3_3
H_0_4_0
H_0_4_1
H_0_4_2
H_0_4_3
H_0_4_4

H0 Moments


H_1_0_0
H_1_1_0
H_1_1_1
H_1_2_0
H_1_2_1
preliminary
H_1_2_2
H_1_3_0
H_1_3_1
H_1_3_2
H_1_3_3
H_1_4_0
H_1_4_1
H_1_4_2
H_1_4_3
H_1_4_4