A 3-body formalism for determining the nature of X(3872)

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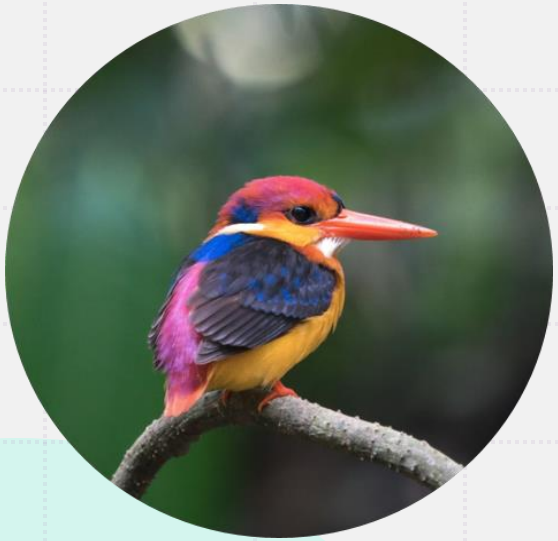
Outline

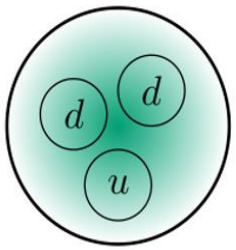
- The exotic nature of the X(3872)
- 3-body formalisms developed:
  - N/D version I
  - Reasonable models
- Conclusions

The advent of the exotics

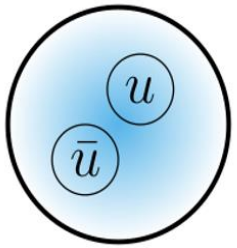


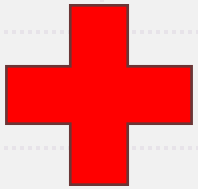
- Before **2003**: mesons as q-anti q and baryons as qqq
- But then: **X(3872)** first particle beyond ordinary quark model & a plethora of other "exotics" discovered since then (supernumerary states, spin-parity not allowed, **cryptoexotics**, explicit quark content from different flavors)

Ordinary baryon

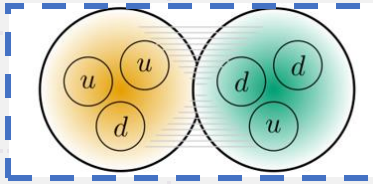


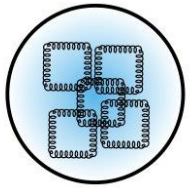
Ordinary meson



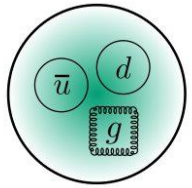


Hadronic molecules

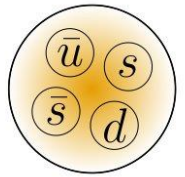




glueball



hybrid



tetraquark

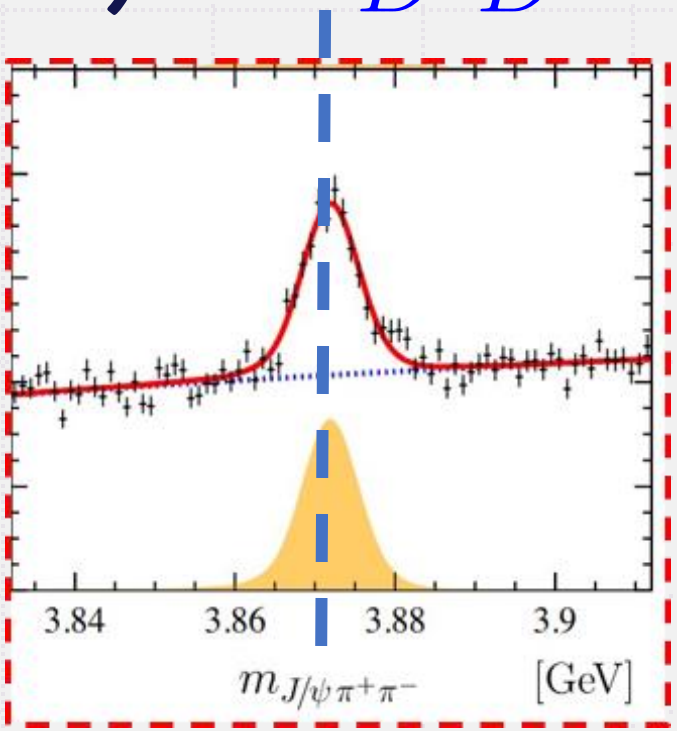
The "exotic-ness" of the X(3872)

It has allowed quantum numbers (axial vector, **$1^{++}(0^+)$**) but:

- Proximity with **D^0D^{0*} threshold**
- **Isospin violation** larger than expected for an ordinary charmonium

$$X \to J/\psi\ \omega$$
$$X \to J/\psi\ \rho$$

$$I_\omega = 0$$
$$I_\rho = 1$$
$$I_{J/\psi} = 0$$



$$m[X(3872)] - [m(D^{0*}) + m(\bar{D}^0)] = -0.17 \pm 0.26\ \text{MeV/c}^2$$

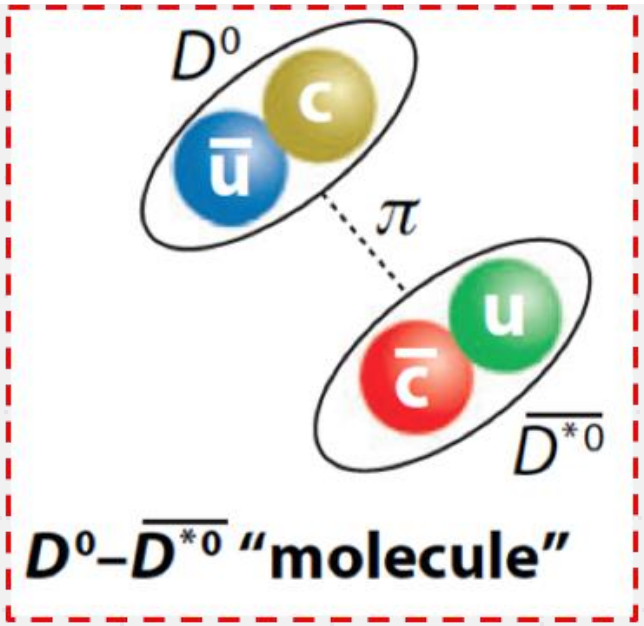
$$\frac{g_{X(3872)\to\rho^0 J/\psi}}{g_{X(3872)\to\omega J/\psi}} = 0.29 \pm 0.04 \quad >> \quad \frac{g_{\psi(2S)\to\pi^0 J/\psi}}{g_{\psi(2S)\to\eta J/\psi}} = 0.045 \pm 0.001$$

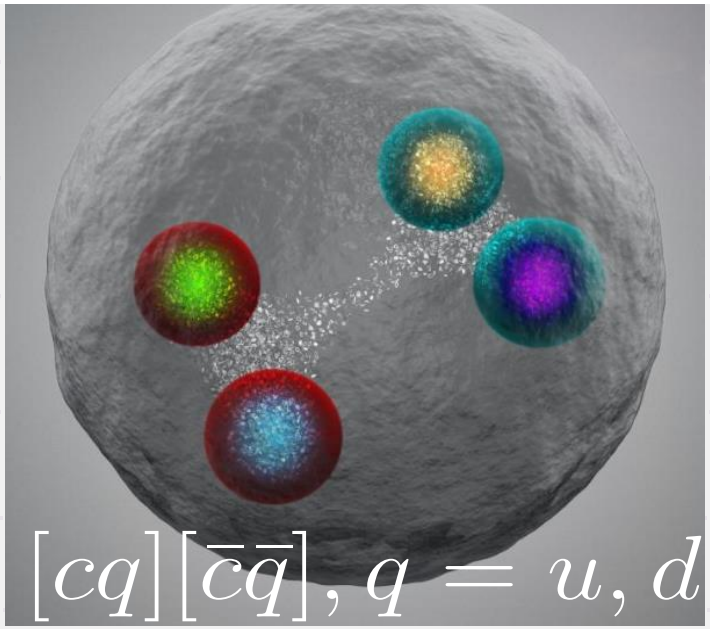
$[\ldots][\ldots], q = u, d$

Viable explanations on the market

In principle the peculiar behavior of the X(3872) can be explained in many ways, like the **diquark-antidiquark picture, triangle singularities** etc. Maybe it's a mixture of them. It should have a large ordinary **charmonium** component. Our interest is to concentrate on the **molecular view**, to prove if this component alone is enough to produce a bound state

$$|X(3872)\rangle = c_1|D^0\bar{D}^{*0}\rangle + c_2|J/\psi\pi\pi\rangle + c_3|D^+\bar{D}^{*-}\rangle + c_4|c\bar{c}\rangle + \dots$$



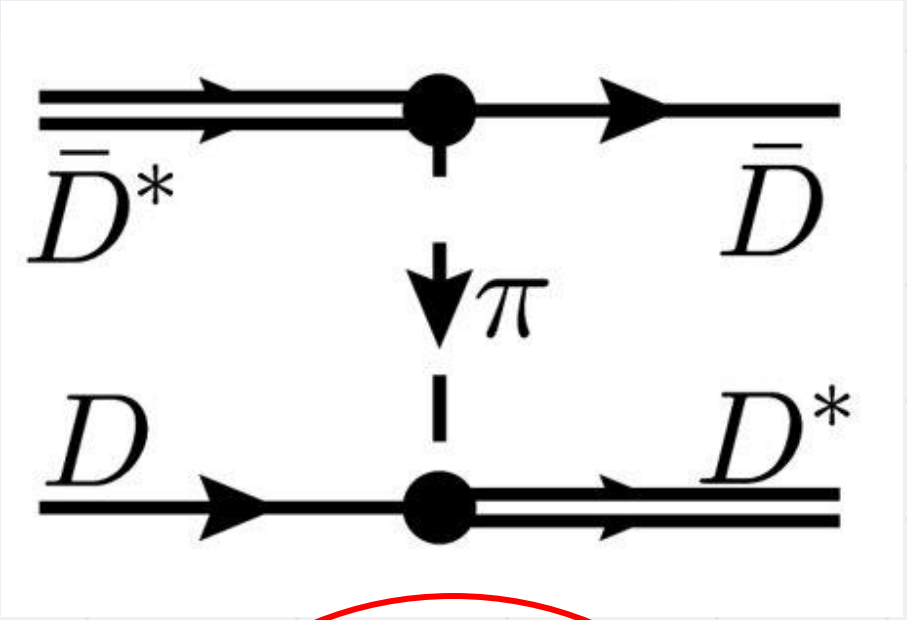


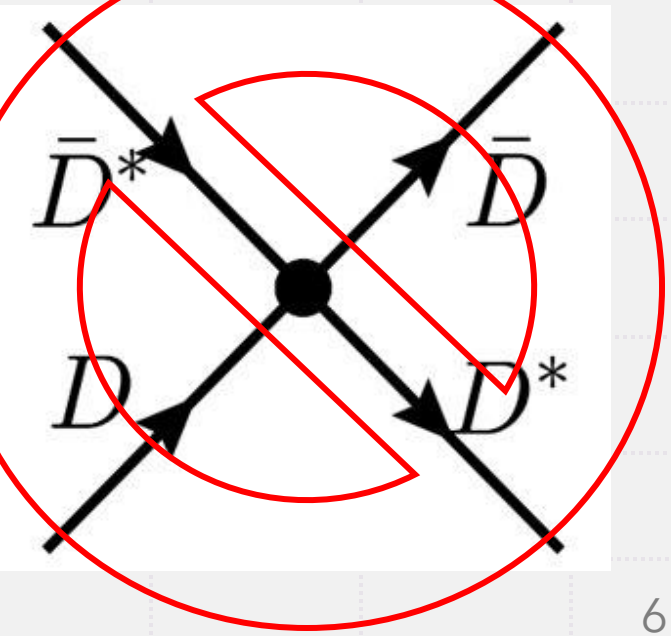
Can π exchange alone bind D and D*?

Pursued approaches to tackle this problem:

- Effective field theories
- NREL quark model (e.g. in Born-Oppenheimer approximation)
- EFT-inspired- and potential models on lattice
- Schwinger-Dyson type equations
- ...

Among all these approaches, we resort to S-matrix theory. Usually, EFT-driven approaches use contact terms together with pion exchange, making difficult discern the actual contribution of pion exchange from that one of the short-range dynamics. Our input in this case is **OPE only, without adding "contact terms"**





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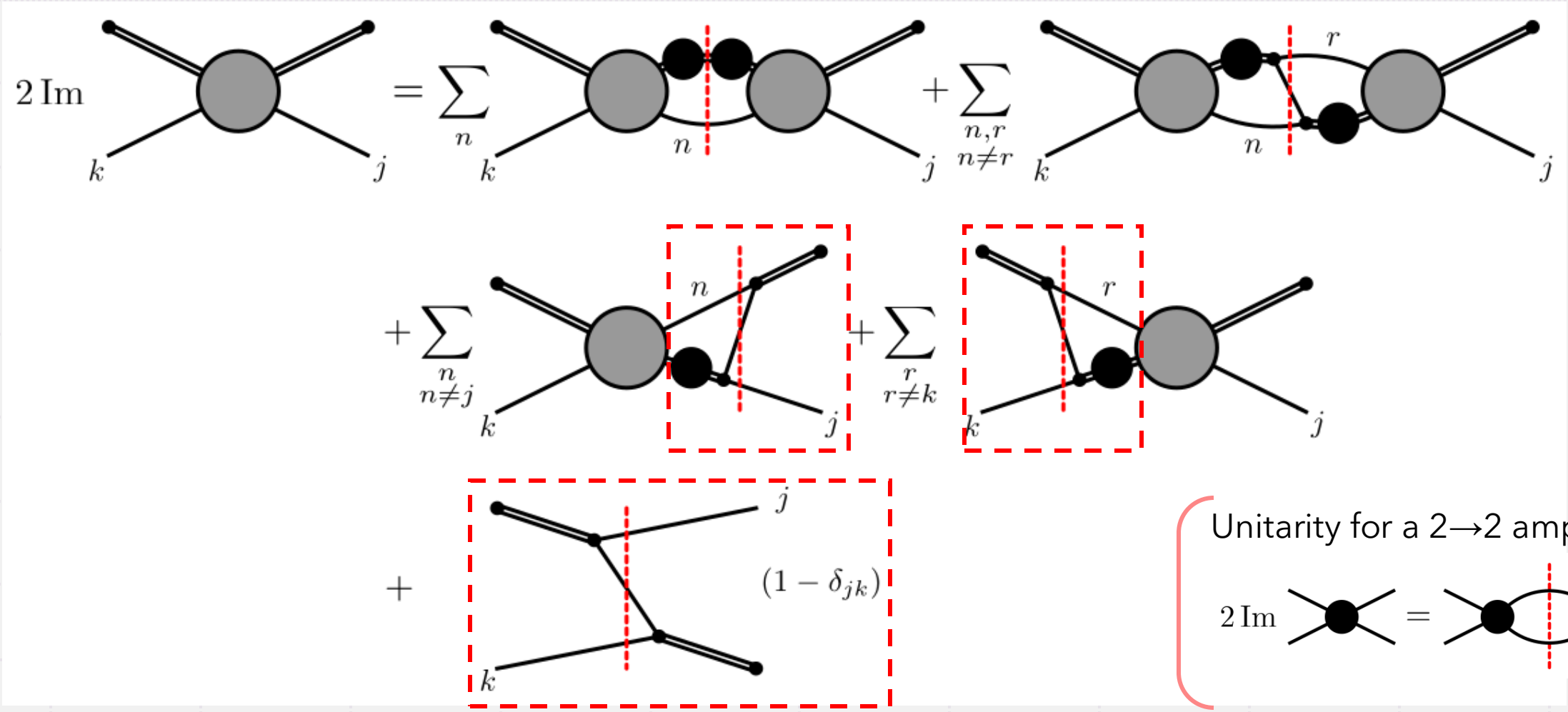
Starting point of all our formalisms: 3-body unitarity

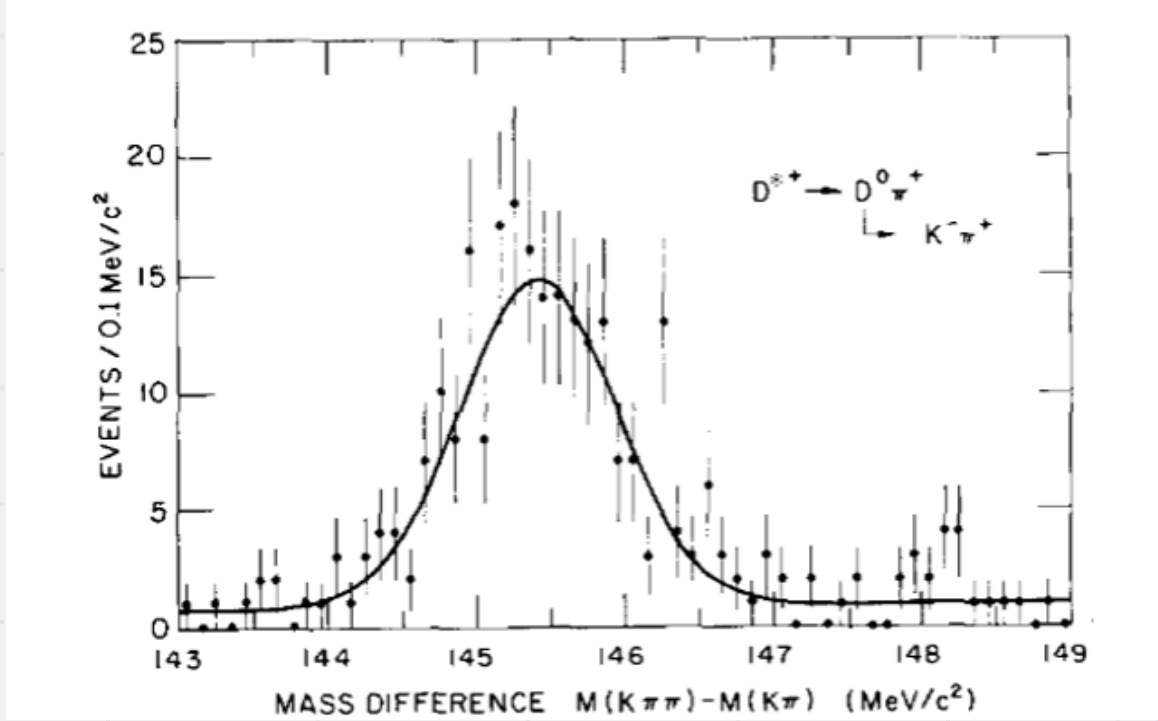


Unitarity for a 2→2 amplitude:

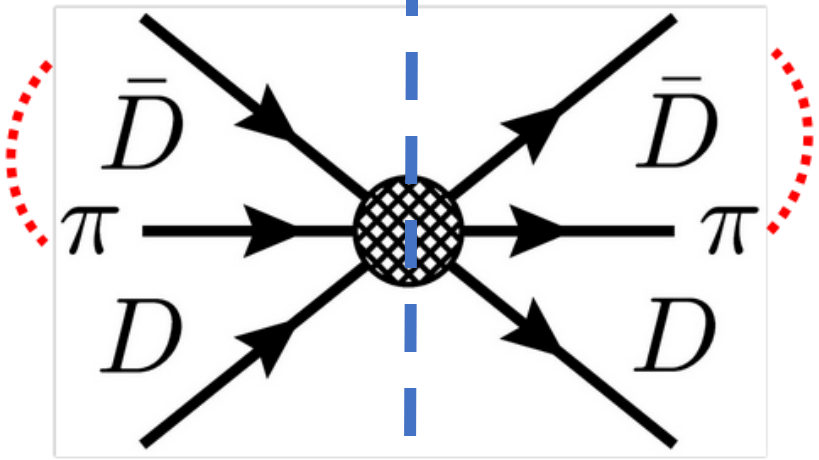
Two independent 3-body partial wave amplitudes

OPE is an explicit piece of its imaginary part of a, and this is what make it independent on b, which has instead the box as low order contribution.

We are treating them in the **isobar approximation** (D* infinitely narrow + neglect the DD subsystem interation)

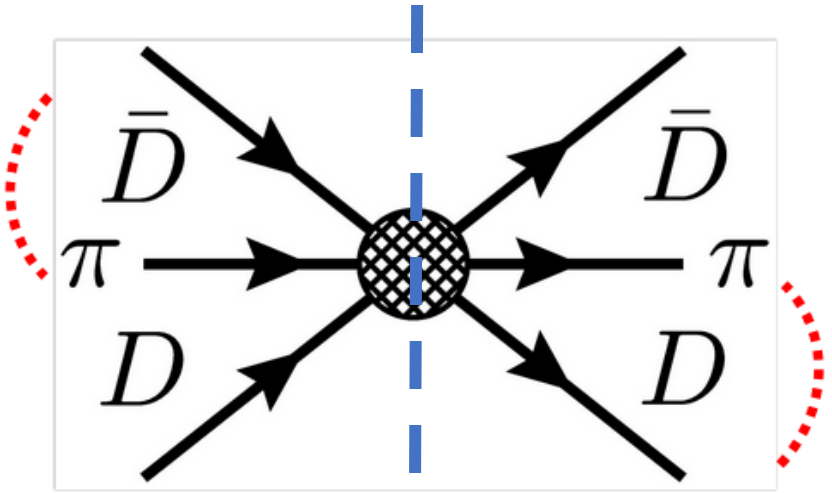


$b(\sigma', s, \sigma)$



VS

$a(\sigma', s, \sigma)$



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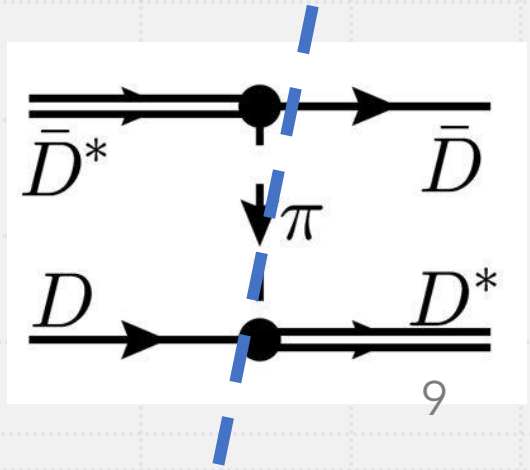
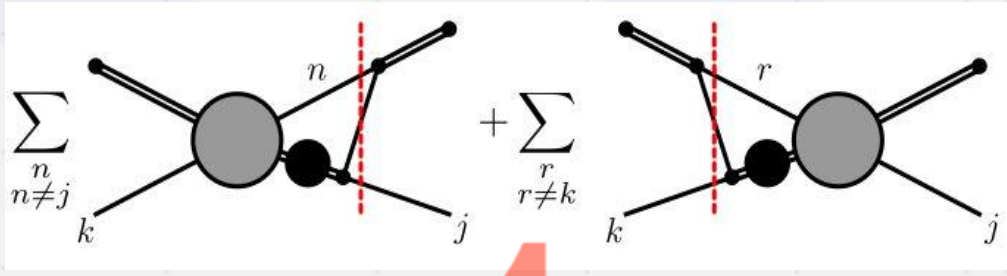
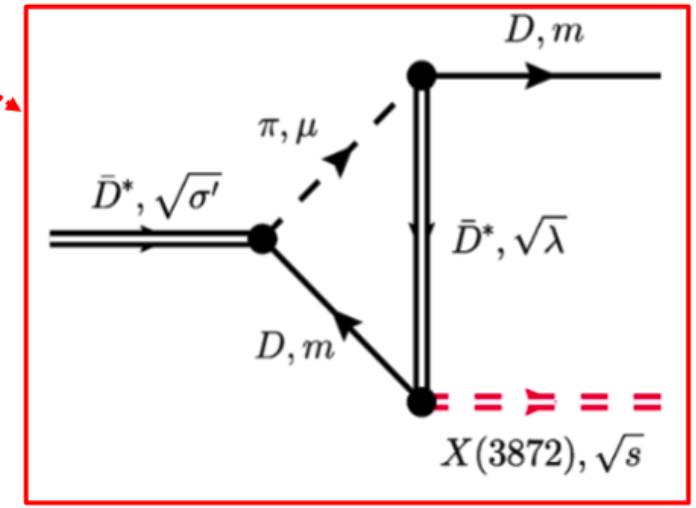


From the discontinuities we can dispersively reconstruct the full amplitudes



$$a(\sigma', s_{+-}, \sigma_+) = \int_{(m+\mu)^2} d\lambda^2 T(\sigma', s_{+-}; \lambda^2) \Delta_{\lambda^2}[\hat{f}b(\lambda^2, s, \sigma)] + C(s, \sigma)$$



$$\begin{cases} a_{\sigma'\sigma} = \sum_\lambda T_{\sigma'\lambda}[\Delta_\lambda(fb)]_{\lambda\sigma} + C_{\sigma'\sigma} \\ b_{\sigma'\sigma} = \sum_\lambda T_{\sigma'\lambda}[\Delta_\lambda(fa)]_{\lambda\sigma} + D_{\sigma'\sigma} \end{cases}$$

Now we can solve the equations with N/D method

This method is based on the assumption (mathematically proven for 2-body amplitudes) that the amplitude can be written as a ratio between a denominator that contains just the singularities governed by unitarity ("right hand singularities") and a numerator that contains all the other singularities ("left hand singularities")

$$a = \frac{N_a}{1 - D_a}, \; b = \frac{N_b}{1 - D_b}$$

Main point: in our model there are no subtraction constants (so everything is really driven by OPE only). If not, they would have act as contact terms

$$\Delta N_a = (1 - D_a)\Im \tilde{a}_{OPE}$$

$$\Delta D_a = \left(1 + \frac{N_b^2}{N_a^2}\frac{|1 - D_a|^2}{|1 - D_b|^2}\right) N_a \rho_{KR}(s) + |1 - D_a|^2 2Re\left(\frac{1 - D_a}{1 - D_b}\right) N_b \rho_{RK}(s)$$

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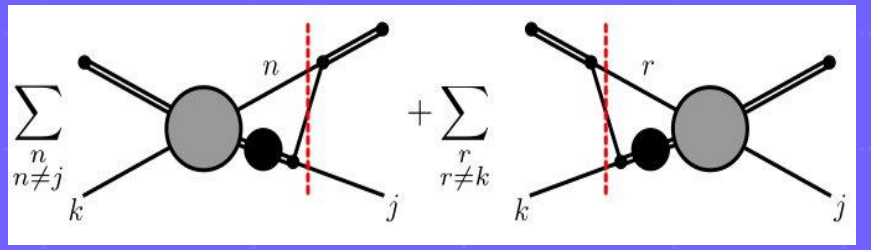
$$N_a = \tilde{a}_{OPE} + \frac{1}{\pi}\int_L ds' \frac{\Im \tilde{a}_{OPE}(s') D_a(s')}{s' - s} \qquad N_b = g^2$$

But now come the problems: spourious singularities of the pws

- In the modified phase-spaces there are modulus squares that imply the presence of pairs of complex conjugate singularities
- In particular, α-terms (α ~ ΔT ~ B=Q_0) coming from diagrams having the Im(OPE) at the endcaps, will produce short branch-cuts above and below the integration contour that run along the real axis: no possibility to avoid their pinching branch points. Then you cannot define proper denominators that have just RH singularities.



$$\rho_{KR}(s) \equiv K(s)\frac{1+|\alpha(s)|^2}{(1-|\alpha(s)|^2)^2} + R(s)\frac{2Re\alpha(s)}{(1-|\alpha(s)|^2)^2}$$

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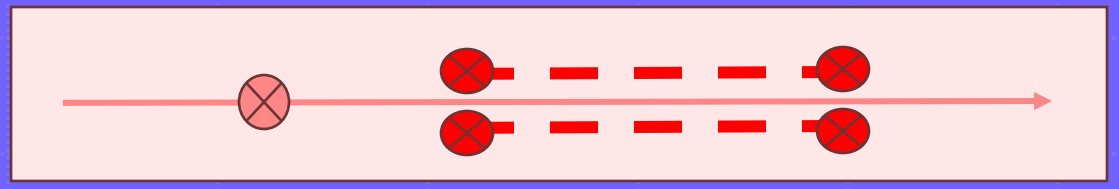
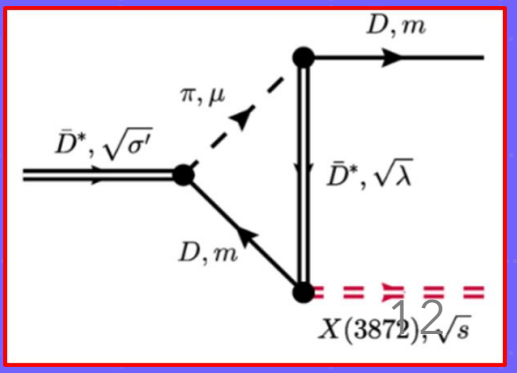
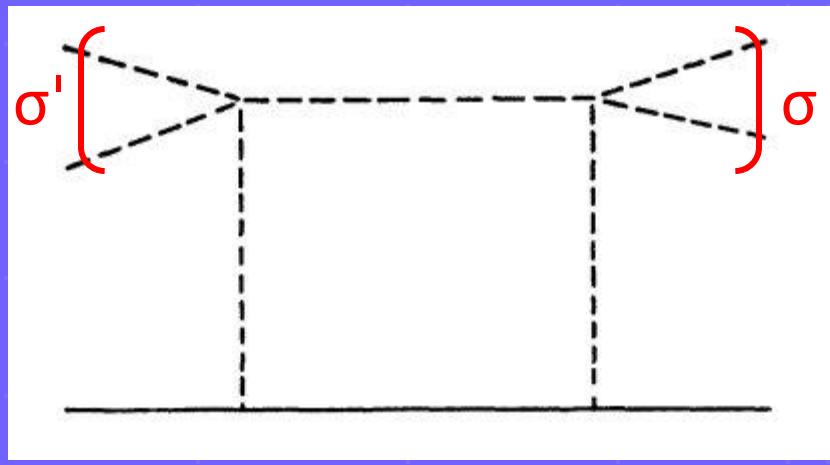


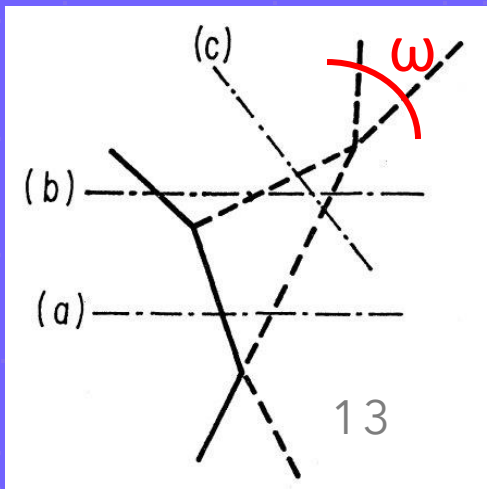


$$\alpha \equiv 2i\Delta_{\sigma'}T(\sigma' = R, s_{+-}, R_+) = 2i\rho_3(R, \mu^2, m^2)B(R, s, R)$$

But now come the problems: spourious singularities of the pws

- E.g. Box diagram [A], contributing to 3-body amplitude (b): its S-wave partial wave has some singularity in the (σ,σ') plane that the full box doesn't have
- Even in the simpler case of triangle, there's a non-trivial dependence on the invariant mass of the external isobar ω [B]
- Understand this dependence on isobars' mass is essential for writing a coherent formalism

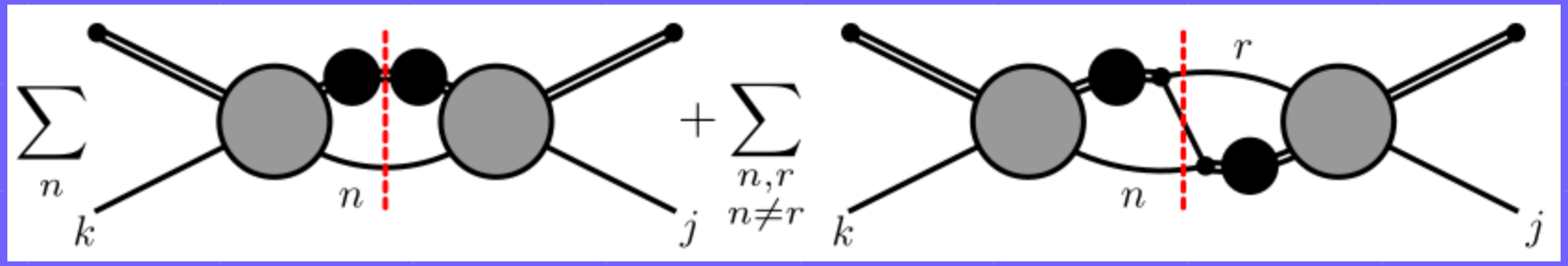


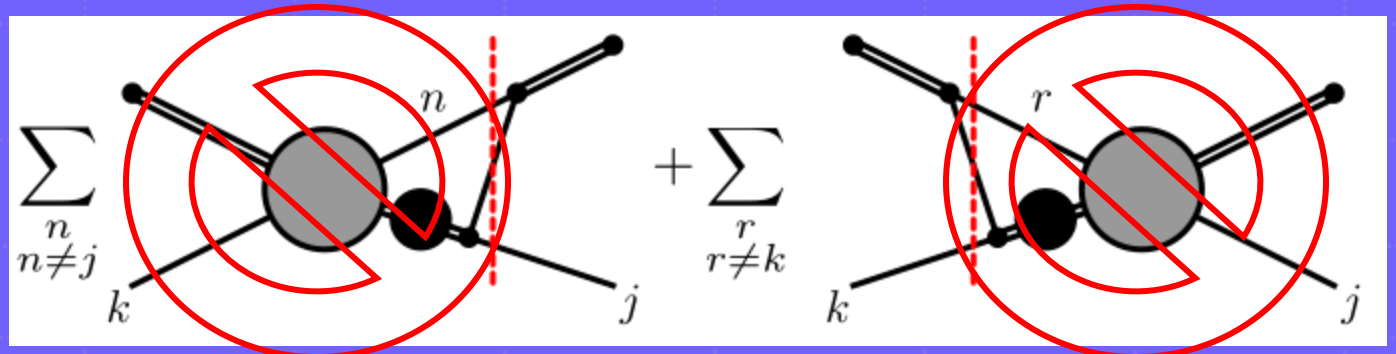


[A]: "Three-Particle Contributions to Elastic Scattering", Grisaru, 1966
[B]: "Scattering and Production Amplitudes with Unstable Particles", Ball et al., 1962

A reasonable workaround

Negleting α-terms it's a reasonable choice to avoid this spourious, unavoidable singularities, though we will loose the full generality of the procedure. It corresponds to a model choice in which we are dropping the diagrams with Im(OPE) at the endcaps from the unitarity equation, to keep just the following loop diagrams





Scheme of the numerical solution

- 0^{th} iteration: $N_a = a_{OPE}$, $N_b = g^2$, Da = 0→a = a_{OPE}, $D_b = 0 \rightarrow b = g^2$
- 1^{st} iteration: disperse ΔD_a, ΔD_b between threshold $(DD\pi)^2$ and $+\infty$ and ΔN_a on the LHC = $]-\infty,0] \cup [s_+, s_-]$
- Successive iterations until convergence (L^2 or L^∞ norm)

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Main point: in our model there are no subtraction constants (so everything is really driven by OPE only). If not, they would have act as contact terms

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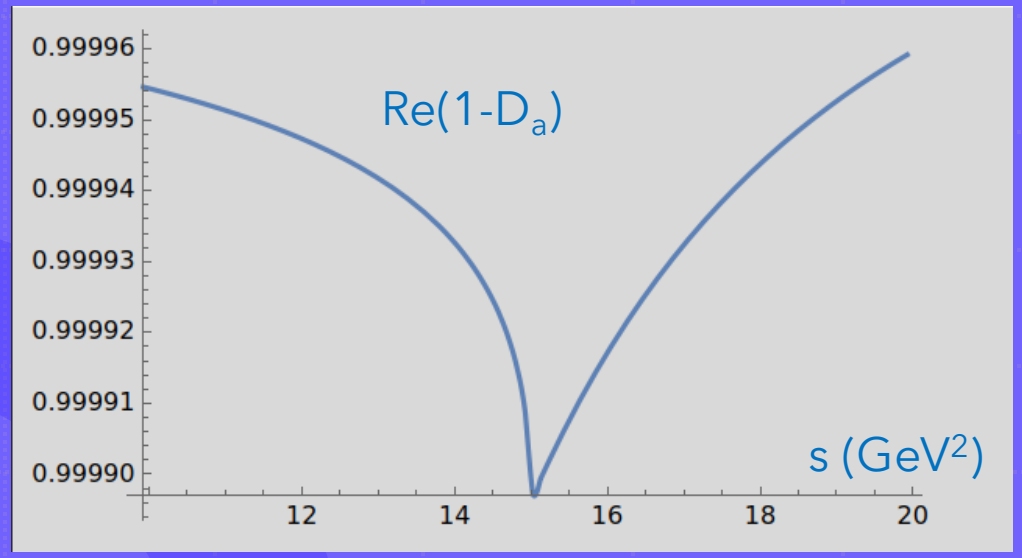
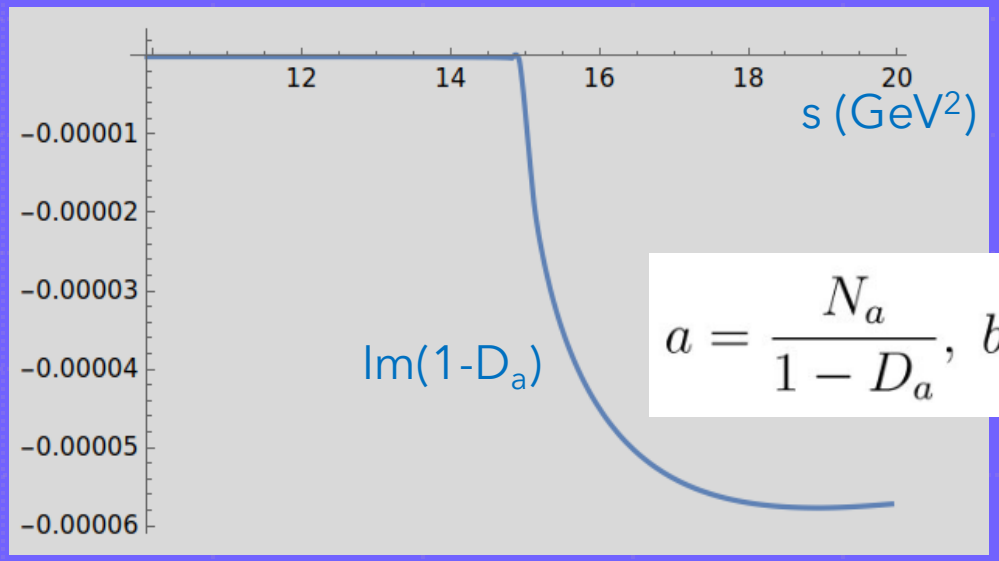
Numerical results for all-scalar particles case

- Convergence reached in few iterations
- These preliminary results of these N/D equations tell us that the one pion exchange alone cannot in any way manage to bind D and D*: the solution is just a little correction to the starting point, a_{OPE}
- We have to check still the spin case, even if our gut feeling is that nothing will change dramatically (dynamics doesn't stay in the spin)





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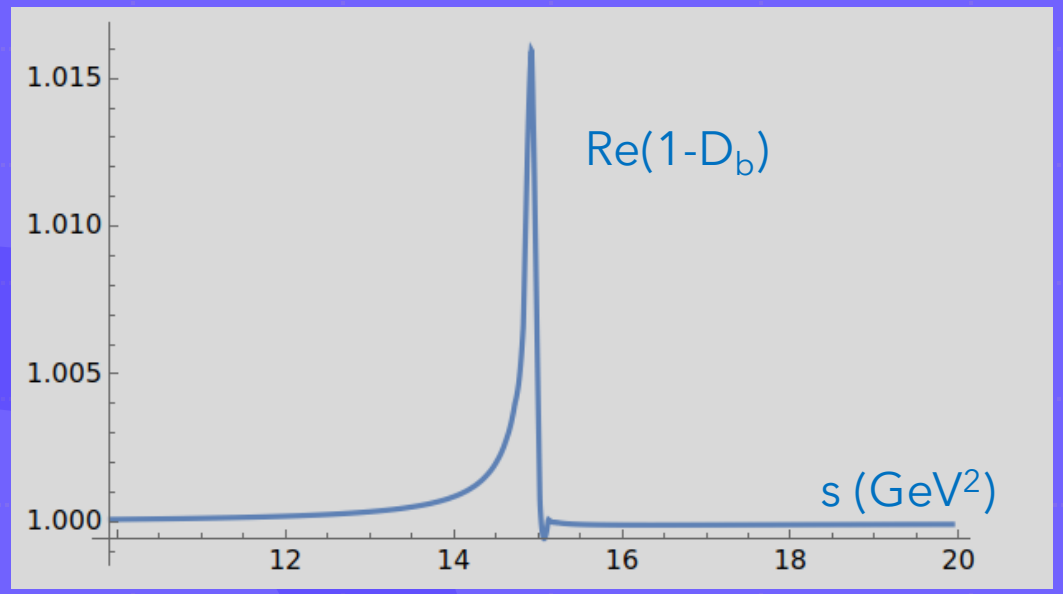

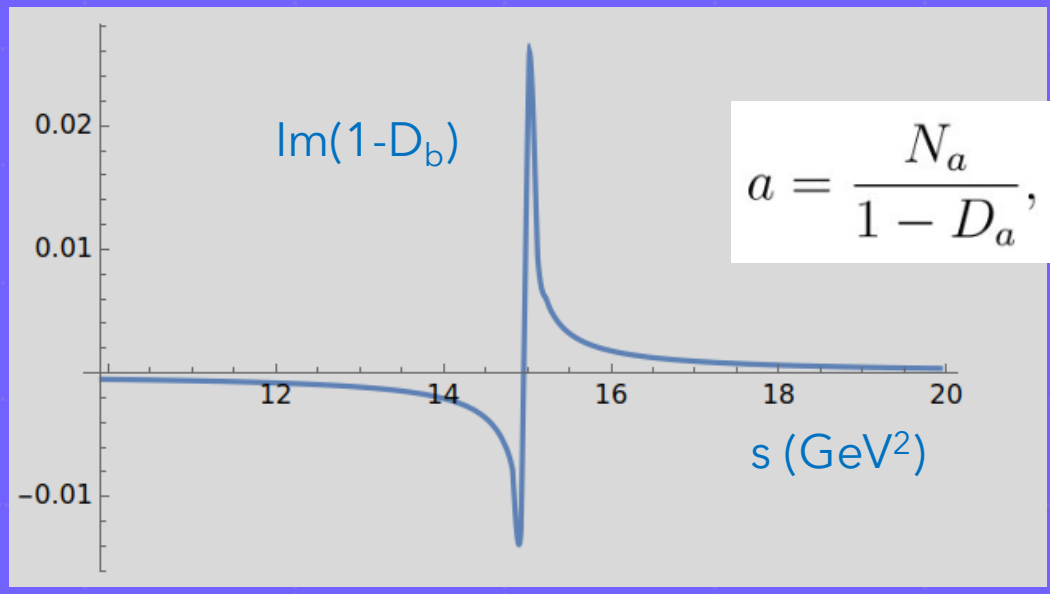

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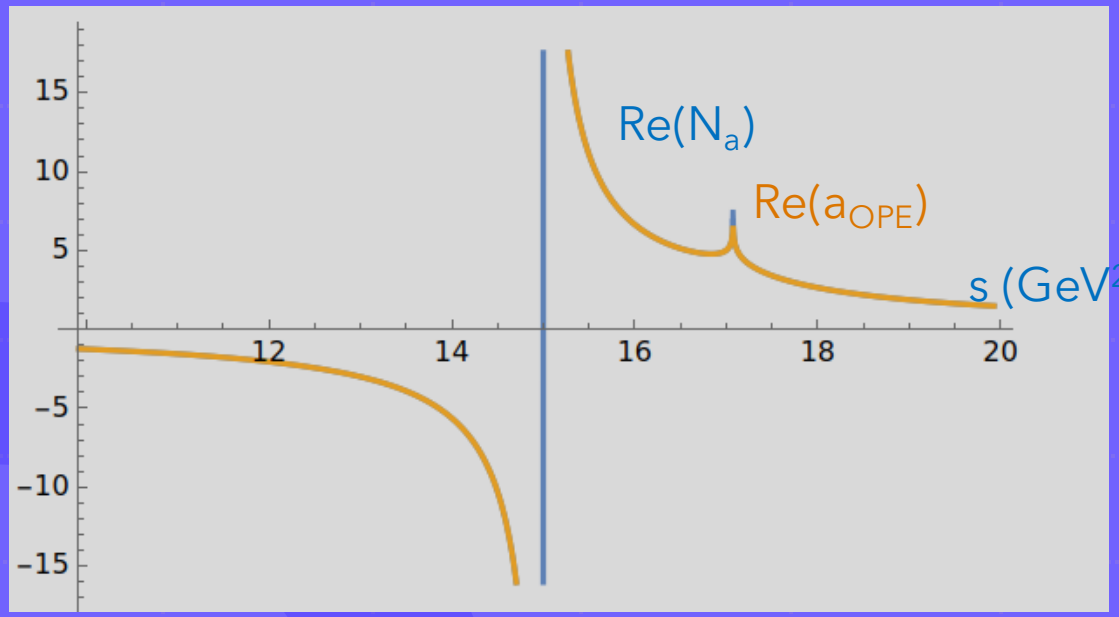

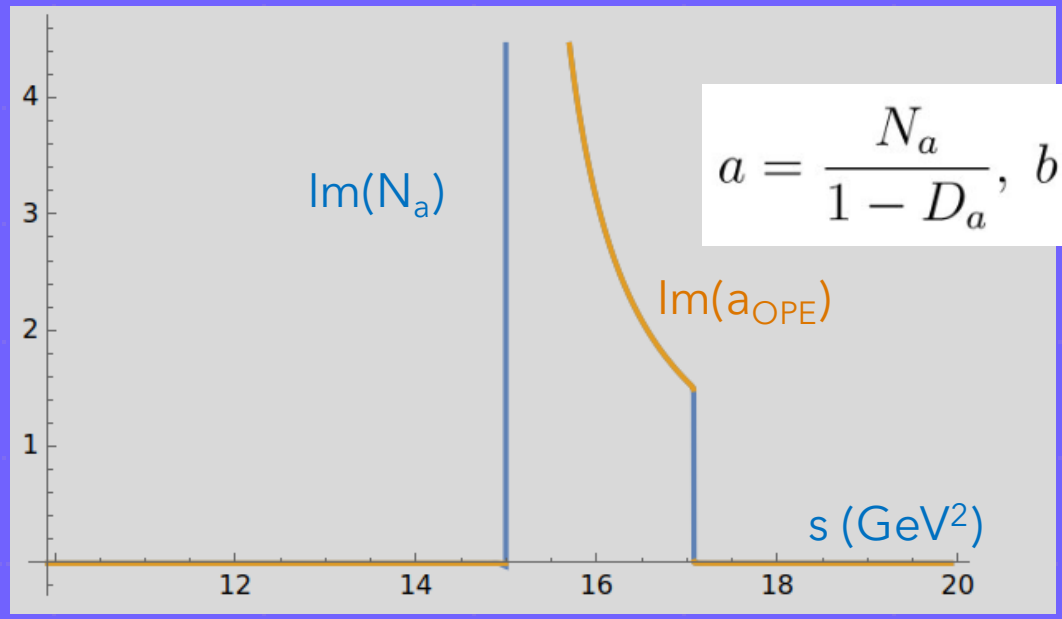

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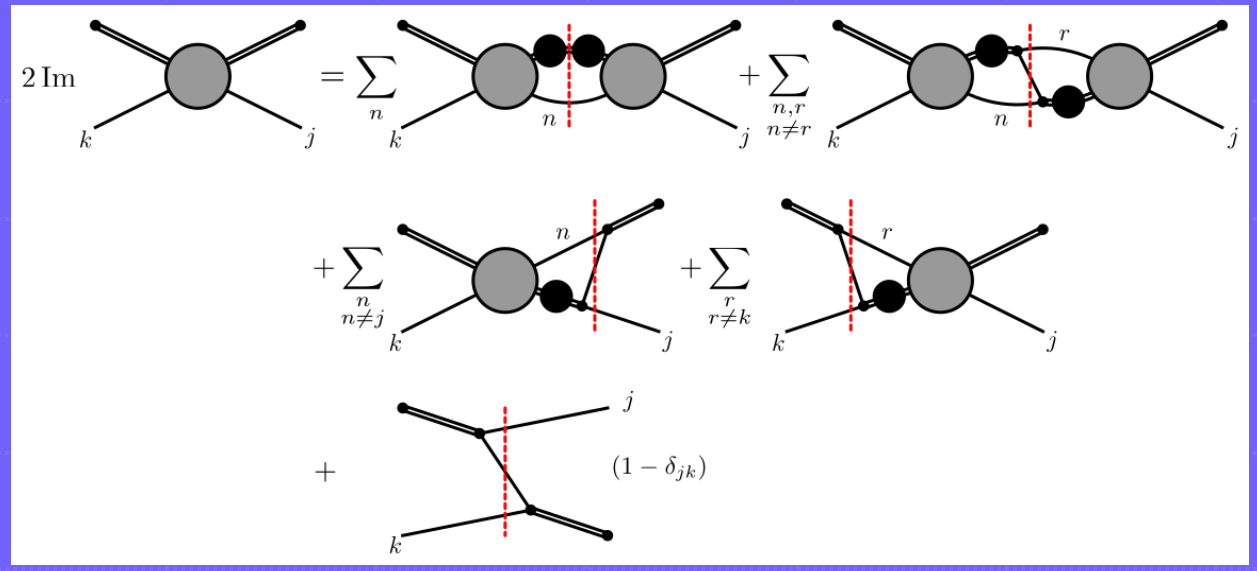
Very preliminary results from a reasonable model

- Let us start again from the unitarity equation, written for the sum of the two amplitudes, A = a + b
- Remap A into a form in which there is an explicit contributions pion exchanges + a remainder that we assume with a precise dependance in s and in the isobars' invariant mass σ,σ'

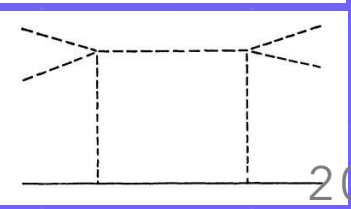


$$A = \tilde{a}_{OPE} + \alpha(\sigma', s)\hat{a}(\sigma', s, \sigma)\alpha(s, \sigma)$$

$$A = \tilde{a}_{OPE}(\sigma', s, \sigma) + B(\sigma', s, \sigma) + \alpha(\sigma', s)\hat{a}(\sigma', s, \sigma)\alpha(s, \sigma)$$

...up to the n-th exchange

B is the pw of the double pion exchange = box:

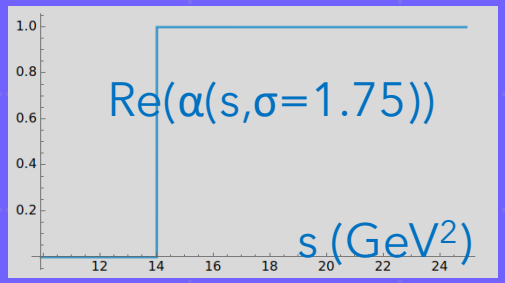


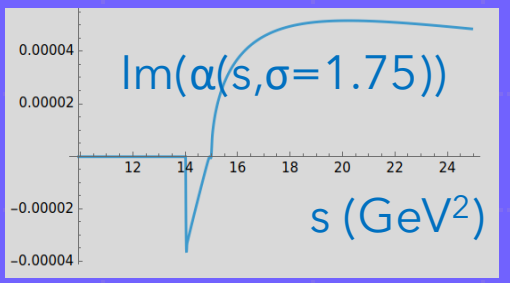
Very preliminary results from a reasonable model

- Plug this form for A into the unitarity: unitarity equations will link terms of different orders. Here comes the model: get rid of the terms until each term will appear at the same order (a sort of perturbative unitarity). It seems that all this models boil down to the same equations
- Analysing the asymptotic behavior of α and L in s, is quite clear that solving the equation for â dispersively will require subtractions to make it converge, but subtractions constants are equivalent to contact terms used in the EFT

$$\alpha(\sigma,s) = g(\sigma) + \int d\nu[\tilde{a}_{OPE}(\sigma,s,\nu)\tau_3(s,\nu)\alpha(\nu,s) \equiv g(\sigma) + \int d\nu K(\sigma,\nu;s)\alpha(\nu,s)$$

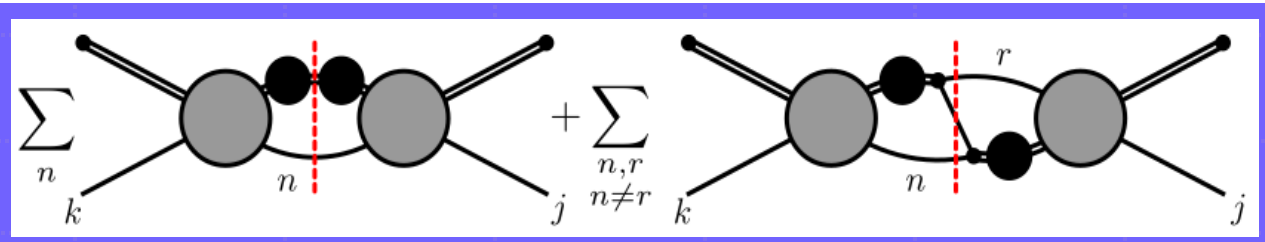
$$Im\hat{a}(s) = \hat{a}^*(s)L(s)a(s)$$





L comes from the same sum of the loops we saw before in the N/D formalism

$$L(s) = \int d\nu\alpha(s,\nu)\,\mathrm{Im}\,\tau_3(s,\nu)\alpha(\nu,s) + \int d\nu d\tau\alpha(s,\nu)K(\nu,s,\tau)\alpha(\tau,s)$$



Outline

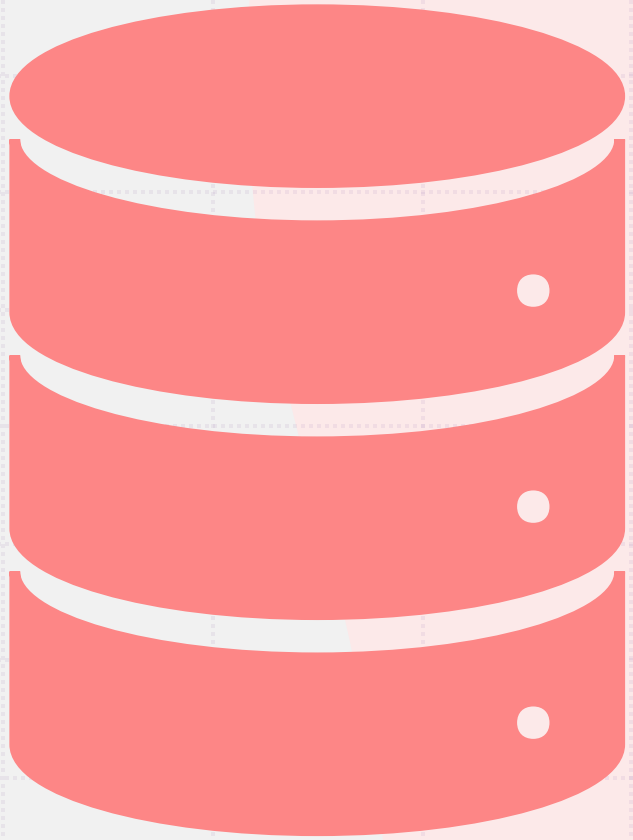
- The exotic nature of the X(3872)
- 3-body formalisms developed:
  - N/D version I
  - Reasonable models
- Conclusions

Summary

- We developed various 3-body formalisms to test the hypothesis that pion exchange only can produce a bound state compatible with the X(3872) in our S-matrix framework
- From our N/D formalism we saw that pion exchange only doesn't manage to produce sizable effects; note that this formalism does not have by construction free parameters, that would be equivalent to contact terms
- From the reasonable model with n-th exchange and "perturbative" unitarity emerge the necessary of performing subtractions (i.e. including contact terms); note that in this case pion exchanges appear explicitly as additive piece of the final solution, so are in a certain sense separated from the remaining interaction
- Without making any bold, final statement, these independents results (even though not including spin, yet) seem to point to a conclusion: in our S-matrix framework and under physically reasonable assumptions and approximations, long interaction mediated by pions seems to be not enough to bind D and D*, then the X(3872) would appear as a product of the short-range interaction mainly, despite of the molecular view emerging e.g. in the previous EFT works


Thanks for your attention!

Backup



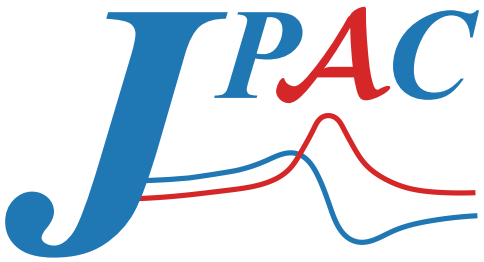
Diffusion models for physics extraction

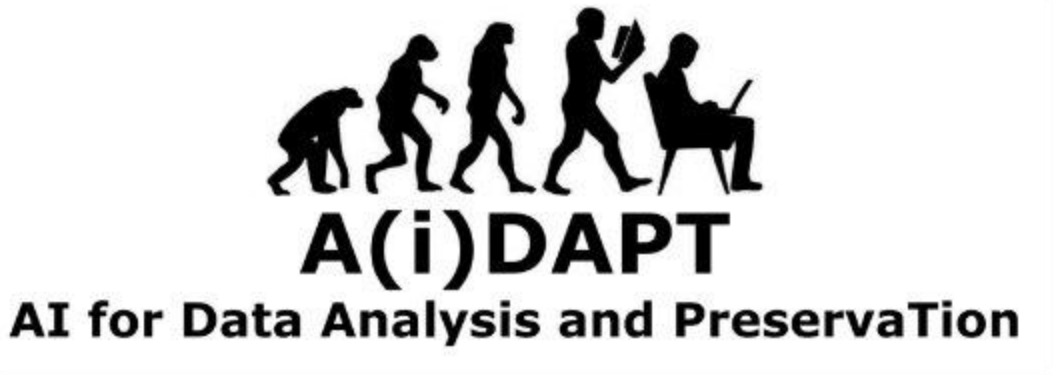
GIORGIO FOTI (UNIME & INFN CATANIA), ON BEHALF OF A(I)DAPT COLLABORATION

"DIGITAL TWINS FOR NUCLEAR AND PARTICLE PHYSICS - NPTWINS 2025"

OCT 6 – 8, 2025

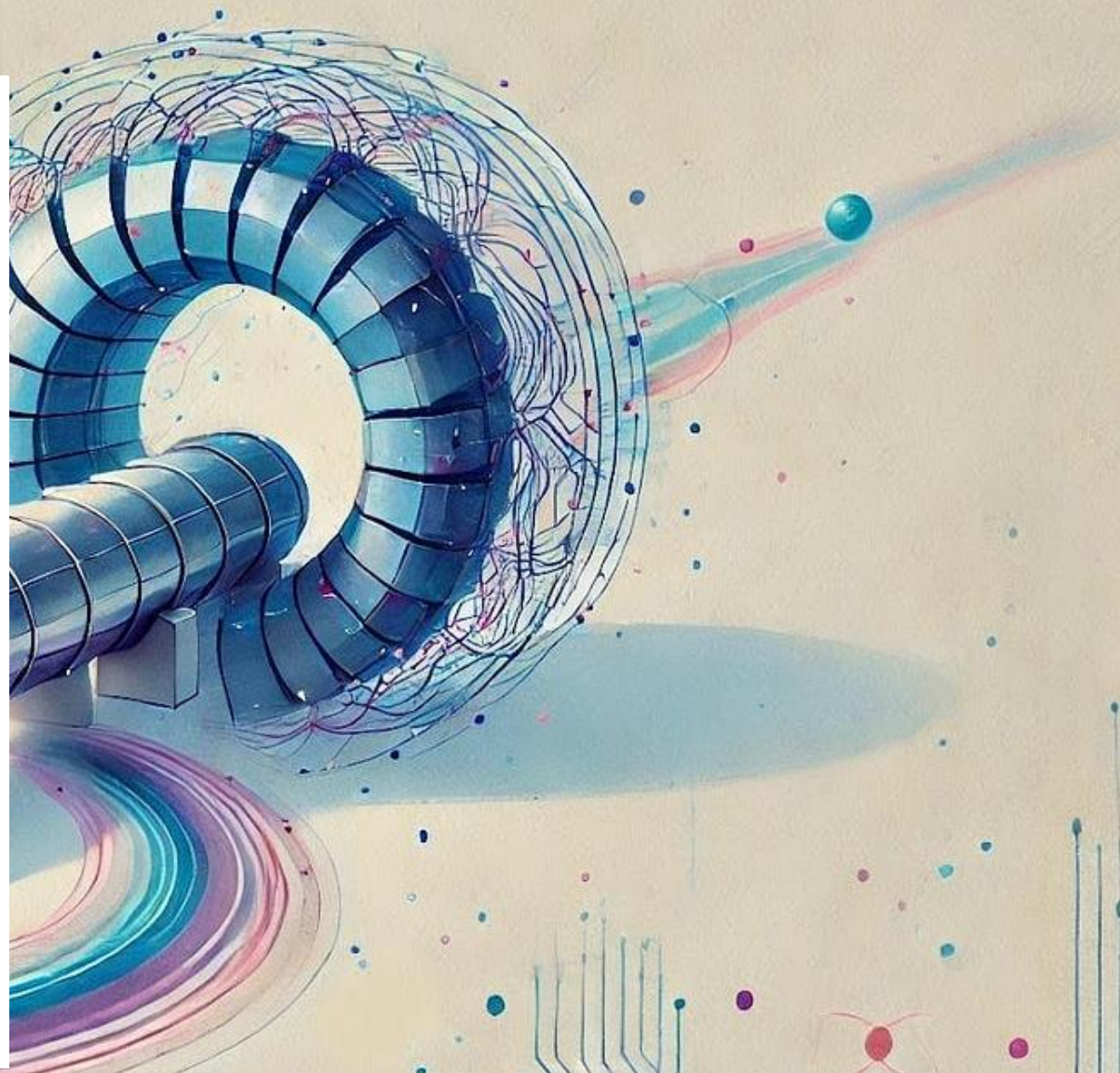
UNIVERSITÀ DI MESSINA





Summary

- Unfolding the detector effect through generative AI
- How a diffusion model work
- Status of closure test for CLAS detector
- Conclusions & Next directions

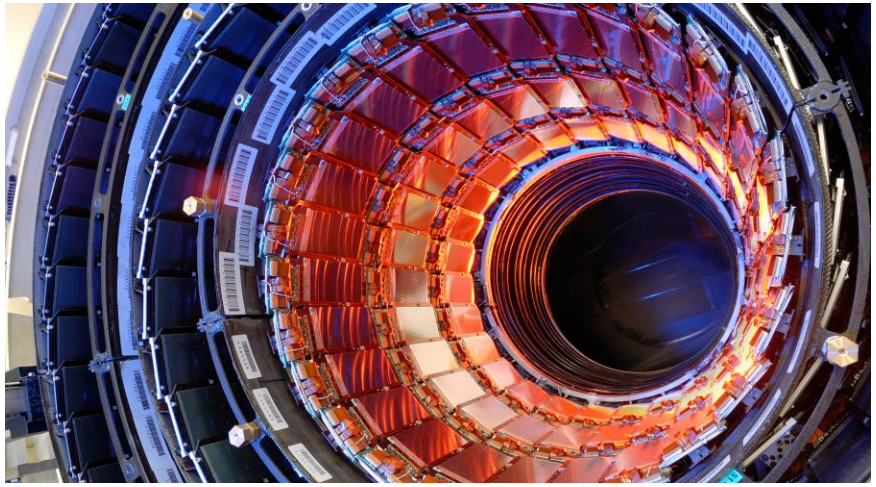


The response of a detector

Detector introduce resolution effects, acceptance gaps and inefficiencies that distort the true particle distributions.

$$\det(y) = \int_x P(y|x)\,\mathrm{true}(x)$$

Ground truth ("vertex" level):
E = **10.0 GeV**



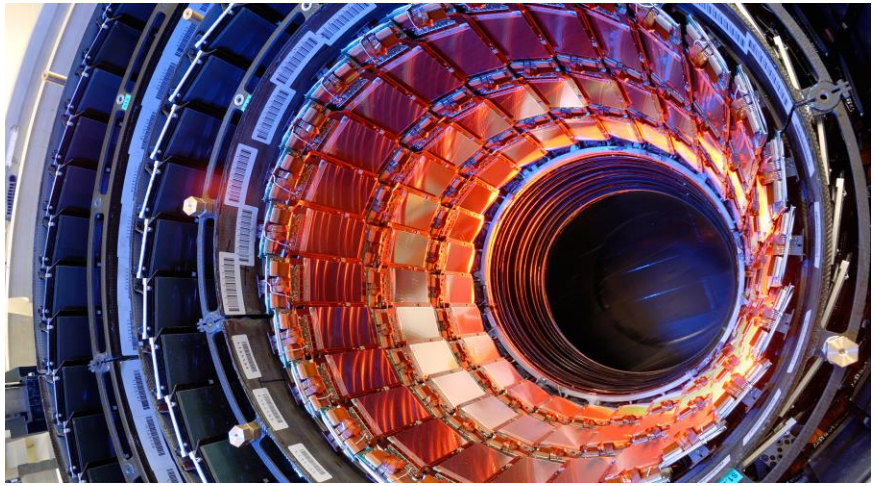
Detector level:
E=**10.1 GeV**

Unfolding the detector effect

We want to infer the posterior probability P(x|y) given the measured det(y) (**"unfolding"**). Prior true(x) is unknown: this is an **ill-posed inverse problem**

$$P(x|y) = \frac{P(y|x)\,\mathrm{true}(x)}{\det(y)}$$

Ground truth ("vertex" level): E = **10.0 GeV**



Detector level: E=**10.1 GeV**

Traditional Unfolding = matrix inversion

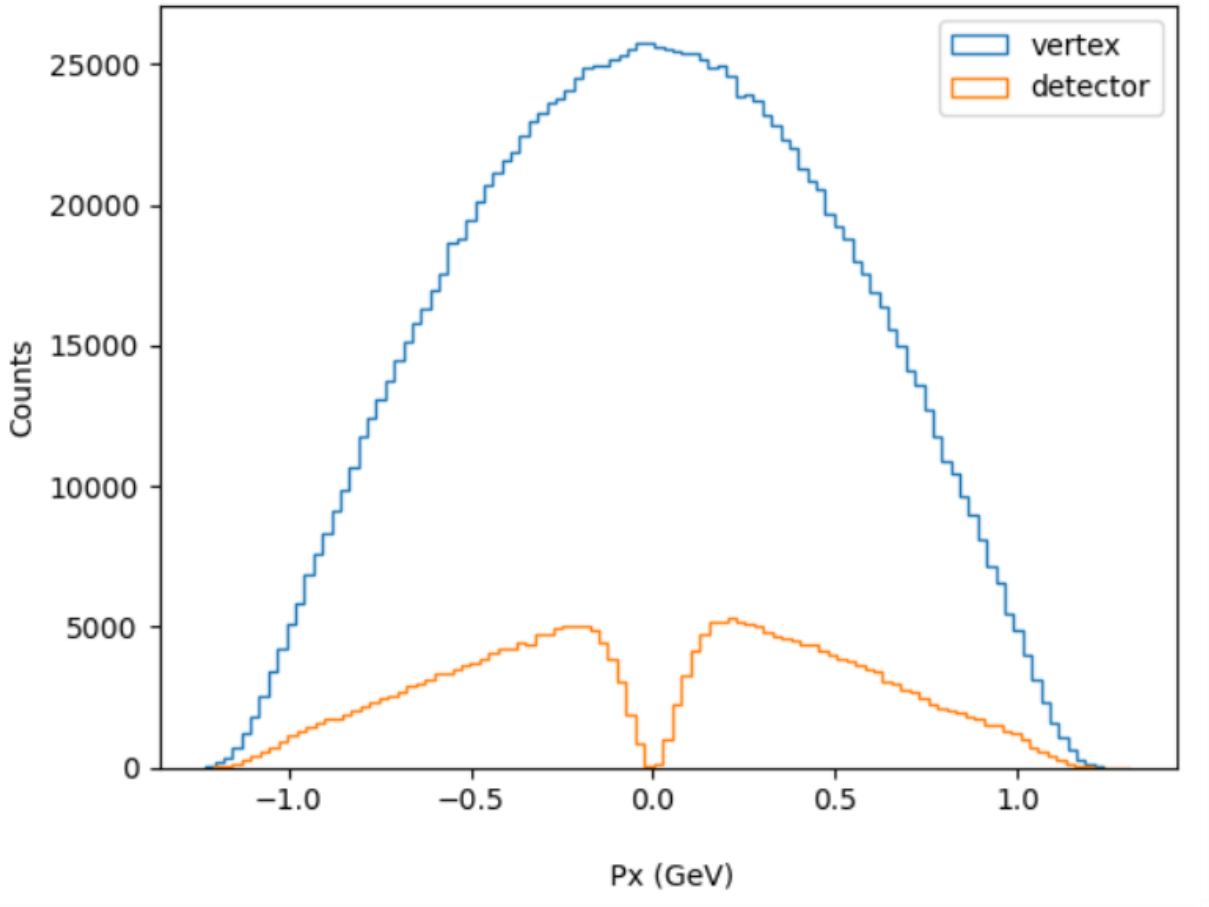
$$\det(y) = \int_x P(y|x)\,\mathrm{true}(x)$$

Binning

$$det_i = \sum_j M_{ij}\, true_j$$

Response matrix inversion

$$true_i = \sum_j M^{-1}_{ij}\, det_j$$



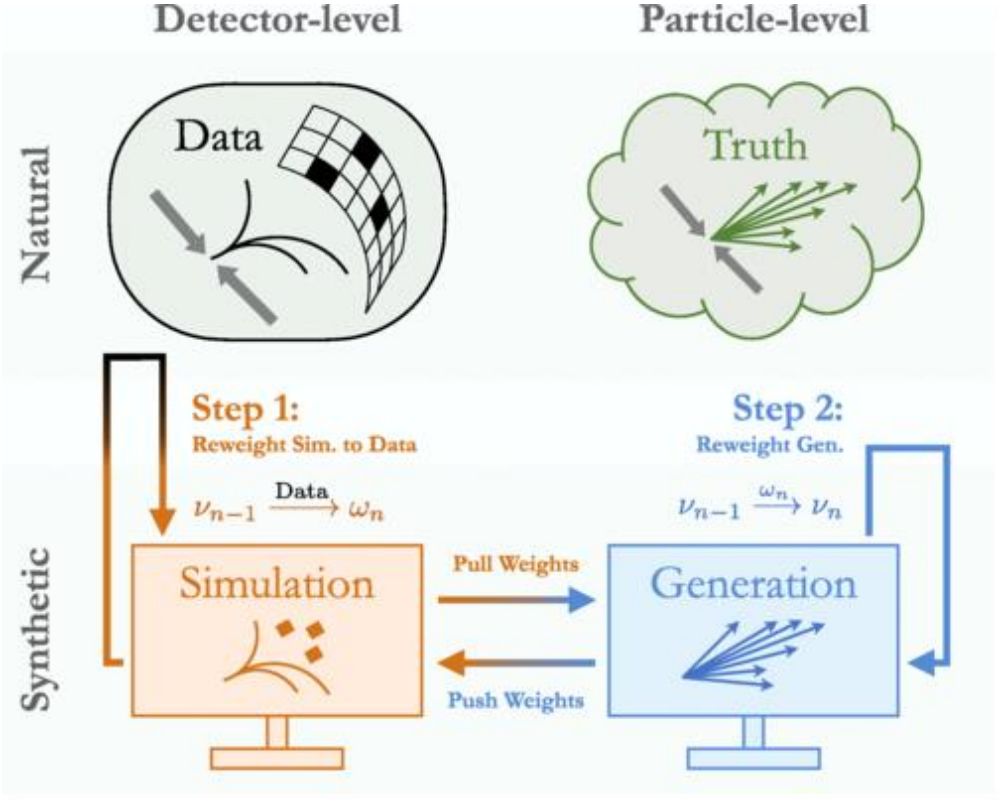
Limitations of Traditional Unfolding

[Unfolding Methods in Particle Physics for Quantum Information Observables Michele Pinamonti (https://agenda.infn.it/event/34555/contributions/212423/attachments/112947/161553/Unfolding_Pinamonti10nov2023.pdf]

- Standard techniques typically requires **binning** data and assume a **specific response matrix** (e.g. linear)
- **Background** need to be subtracted before unfolding (misidentification & other reducible sources – irreducible ones are eliminated with the successive physical analysis)
- Struggle with **systematics uncertainties**: they enter in the response matrix choice and in the background estimation
- Bad times with unfolding **many correlated observables**
- For many algorithms, different hypotheses for **underlying physics** could in general change the response function
- Large **"oscillations" in the inversion**: amplification of statistical fluctuations due to off-diagonal entries of M ((many different ways to mitigate them)

These challenges motivate alternative approaches, like **AI-based unfolding**, which can handle high-dimensional input (e.g. full event kinematics or images) and learn complex detector effects without manual binning.

ML-based alternative on the market



E.g. **OmniFold**: A Method to Simultaneously Unfold All Observables

Pros:

- Based on iterative reweighting of a simulated dataset, using machine learning to capitalize on all available information
- Unbinned approach
- Works for arbitrarily high-dimensional data, and naturally incorporates information from the full phase space

Cons:

- Tuned on LHC experiments, typically much more inclusive than ours of hadronic physics
- We need something more suitable for exclusive experiments like CLAS (GlueX, Compass…)

Generative AI-Based Unfolding Approaches

Toward a generative modeling analysis of CLAS exclusive 2π photoproduction

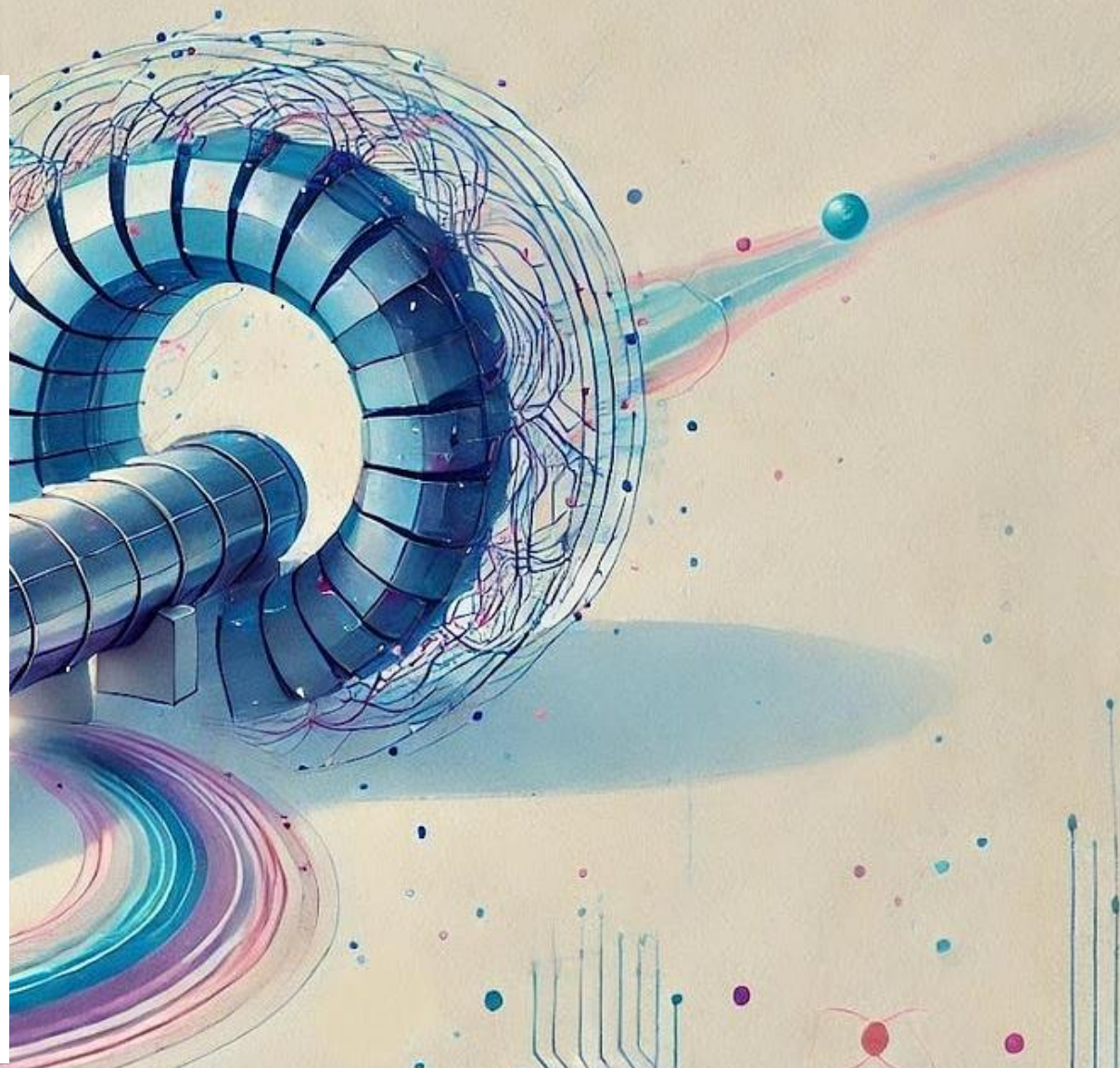
T. Alghamdi,[1,2,*] Y. Alanazi,[3] M. Battaglieri,[4] Ł. Bibrzycki,[5] A. V. Golda,[6]
A. N. Hiller Blin,[7] E. L. Isupov,[8] Y. Li,[1] L. Marsicano,[4] W. Melnitchouk,[3,9] V. I. Mokeev,[3]
G. Montaña,[3] A. Pilloni,[10,11] N. Sato,[3] A. P. Szczepaniak,[3,12,13] and T. Vittorini[4,14]

- Adversarial and likelihood-based models have been applied: e.g. in A(i)DAPT, generative adversarial networks (GANs) (**see Tommaso's talk**), (conditional) normalizing flows
- Diffusion models: recent work show that they are **outperforming GANs**. These can, in principle, model complex posteriors and provide uncertainty quantification.

Each AI method has trade-offs; diffusion models are attractive for their strong theoretical foundations, **flexibility and bijectivity**

Summary

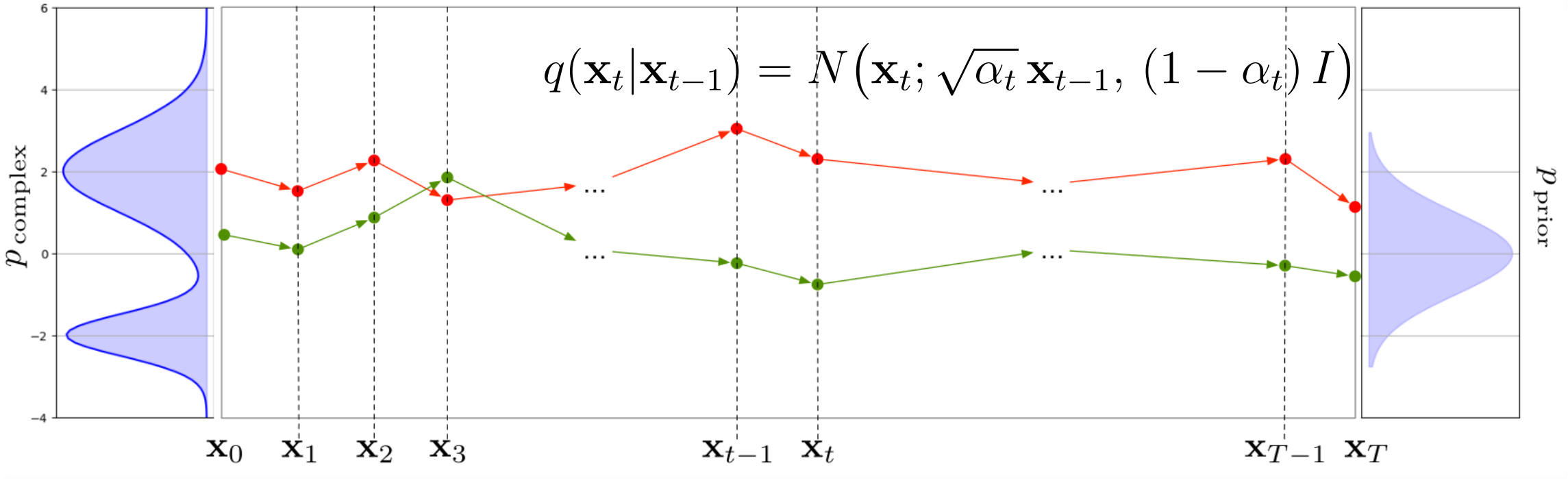
- Unfolding the detector effect through generative AI
- How a diffusion model work
- Status of closure test for CLAS detector
- Conclusions & Next directions



Denoising Diffusion Probabilistic Models (DMs)

A DM is a generative model is trained in two phases: a **deterministic forward diffusion** (noise addition) process and a **learned reverse** (denoising) process:

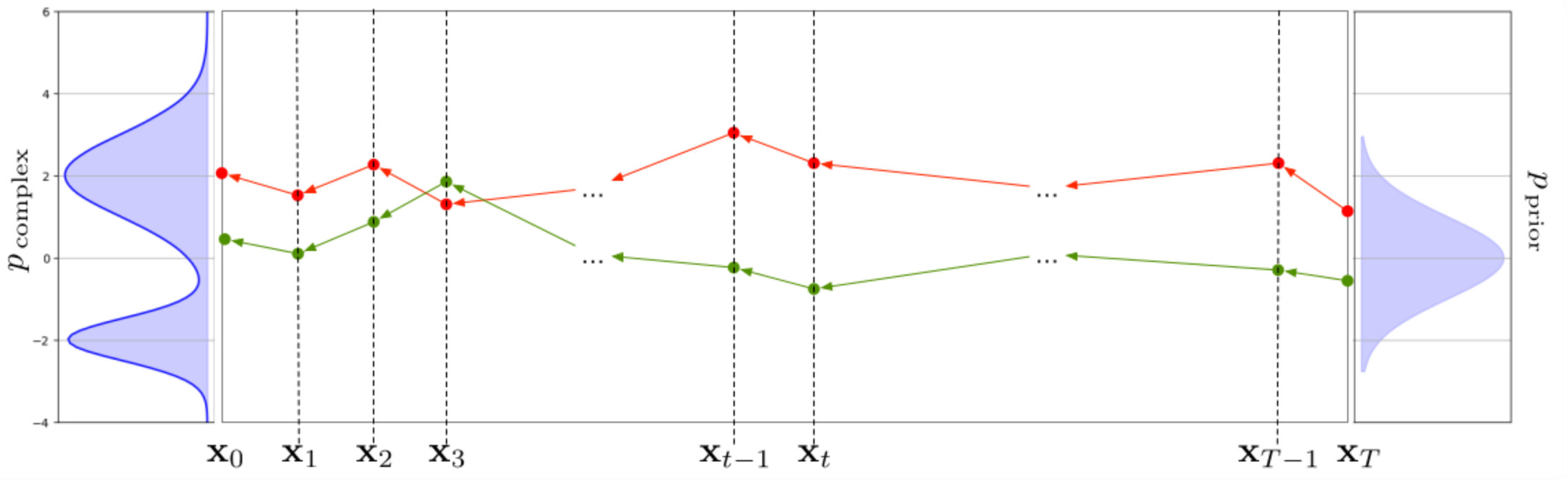
- In the forward pass, one gradually corrupts a clean data sample x0 by injecting Gaussian noise over T timesteps, producing xT≈N(0,I) (αt is a variance schedule):



Denoising Diffusion Probabilistic Models (DMs)

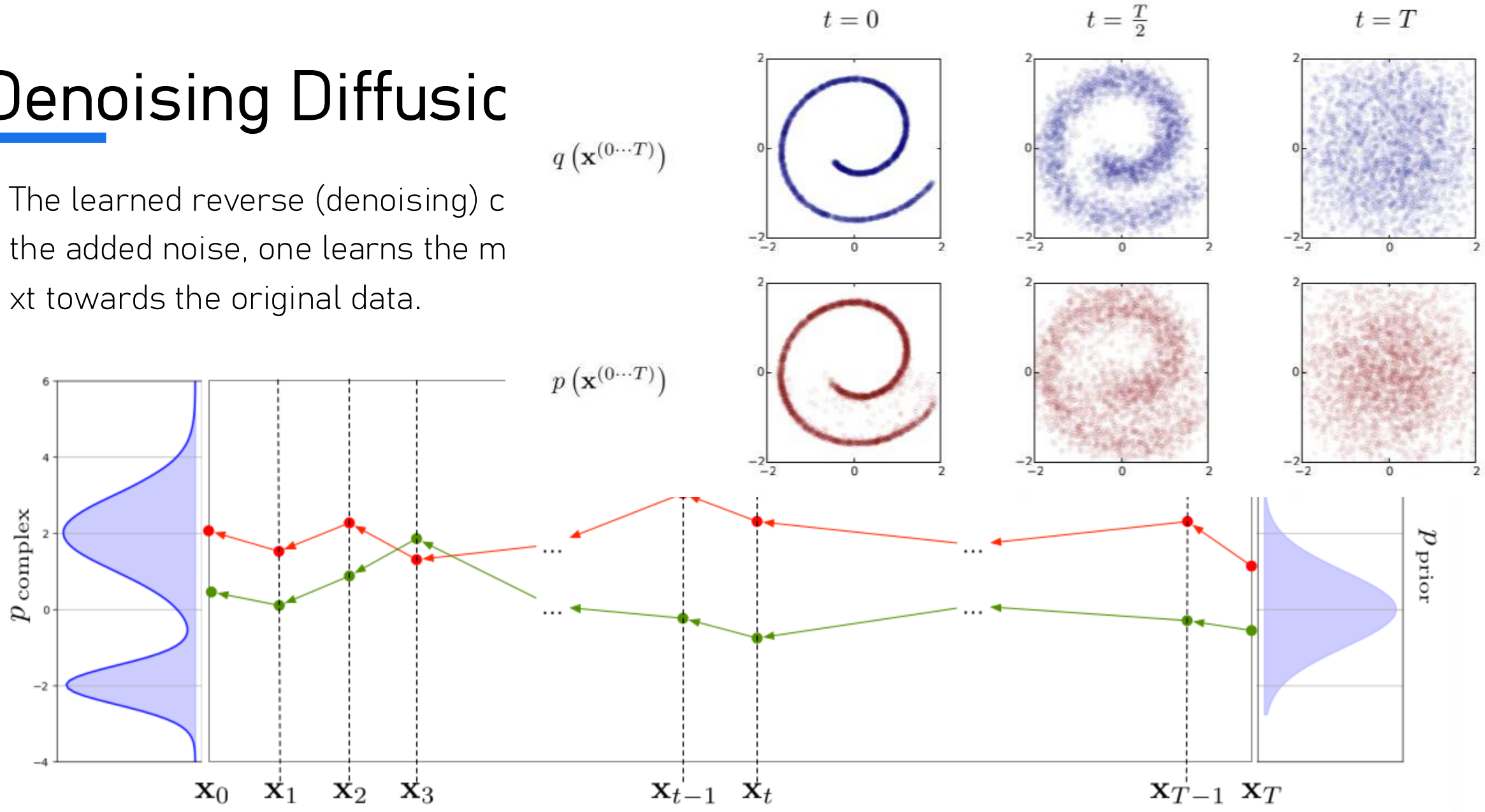
- The learned reverse (denoising) chain is parameterized by θ: by training the model to predict the added noise, one learns the mean of the Gaussian transition, μθ(xt, t), that maps any noisy xt towards the original data.

$$p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t) = N\left(\mathbf{x}_{t-1}; \mu_\theta(\mathbf{x}_t, t),\ \sigma_t^2 I\right)$$



Denoising Diffusic

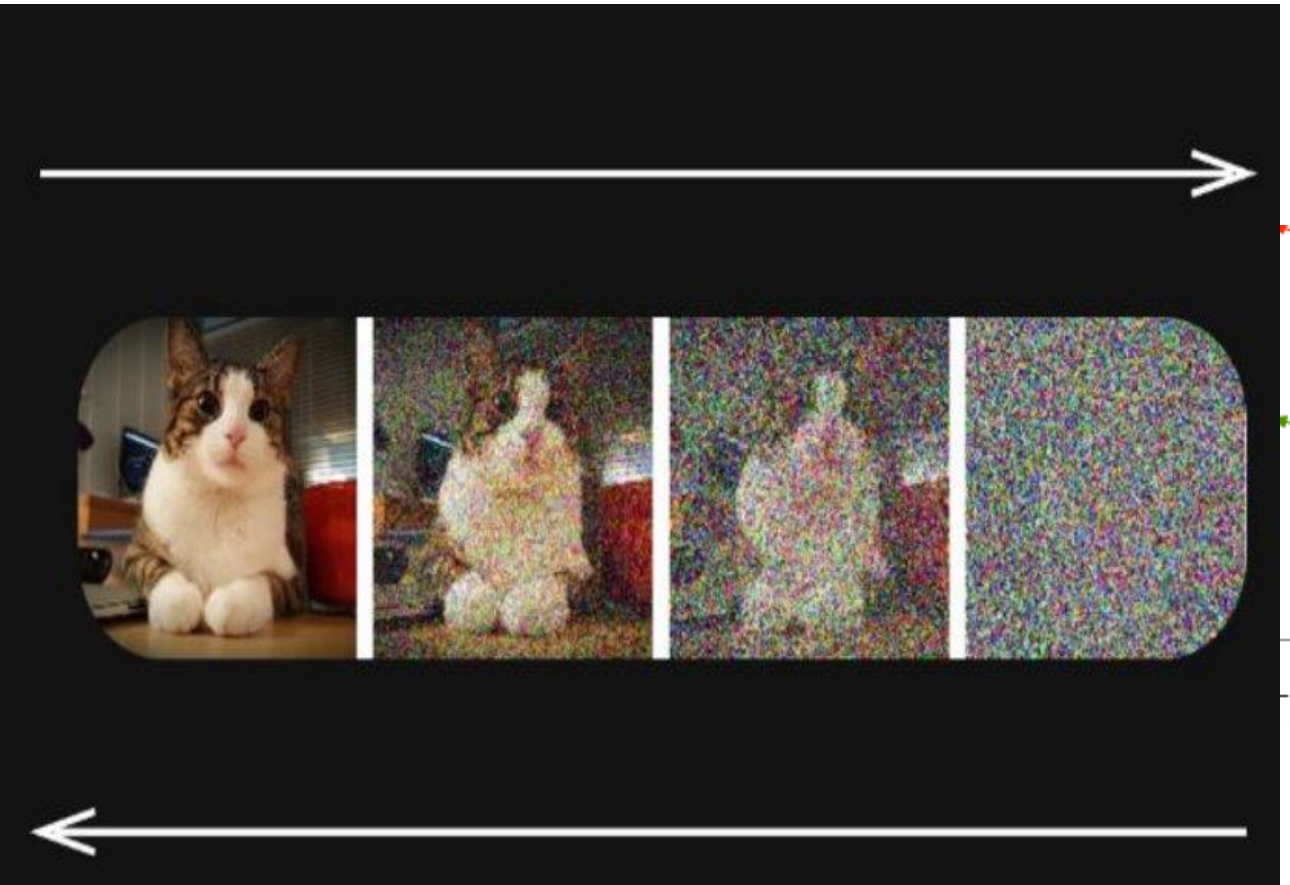
- The learned reverse (denoising) c
  the added noise, one learns the m
  xt towards the original data.

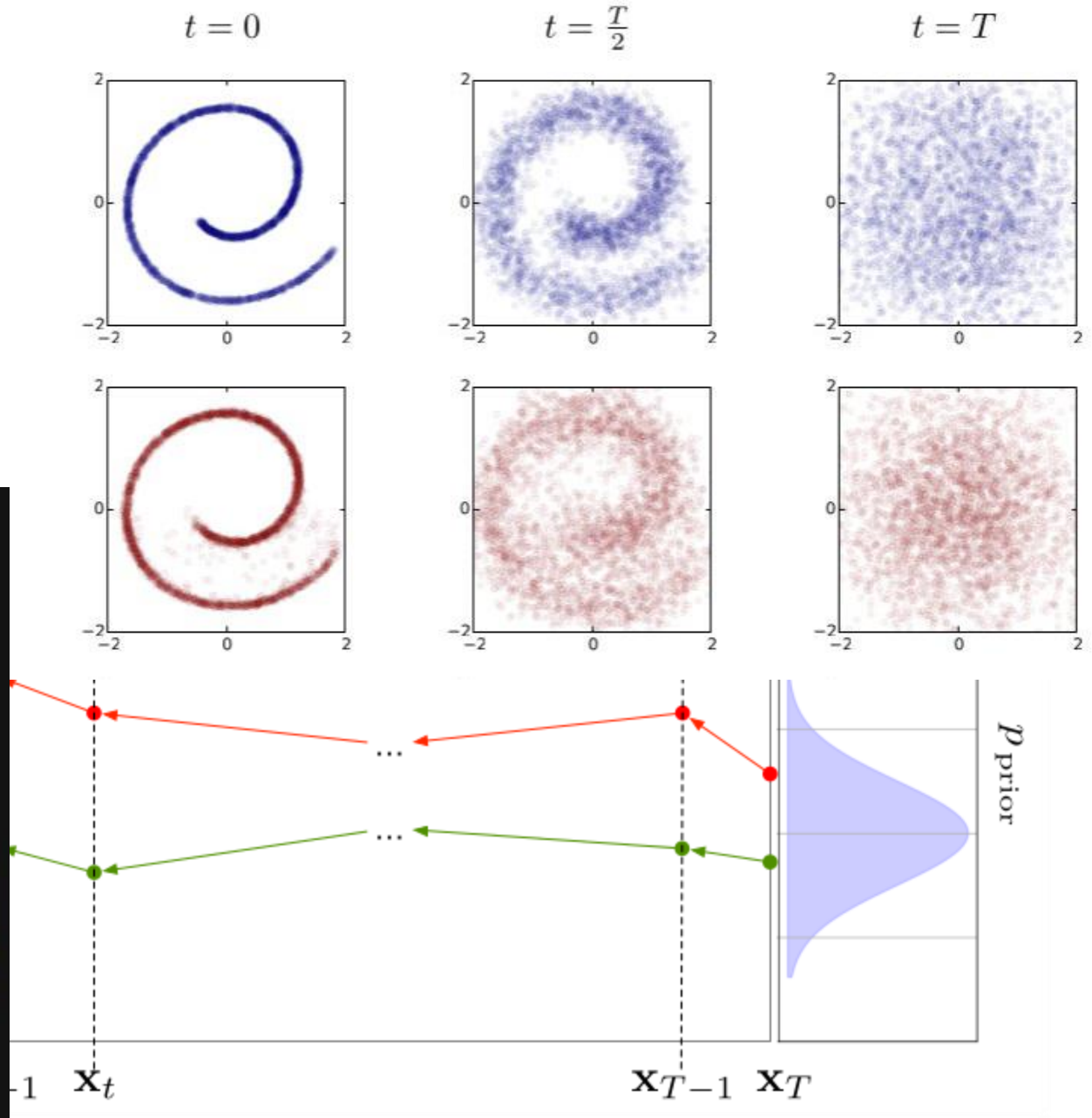


Denoising Diffusic

$q\left(\mathbf{x}^{(0\cdots T)}\right)$

- The learned reverse (denoising) c
  the added noise, one learns the m
  xt towards the original data.





Diffusion for Unfolding: conditioning

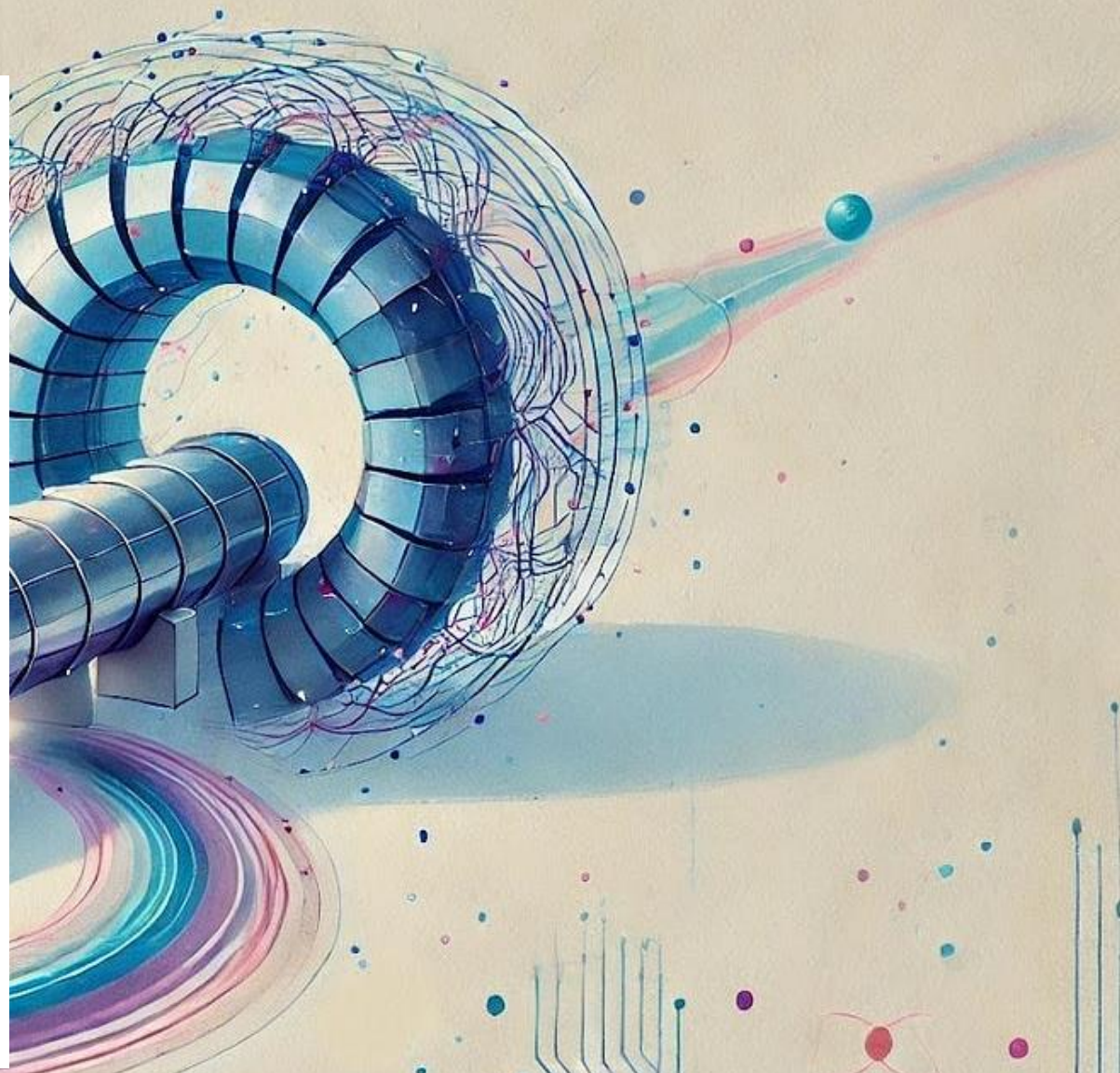
To apply diffusion to unfolding, we use a **conditional diffusion model**. Here the reverse process is conditioned on the detector level y:

$$p_\theta(x_{t-1}|x_t, y) = N(x_{t-1}; \mu_\theta(x_t, y, t), \sigma_t^2 I)$$

- The forward noising process remains unchanged, but **the denoiser network now takes y as an input or context** (this context can be given by labels, features etc.)
- In practice, the output x0 is a sample from the posterior P(x|y). Conditioning directly on y makes the **generative process data-dependent** and enables it to **approximate the Bayesian inverse** P(x|y) without explicitly computing the prior

Summary

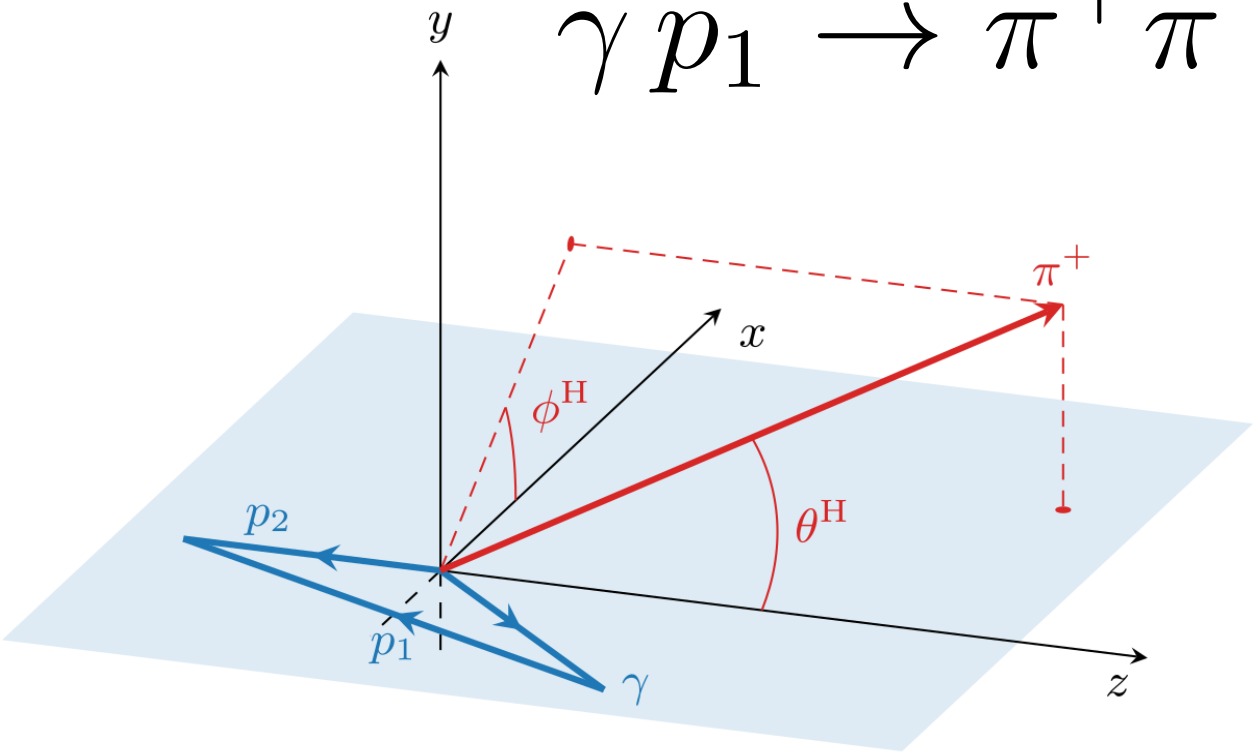
- Unfolding the detector effect through generative AI
- How a diffusion model work
- Status of closure test for CLAS detector
- Conclusions & Next directions

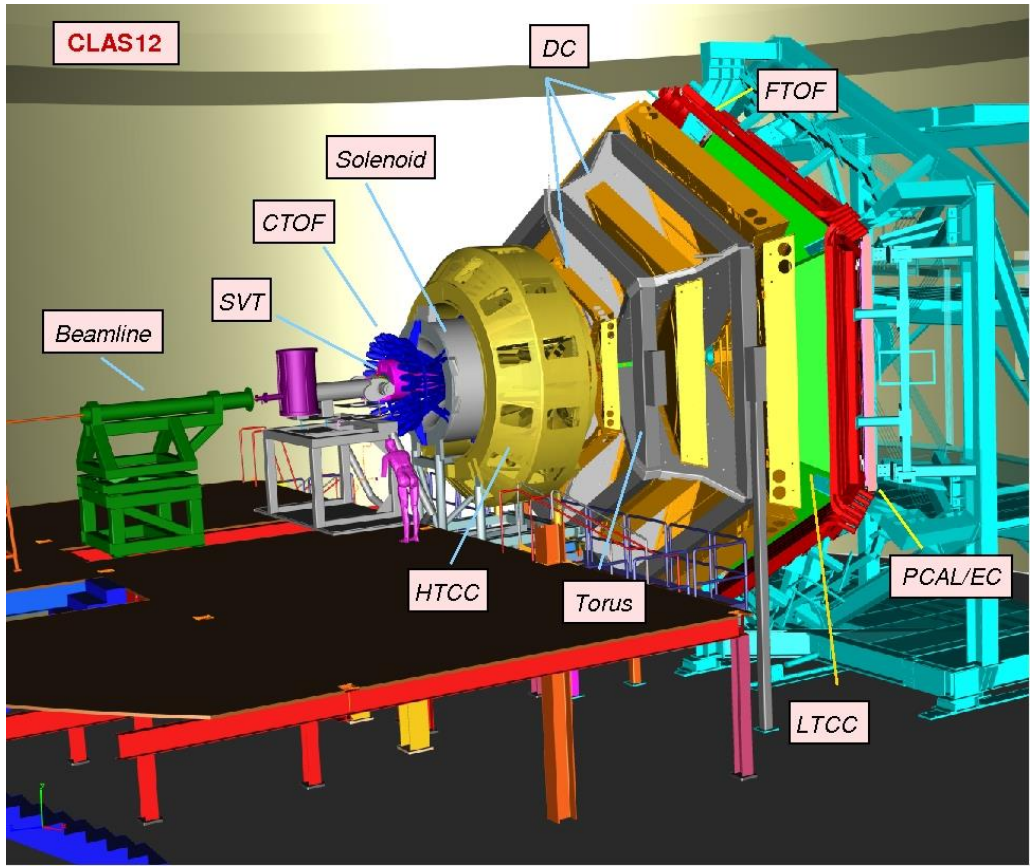


A closure test for unfolding CLAS detector

Photoproduction of two pions on the proton, measured by CLAS

$$\gamma\, p_1 \rightarrow \pi^+\pi^-\, p_2$$





Let's define the moments

$$\frac{d\sigma}{dtd\sqrt{s_{12}}d\Omega^{\mathrm{H}}} = \kappa \sum_{\lambda_1\lambda_\gamma\lambda_2} |\mathcal{M}_{\lambda_\gamma\lambda_1\lambda_2}(s,t,s_{12},\Omega^{\mathrm{H}})|^2,$$

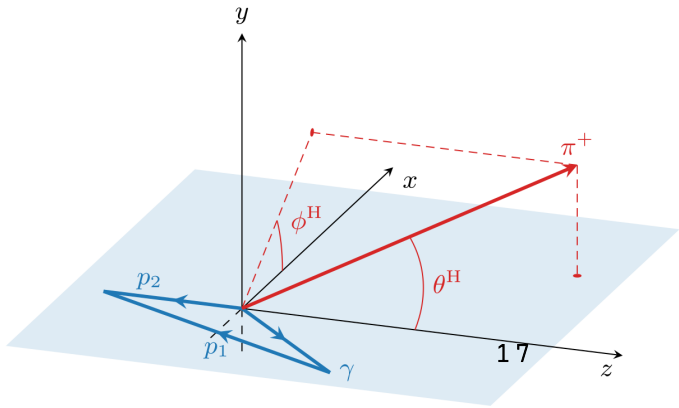
$$\langle Y_M^L \rangle = \sqrt{4\pi} \int d\Omega^{\mathrm{H}} \frac{d\sigma}{dtd\sqrt{s_{12}}d\Omega^{\mathrm{H}}} \mathrm{Re}\{Y_M^L(\Omega^{\mathrm{H}})\}$$

$$\mathcal{M}_{\lambda_\gamma\lambda_1\lambda_2}(s,t,s_{12},\Omega^{\mathrm{H}}) = \sum_{lm} \mathcal{M}^l_{\lambda_\gamma\lambda_1\lambda_2 m}(s,t,s_{12}) Y^l_m(\Omega^{\mathrm{H}}),$$

$$\langle Y_M^L \rangle = \sqrt{4\pi}\kappa \sum_{lml'm'} A^{Lll'}_{Mmm'} \times \sum_{\lambda_\gamma\lambda_1\lambda_2} \mathcal{M}^{l'*}_{\lambda_\gamma\lambda_1 m'\lambda_2}(s,t,s_{12}) \mathcal{M}^l_{\lambda_\gamma\lambda_1 m\lambda_2}(s,t,s_{12}),$$

where

$$A^{Lll'}_{Mmm'} = \int d\Omega^{\mathrm{H}} Y^l_m(\Omega^{\mathrm{H}}) Y^{l'*}_{m'}(\Omega^{\mathrm{H}}) \mathrm{Re}\{Y^L_M(\Omega^{\mathrm{H}})\}.$$



Let's define the moments

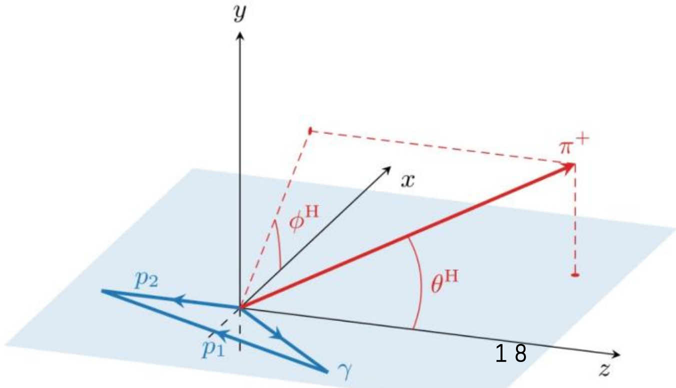
Moment extraction from real data by ...llaboration

$$\frac{d\sigma}{dtd\sqrt{\ldots}} \ldots \frac{\ldots}{\ldots{}_2 d\Omega^{\mathrm{H}}}\mathrm{Re}\{Y^L_M(\Omega^{\mathrm{H}})\},$$

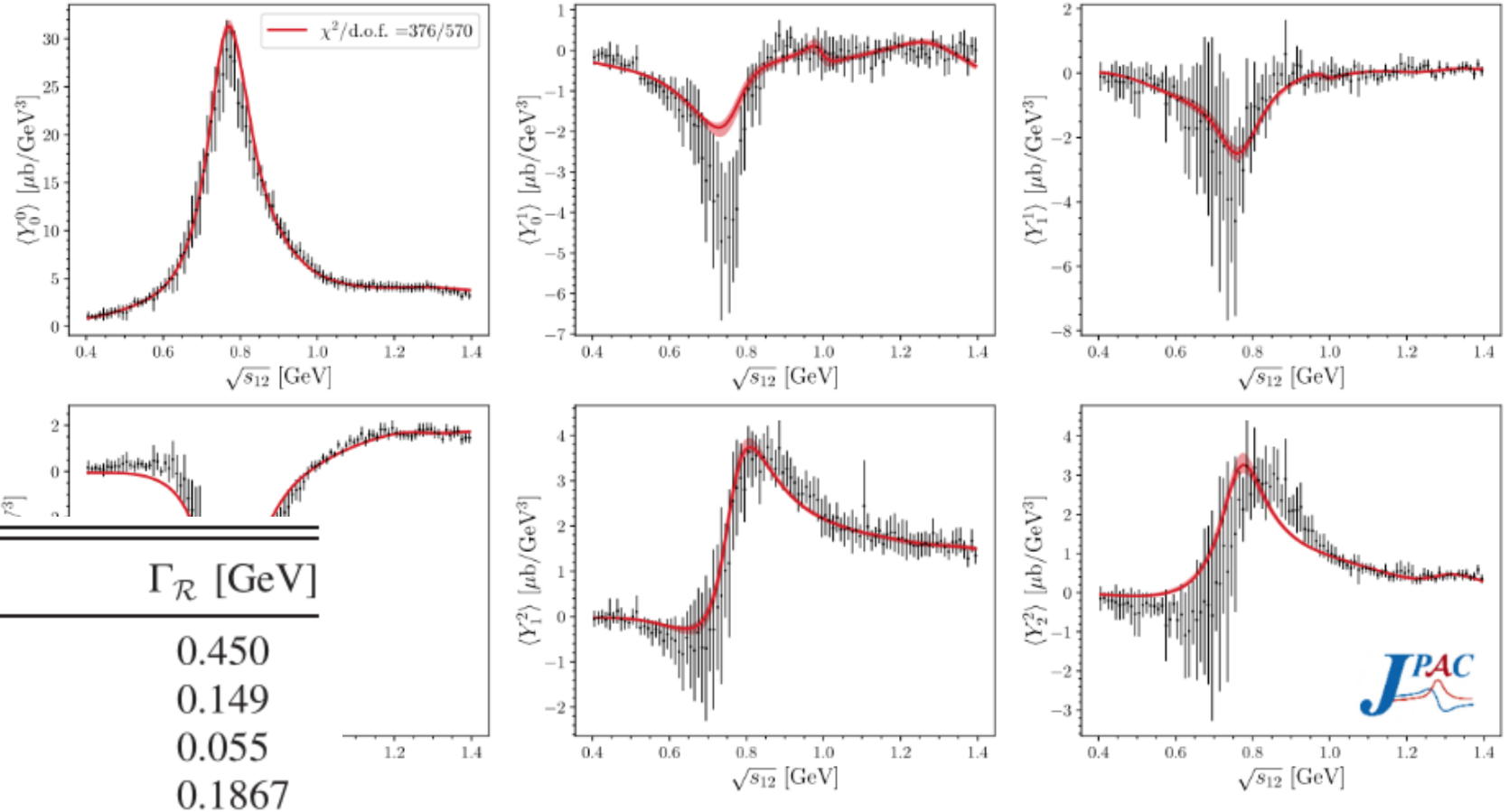
Photoproduction of $\pi^+\pi^-$ meson pairs on the proton

M. Battaglieri,[1] R. De Vita,[1] A. P. Szczepaniak,[2] K. P. Adhikari,[35] M.J. Amaryan,[35] M. Anghinolfi,[1] H. Baghdasaryan,[45] I. Bedlinskiy,[22] M. Bellis,[7] L. Bibrzycki,[29] A.S. Biselli,[13,36] C. Bookwalter,[15] D. Branford,[12] W.J. Briscoe,[16] V.D. Burkert,[42] S.L. Careccia,[35] D.S. Carman,[42] E. Clinton,[28] P.L. Cole,[18] P. Collins,[4] V. Crede,[15] D. Dale,[18] A. D'Angelo,[20,38] A. Daniel,[34] N. Dashyan,[47] E. De Sanctis,[19] A. Deur,[42] S. Dhamija,[14] C. Djalali,[41] G.E. Dodge,[35] D. Doughty,[10,42] V. Drozdov,[1] H. Egiyan,[30,42] P. Eugenio,[15] G. Fedotov,[40] S. Fegan,[17] A. Fradi,[21] M.Y. Gabrielyan,[14] L. Gan,[32] M. Garçon,[9] A. Gasparian,[33] G.P. Gilfoyle,[37] K.L. Giovanetti,[24] F.X. Girod,[9,*] O. Glamazdin,[26] J. Goett,[36] J.T. Goetz,[5] W. Gohn,[11] E. Golovatch,[40,1] R.W. Gothe,[41] K.A. Griffioen,[46] M. Guidal,[21] L. Guo,[42,†] K. Hafidi,[3] H. Hakobyan,[44,47] C. Hanretty,[15] N. Hassall,[17] K. Hicks,[34] M. Holtrop,[30] C.E. Hyde,[35] Y. Ilieva,[41,16] D.G. Ireland,[17] E.L. Isupov,[40] J.R. Johnstone,[17] K. Joo,[11] D. Keller,[34] M. Khandaker,[31] P. Khetarpal,[36] W. Kim,[27] A. Klein,[35] F.J. Klein,[8] M. Kossov,[22] A. Kubarovsky,[35] V. Kubarovsky,[42] S.V. Kuleshov,[44,22] V. Kuznetsov,[27] J.M. Laget,[42,9] L. Lesniak,[29] K. Livingston,[17] H.Y. Lu,[41] M. Mayer,[35] M.E. McCracken,[7] B. McKinnon,[17] C.A. Meyer,[7] K. Mikhailov,[22] T Mineeva,[11] M. Mirazita,[19] V. Mochalov,[23] V. Mokeev,[40,42] K. Moriya,[7] E. Munevar,[16] P. Nadel-Turonski,[8] I. Nakagawa,[39] C.S. Nepali,[35] S. Niccolai,[21] I. Niculescu,[24] M.R. Niroula,[35] M. Osipenko,[1,40] A.I. Ostrovidov,[15] K. Park,[41,27,*] S. Park,[15] M. Paris,[16,42] E. Pasyuk,[4] S.Anefalos Pereira,[19] S. Pisano,[21] N. Pivnyuk,[22] O. Pogorelko,[22] S. Pozdniakov,[22] J.W. Price,[6] Y. Prok,[45,‡] D. Protopopescu,[17] B.A. Raue,[14,42] G. Ri M. Ripani,[1] B.G. Ritchie,[4] G. Rosner,[17] P. Rossi,[19] F. Sabatié,[9] M.S. Saini,[15] C. Salgado,[31] D. Scho R.A. Schumacher,[7] H. Seraydaryan,[35] Y.G. Sharabian,[42] D.I. Sober,[8] D. Sokhan,[12] A. Stavinsky,[22] S. Ste S. S. Stepanyan,[27] P. Stoler,[36] I.I. Strakovsky,[16] S. Strauch,[41,16] M. Taiuti,[1] D.J. Tedeschi,[41] A. Teymur S. Tkachenko,[35] M. Ungaro,[11,36] M.F. Vineyard,[43] A.V. Vlassov,[22] D.P. Watts,[17,§] L.B. Weinstein D.P. Weygand,[42] M. Williams,[7] E. Wolin,[42] M.H. Wood,[41] L. Zana,[30] J. Zhang,[35] B. Zhao,[11,¶] and Z.W
(The CLAS Collaboration)

$$A^{Ll l'}_{Mmm'} = \int d\Omega^{\mathrm{H}} Y^l_m(\Omega^{\mathrm{H}}) Y^{l'*}_{m'}(\Omega^{\mathrm{H}}) \mathrm{Re}\{Y^L_M(\Omega^{\mathrm{H}})\}.$$



Fit results by JPAC collaboration



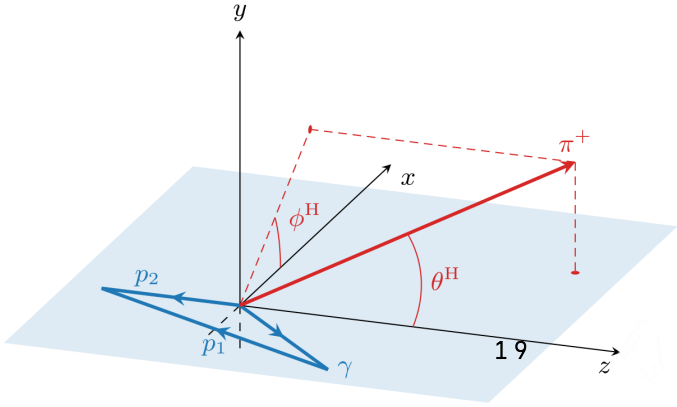
$\mathcal{R}$	J	$m_{\mathcal{R}}$ [GeV]	$\Gamma_{\mathcal{R}}$ [GeV]
$f_0(500)$	0	0.500	0.450
$\rho(770)$	1	0.775	0.149
$f_0(980)$	0	0.990	0.055
$f_2(1270)$	2	1.2755	0.1867
$f_0(1370)$	0	1.370	0.350

te model fitted to experimental measurements from Ref. [21] of two-pion angular moments $\langle Y_M^L \rangle$ fo
$E_\gamma = 3.7$ GeV and $t = -0.45$ GeV^2. Since all data shown here are fit simultaneously, this correspond
to $600 - 30 = 570$ degrees of freedom (d.o.f.).

Studying $\pi^+\pi^-$ photoproduction beyond Pomeron exchange

Łukasz Bibrzycki,[1,*] Nadine Hammoud,[2,3,†] Vincent Mathieu,[3] Robert J. Perry,[3,‡] Alex Akridge,[4,5]
César Fernández-Ramírez,[6] Gloria Montaña,[7] Alessandro Pilloni,[8,9] Arkaitz Rodas,[7,10] Vanamali Shastry,[4,5]
Wyatt A. Smith,[11,12,13] Daniel Winney,[14] and Adam P. Szczepaniak[7,5,4]

(Joint Physics Analysis Center)



Our strategy: generating data

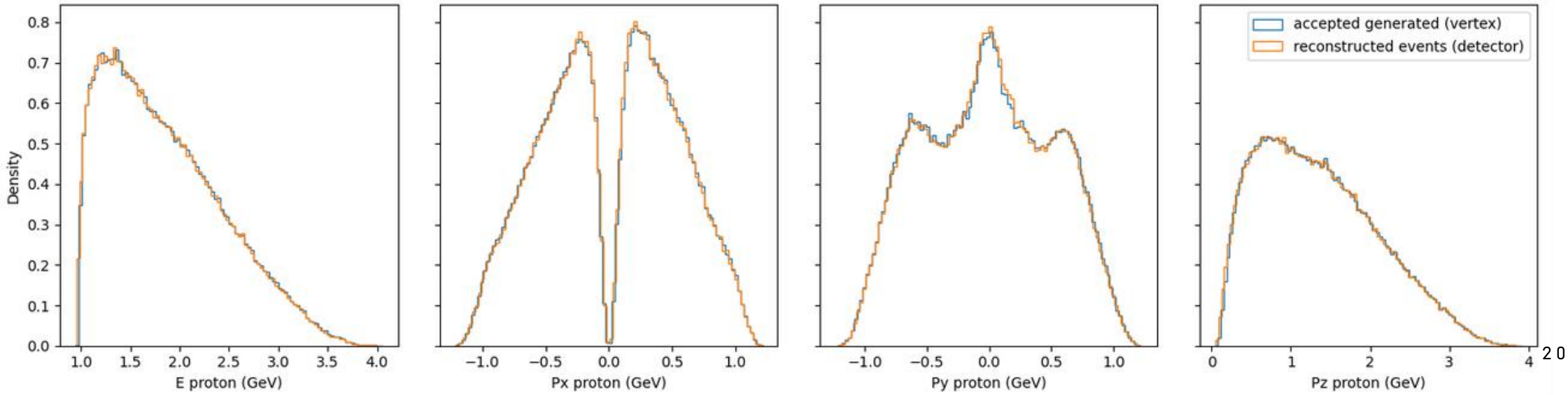
Using MC generated 4-vectors sampled from different models and correspondent GSIM (the *father* of Geant4) reconstructed and train the DM to learn how to do the inverse process:

Phase space MC-generated Vertex level

GSIM (CLAS proxy)

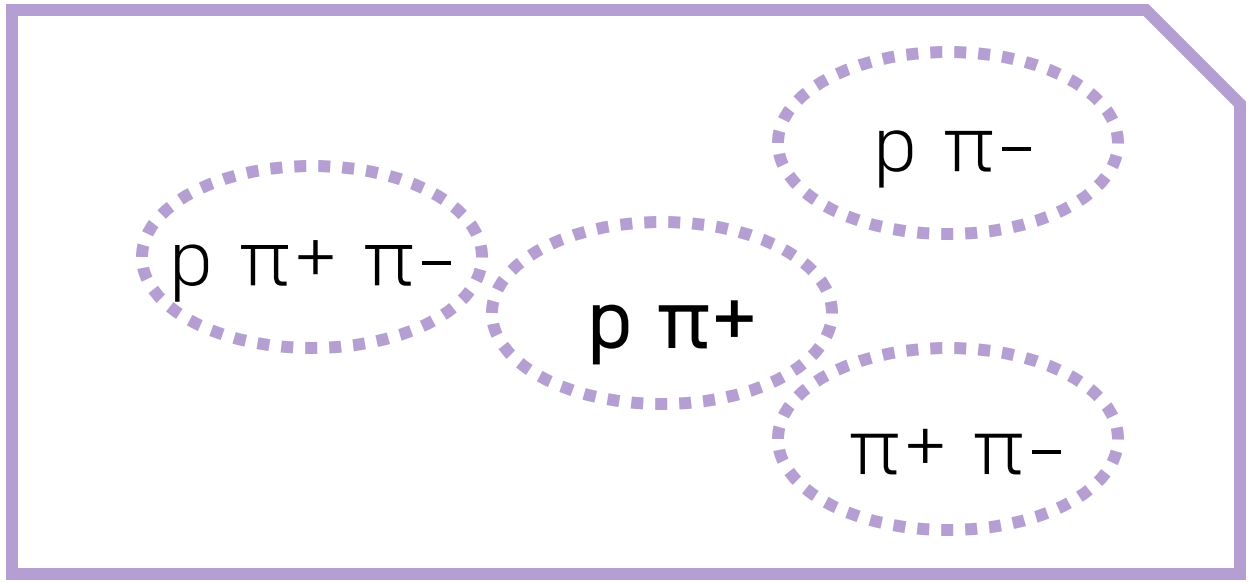
Detector level

Alert: we are learning just smearing, not acceptance



Our strategy: generating data

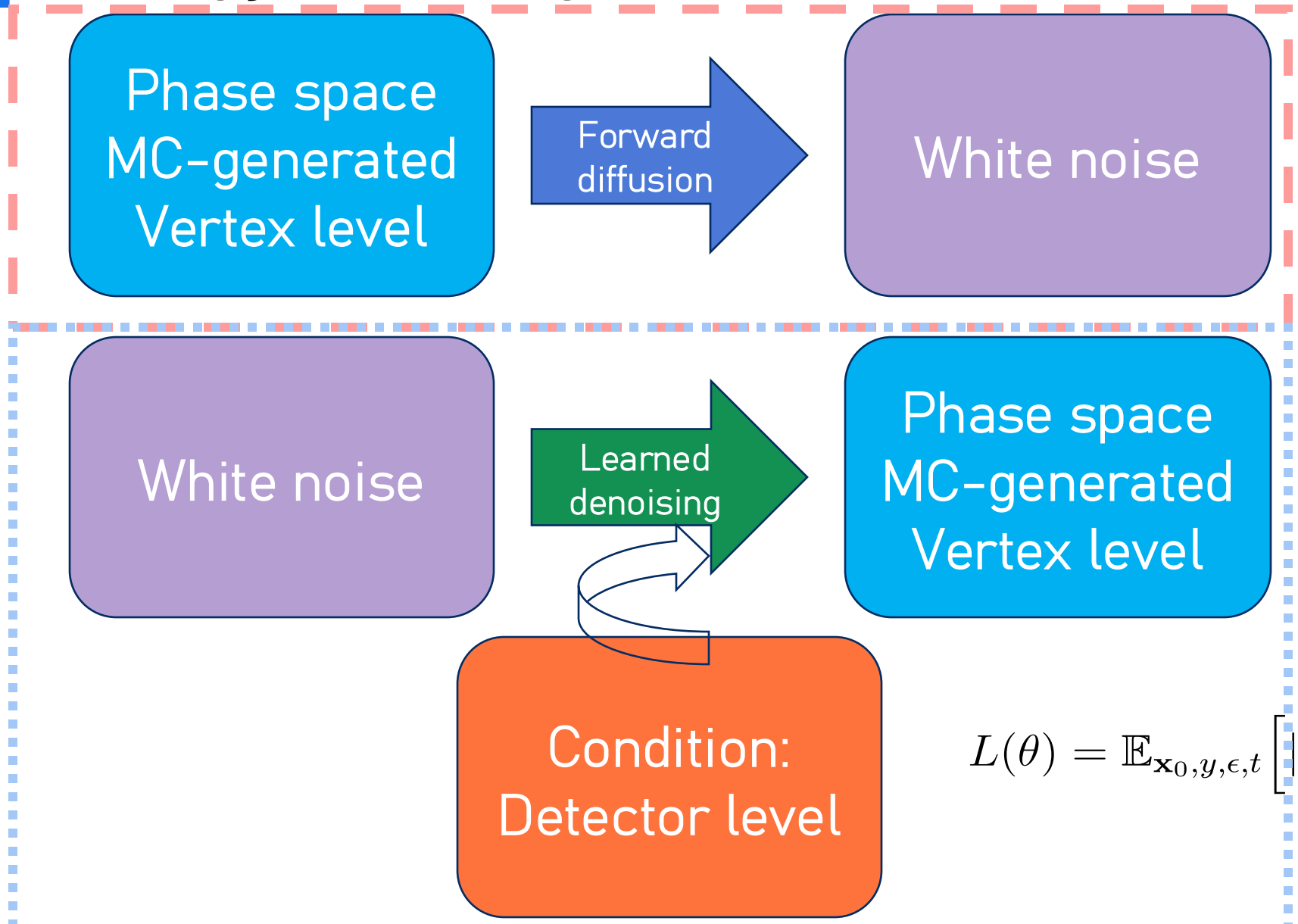
- Let's start from the most populated topology (p π+)



Phase space region covered by CLAS

- Then we will exploit DMs flexibility for moving from one topology to the other

Our strategy: training the Conditional DM



$$L(\theta) = \mathbb{E}_{\mathbf{x}_0, y, \epsilon, t}\left[\|\epsilon - \epsilon_\theta(\mathbf{x}_t, y, t)\|^2\right]$$

Pseudocode for training a DM

- Start with a batch of detector-vertex pairs (x0,y)
- Sample a random t and noise $\epsilon \sim N(0,I)$
- Compute

$$\mathbf{x}_t = \sqrt{\alpha_t}\mathbf{x}_0 + \sqrt{1-\alpha_t}\,\epsilon$$

- The network predicts $\epsilon\theta(xt,y,t)$
- Compute L and backpropagate to update θ

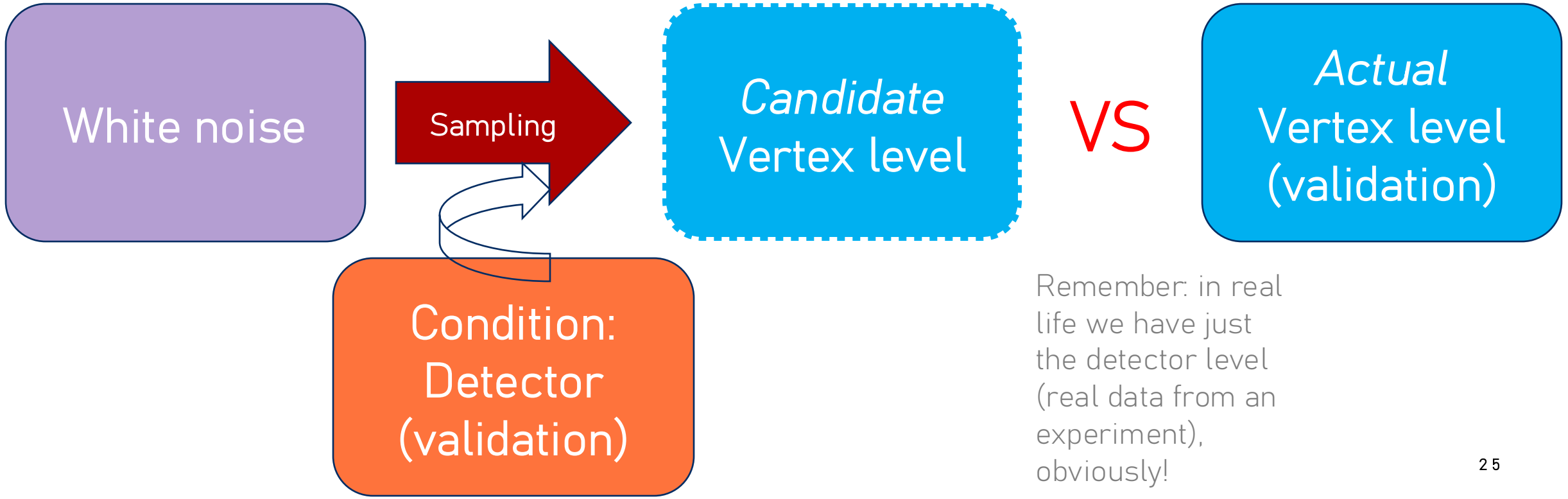
$$L(\theta) = \mathbb{E}_{\mathbf{x}_0,y,\epsilon,t}\left[\|\epsilon - \epsilon_\theta(\mathbf{x}_t, y, t)\|^2\right]$$

- Iterate until convergence

Our strategy: results validation

Once you have a trained model, you can use it to generate as much vertex samples as you want, giving it a validation detector set – independent from the training set – and comparing it with its actual vertex level, to complete the closure test:

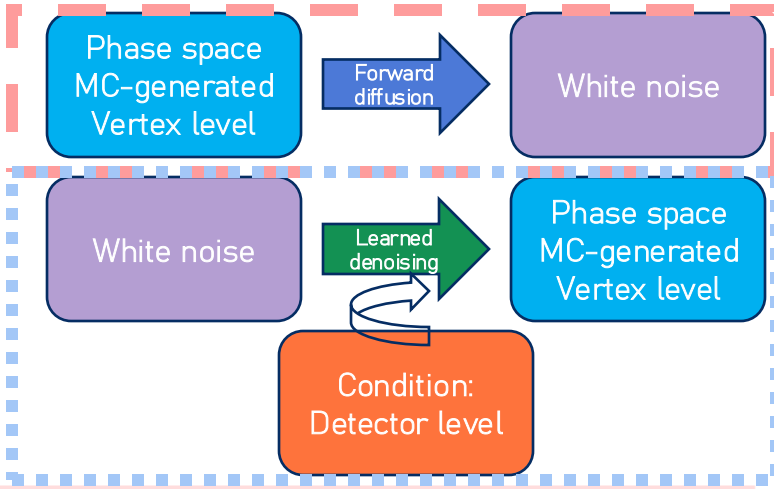
- Compute pulls distributions (pulls = residues normalized to standard deviation)
- Perform statistical tests like Kolmogorov-Smirnov (unbinned! and analogue alternatives)



Remember: in real life we have just the detector level (real data from an experiment), obviously!

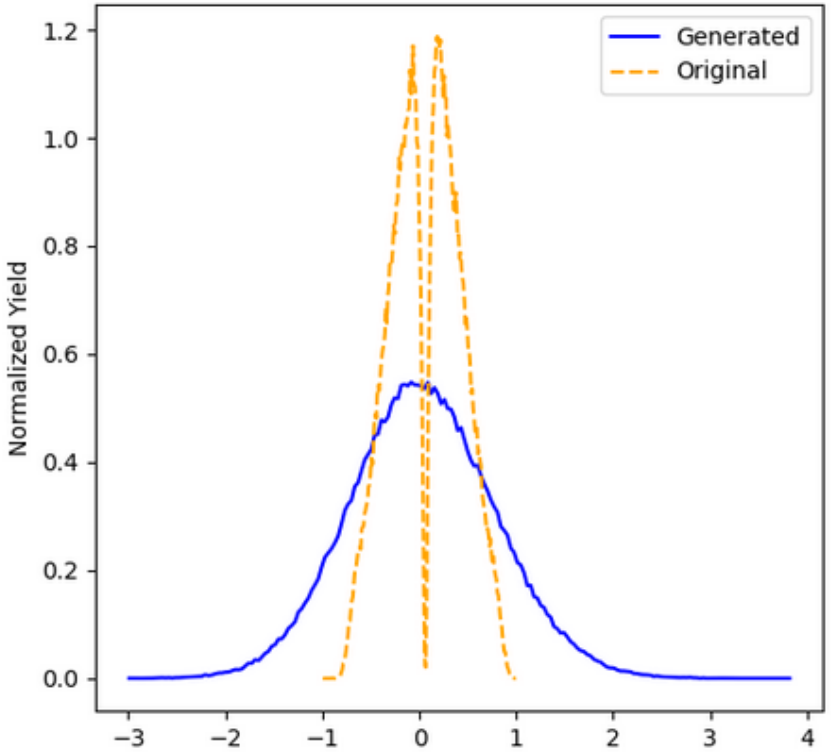
Initial explorative test

- We started training the diffusion model on simulations based on phase space only: generated in the acceptance + reconstructed (gsim detector proxy) (re-mapped into [-1,1] interval)

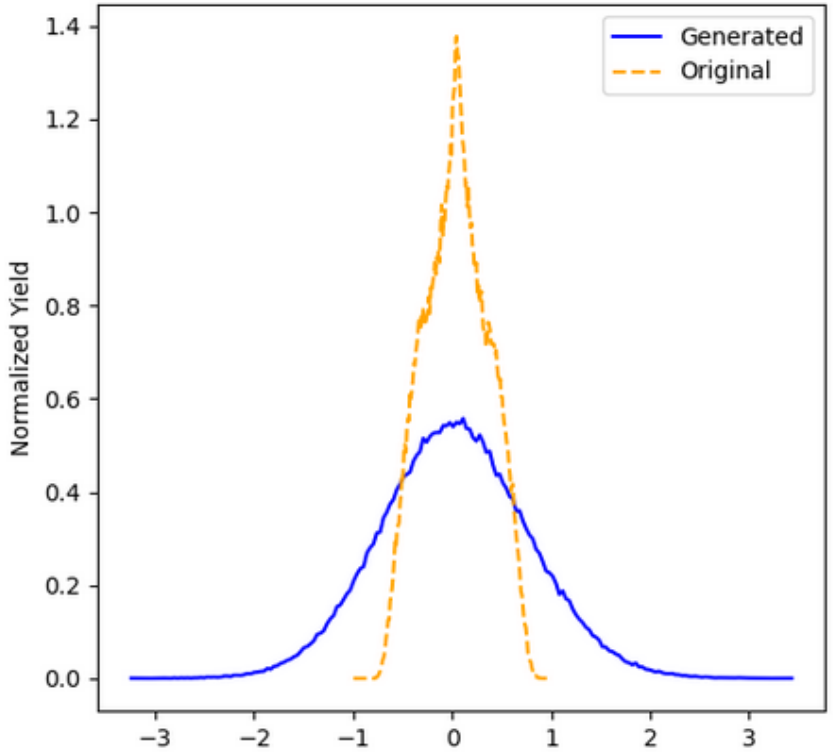



```
Epoch 5/10000 [Loss: 0.209704]
Sampling: 100%|██████████| 100/100 [00:34<00:00,  2.92it/s]
```

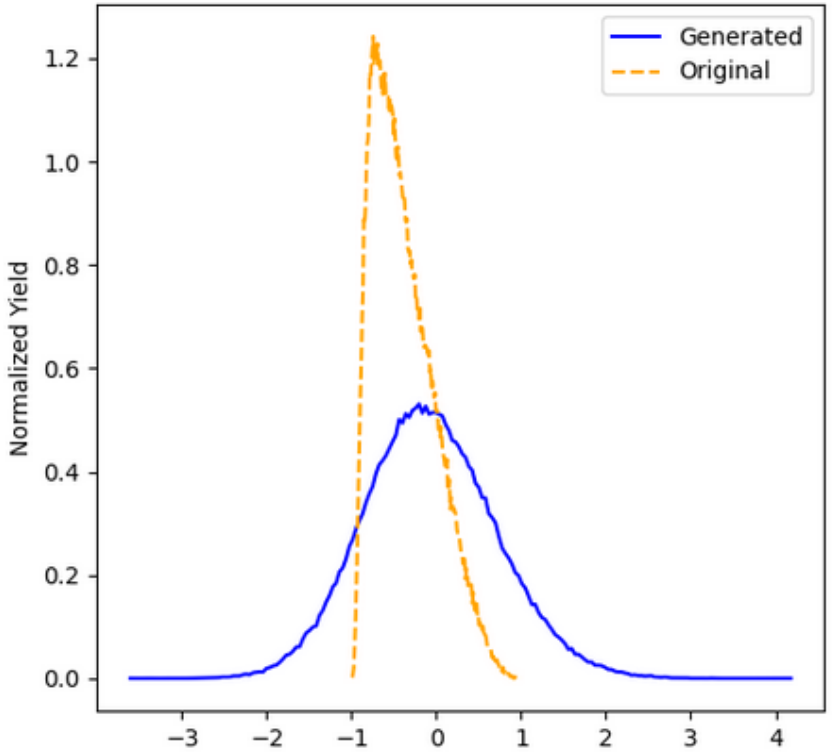
(proton 3-momentum)



px



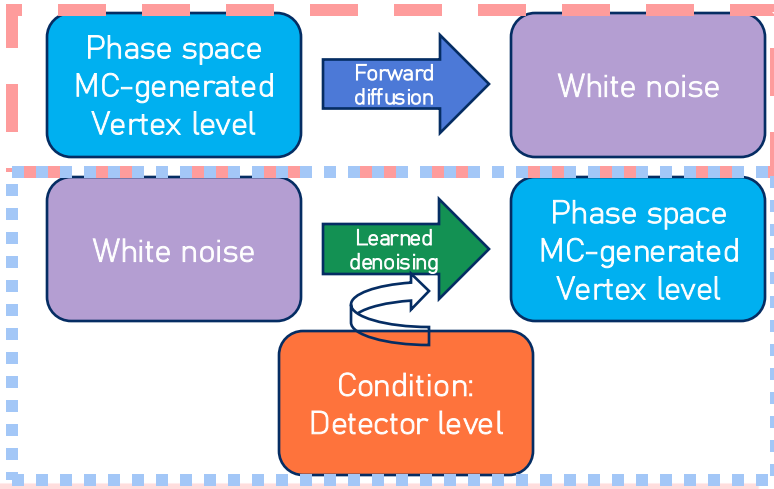
py



pz

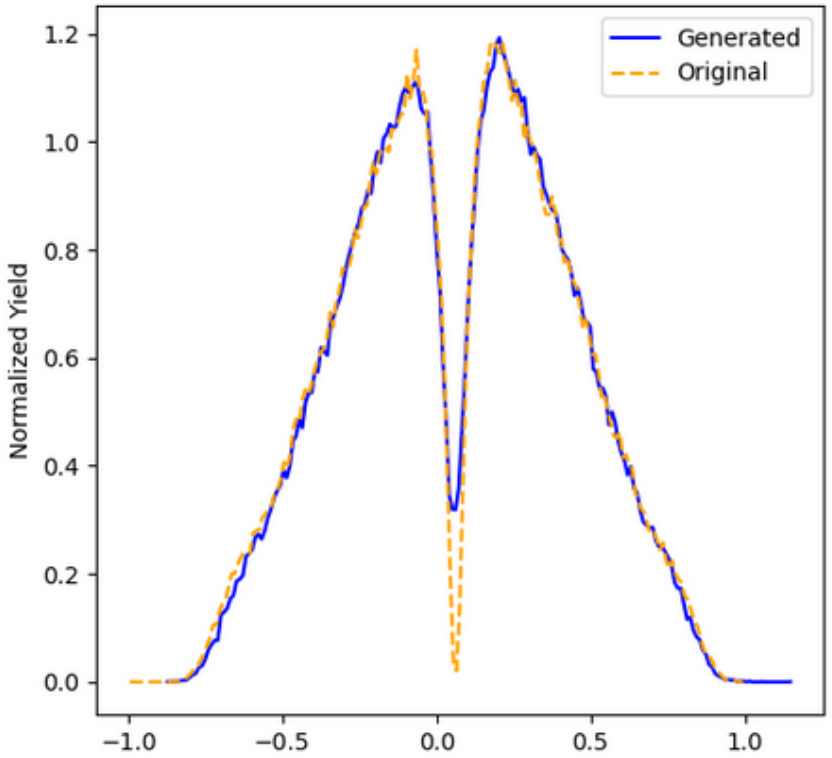
Initial explorative test

- We observed rapid convergence and huge flexibility: training other topologies we mantained the same architecture

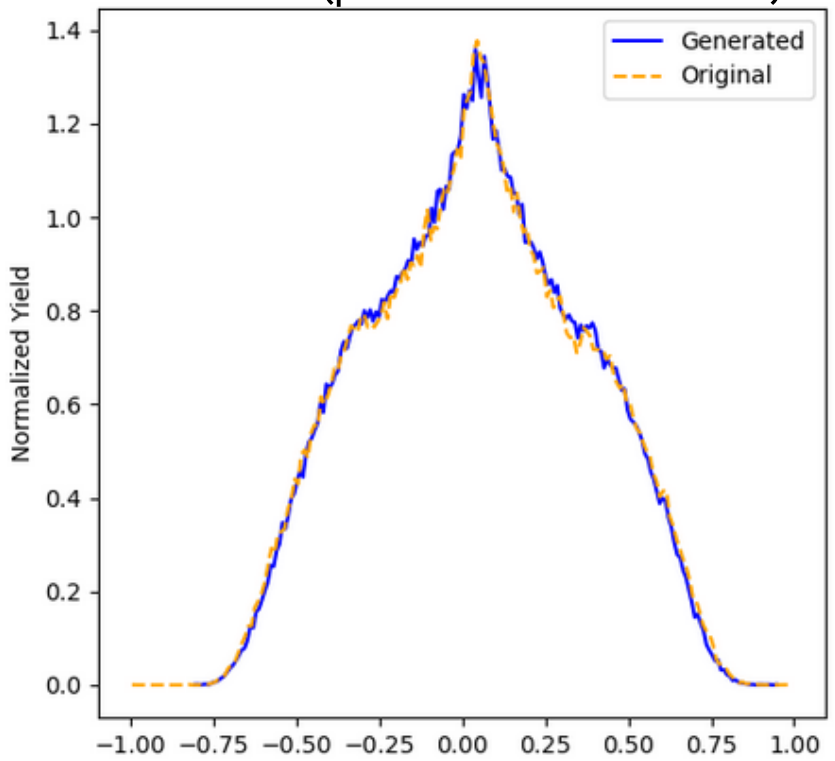



```
Epoch 95/10000 [Loss: 0.018124]
Sampling: 100%|██████████| 100/100 [00:33<00:00,  2.95it/s]
```

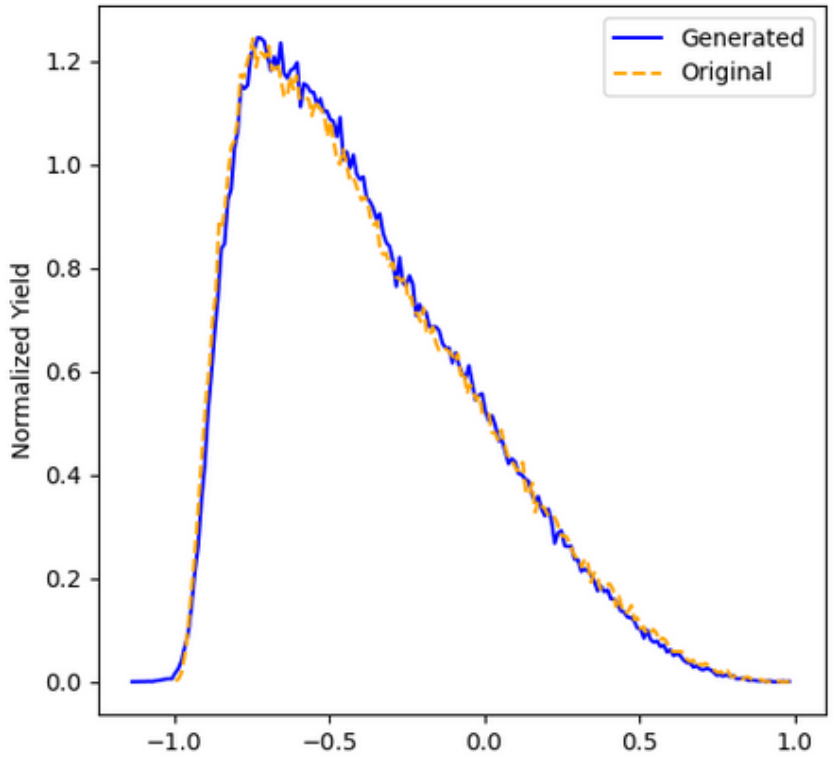
(proton 3-momentum)



p_x



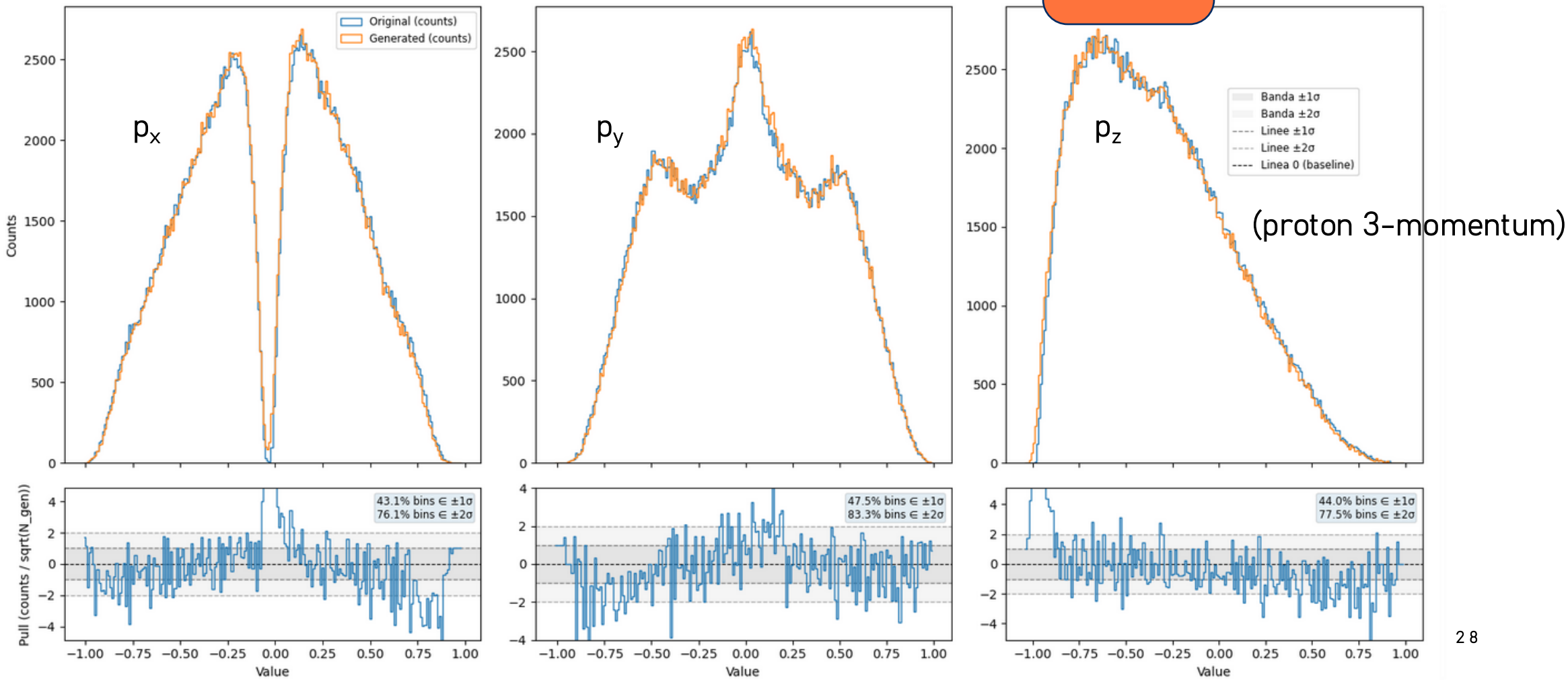
p_y



p_z

Initial explorative test

- We observed rapid convergence and huge flexibility: training other topologies we manteined the same architecture



(proton 3-momentum)

Current status of the project

- We want to have under control the mapping between the vertex and the detector in the **entire measured phase space**, so we are focusing on a complete set of Mandelstam invariants (they are 5, for a 2->3 process)
- Training and validation sets have the same size.
- Training observables:

$$s_{23} = (p_{\pi^-} + p_f)^2$$
$$u_2 = (p_{\pi^-} - p_i)^2$$
$$s_{12} = (p_{\pi^+} + p_{\pi^-})^2$$
$$s_{13} = (p_{\pi^+} + p_f)^2$$
$$u = (p_{\pi^+} - p_i)^2 .$$

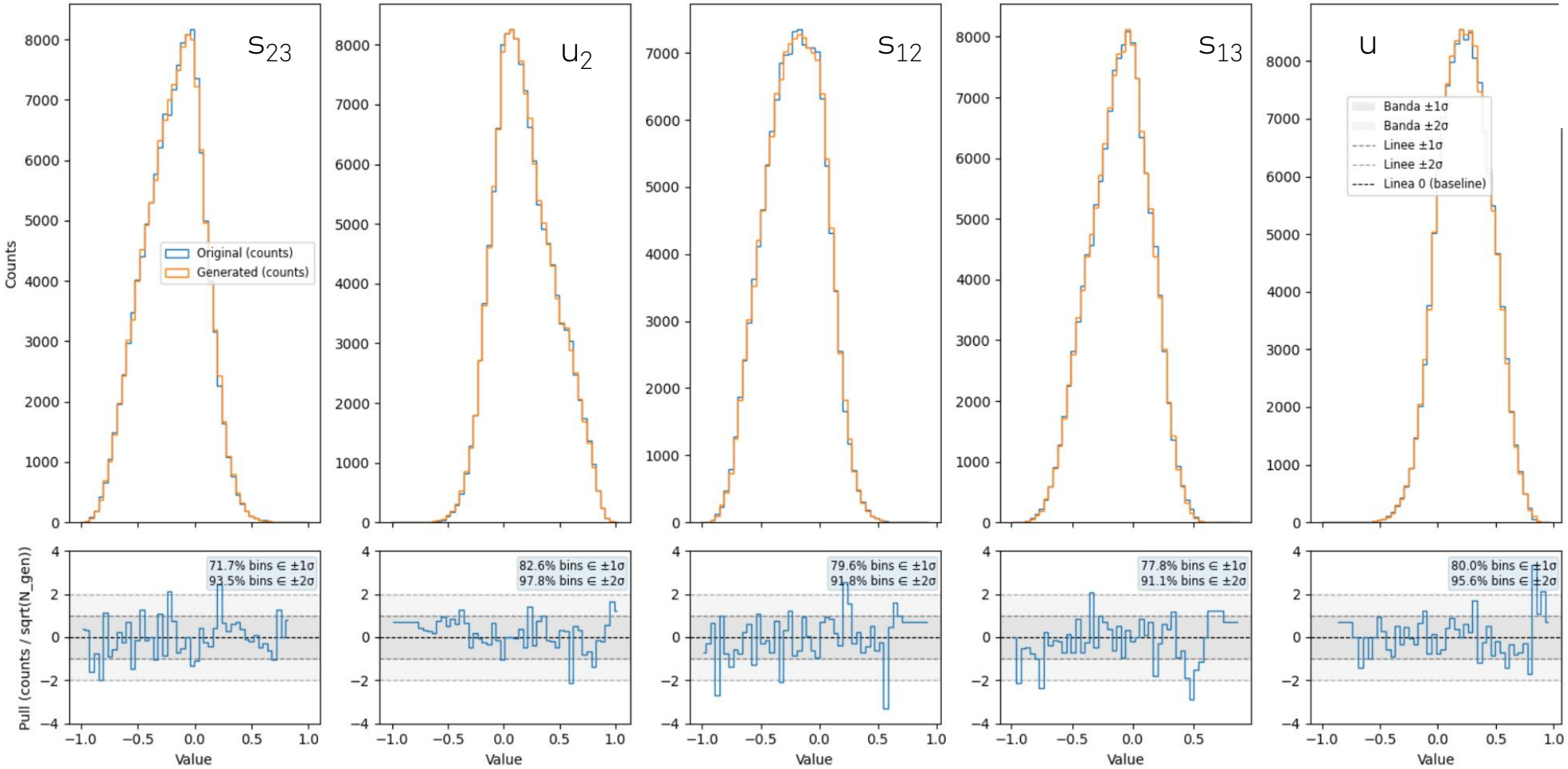
$$\gamma\, p_1 \rightarrow \pi^+ \pi^- p_2$$

- To eliminate outliers, we cut in s in [5.9,8.4] GeV^2. (s_{max} is fixed by the energy of the beam~3.7 GeV).
- Every variable is transformed according to the map $x \rightarrow sign(x) \cdot (1.0 - (1.00000001 - |x|)^{\alpha}) \, , \alpha = 0.4$

to reduce the slope at the borders of the domain.

Current status of the project

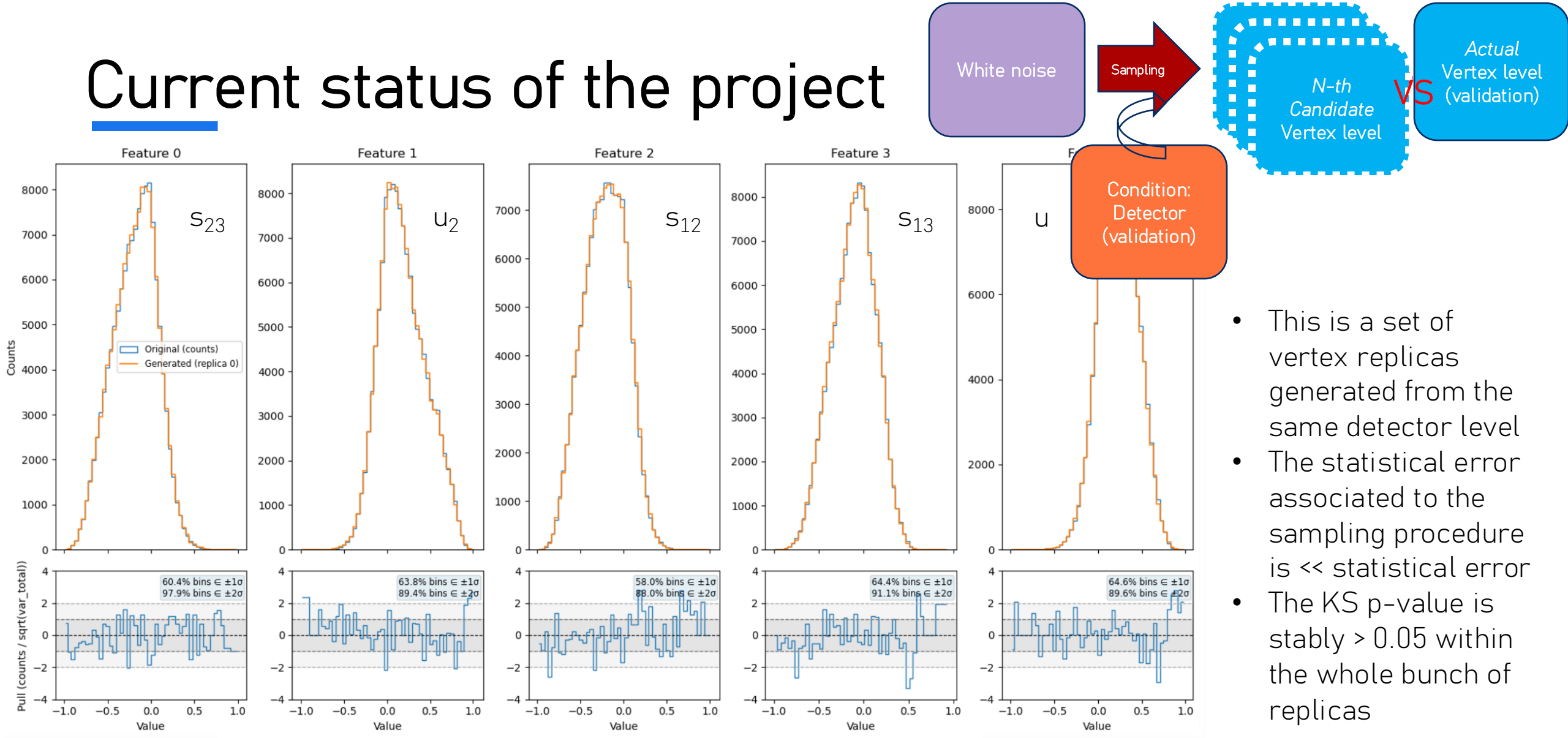
$$s_{23} = (p_{\pi^-} + p_f)^2$$
$$u_2 = (p_{\pi^-} - p_i)^2$$
$$s_{12} = (p_{\pi^+} + p_{\pi^-})^2$$
$$s_{13} = (p_{\pi^+} + p_f)^2$$
$$u = (p_{\pi^+} - p_i)^2 .$$



More than 90% of sampled data **within 2 sigma**, and we haven't included the systematics errors (for sure dominant over the statistical one) yet!

KS-test: p-value = { 0.70, 0.82, 0.23, 0.92, 0.26 }: all greater than the threshold for rejecting the null hypothesis, 0.05!

Current status of the project

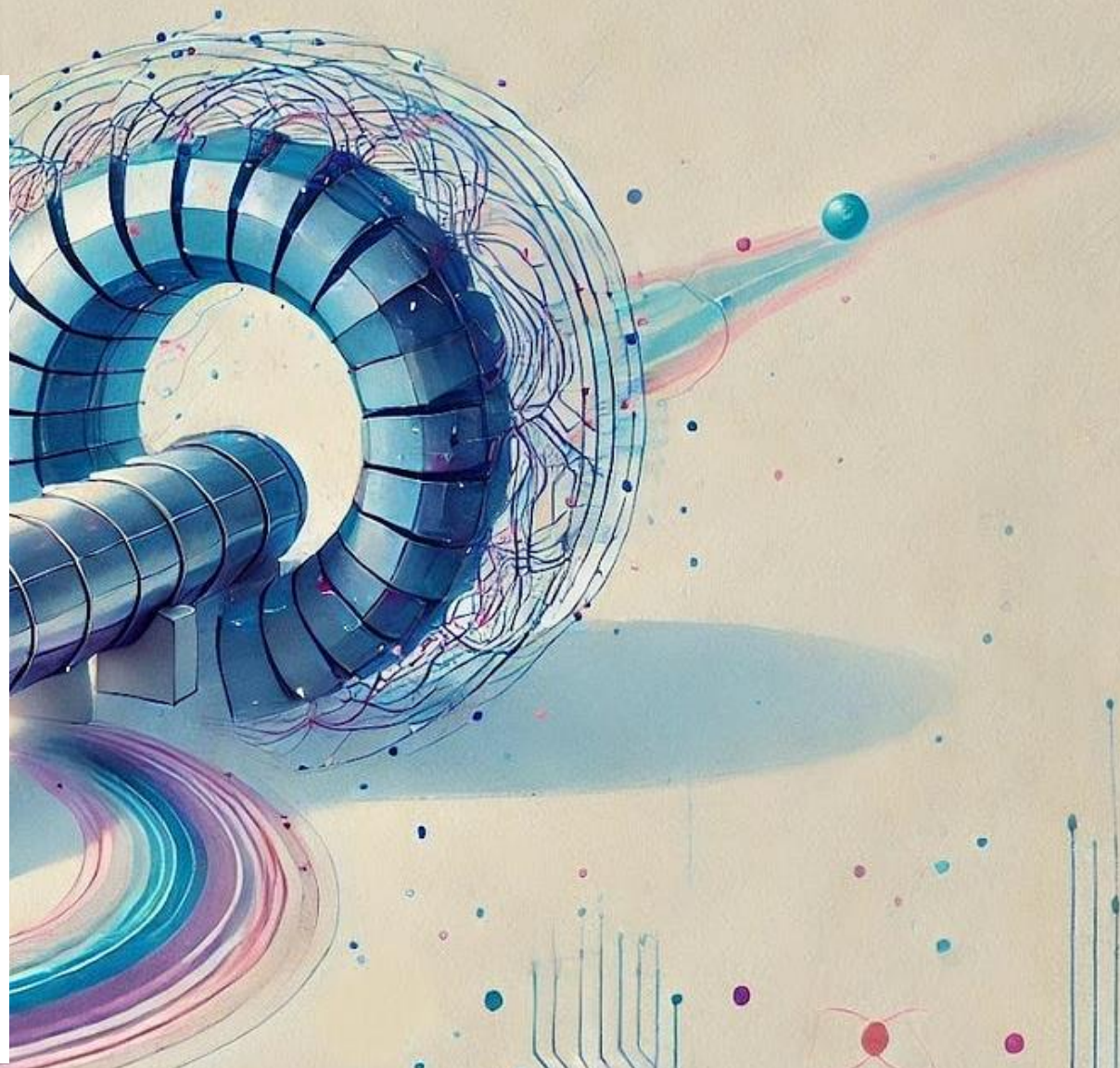



```
Feature 0: KS p-value mean=0.6699, 16-84% = [0.6121, 0.7614]
Feature 1: KS p-value mean=0.6958, 16-84% = [0.6280, 0.7498]
Feature 2: KS p-value mean=0.2347, 16-84% = [0.1418, 0.3123]
Feature 3: KS p-value mean=0.8966, 16-84% = [0.8750, 0.9413]
Feature 4: KS p-value mean=0.4086, 16-84% = [0.3372, 0.4870]
```

- This is a set of vertex replicas generated from the same detector level
- The statistical error associated to the sampling procedure is << statistical error
- The KS p-value is stably > 0.05 within the whole bunch of replicas

Summary

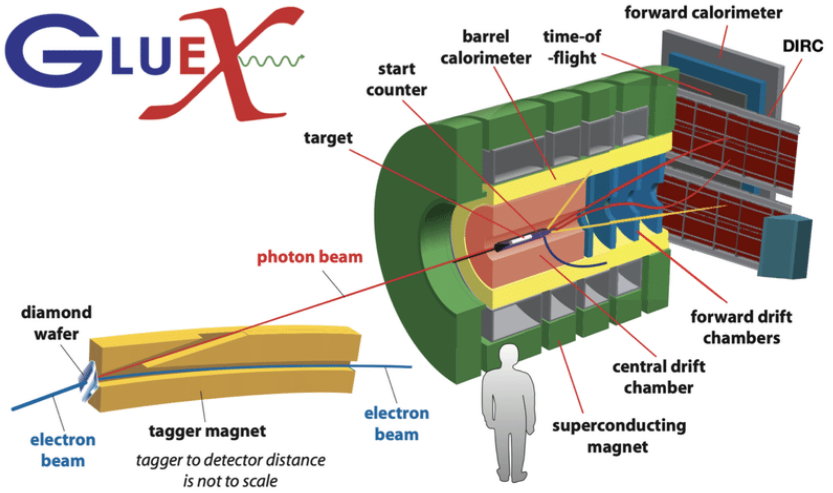
- Unfolding the detector effect through generative AI
- How a diffusion model work
- Status of closure test for CLAS detector
- Conclusions & Next directions



Conclusions

- **We successfully trained** a DM to map vertex and detector level related to the entire phase space simulated for the reaction of photoproduction of a charged pion pair on proton, smeared by the GSIM proxy of CLAS detector
- We can safely substitute the real vertex distribution with the analogue sampled by our DM **within 2 sigma,** including just statistical uncertainties (road to a digital twin) and systematic related to sampling variance (completely negligible)
- **Uncertainty quantification:** to complete this closure test, rigorous systematic uncertainty estimates for unfolded results are needed. Incorporating other sources of **systematic effect** (e.g. systematic errors due to different architectures) is crucial for having a reliable alternative to the traditional unfolding procedures.
- **Challenging events:** special attention must be given to particles outside detector acceptance or **rare phase-space regions** (e.g. the regions at the borders of two different topologies). The model should ideally be able to properly treat also the **outliers**.

Next directions



- Enlarge as much as possible the measured phase space adding **all topologies** through this diffusion model unfolding procedure
- **Application of our DM & Collaboration with GlueX** people to enlarge it way more including the phase space measured by them: note that - independently on the techniques that they will use to unfold GlueX detector effect - we will be able to compare momenta distribution extracted at the vertex level by us and them in the region overlapped by the two detectors -> this is another sanity check that our DM-unfolding procedure works
- This - as far as we know – never tried approach can **reduce a lot the model dependence** that we introduce extrapolating the cross section in the unmeasured phase space region


Thank you for
your attention!

Backup

Structure of the Net