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MOdeling Nuclear STructure and REactions

Nuclear Equation of State: from the Lab to the Stars

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UNIVERSITAT DE BARCELONA

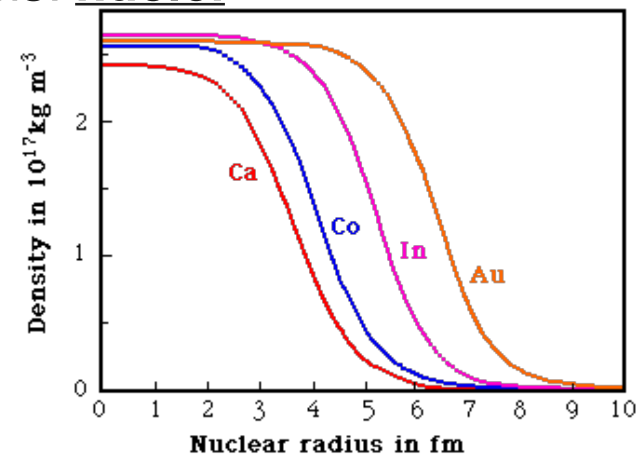
Where can we find neutrons and protons? And in which form? Free? In clusters?

Neutrons and protons in **Earth** are found in cluster systems: **nuclei**

→ The **interior** of all nuclei has **constant density** (10^{14} denser than water) named **saturation density**

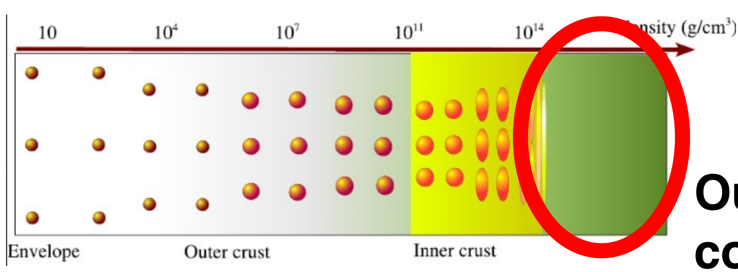
→ Saturation is originated from the short range nature of the **nuclear effective interaction**

→ Neutron in 15 minutes must find a proton or decay

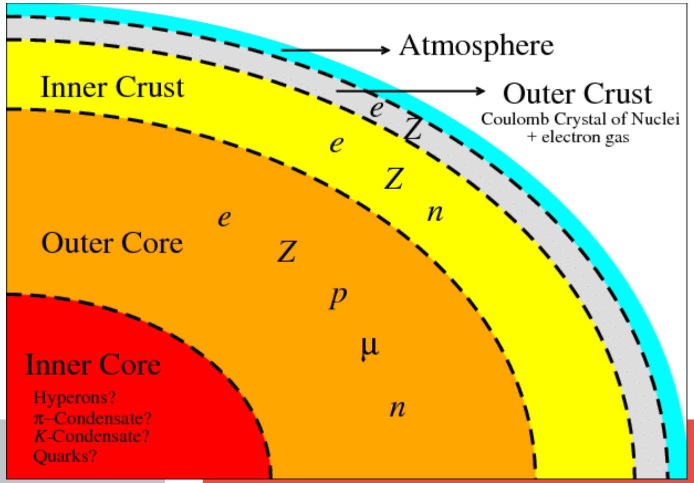


In **heavens**, neutrons and protons can be also found as an interacting sea of fermions (Fermi liquid): **matter in <u>the outer core</u> of a neutron star**

→ Densities can reach several times nuclear saturation



Outer core



Nuclear Equation of State (EoS)

<u>Definition:</u> the **energy** (E) per **nucleon** (A=N+Z), $e \equiv E/A$, of an **uniform system of neutrons** (N) **and protons** (Z) as a function of the **neutron** (ρ_n = N/V) and **proton** (ρ_p = Z/V) **densities …**

→ **… at <u>zero temperature</u>:** room temperature 10^2K→10^{-8} MeV while "cold" neutron stars are at about 10^{10}K→1 MeV. **Separation energy** in stable nuclei (equivalent to ionization energy in atoms) is of **several MeV**.

→ … **<u>unpolarized</u>:** energy favors **couples** of neutrons and protons **occupying the same state but with opposite spins** (equivalent to electrons in atoms)

→ … **<u>isospin symmetric</u>:** neutron-neutron, proton-proton and neutron-proton **nuclear interaction** are very **similar** among them. **Masses** of neutrons and protons are almost **degenerate**. Hence neutrons and protons can be thought as **two states** of the **same particle** with different isobaric spin or **isospin** (in analogy with spin): the **nucleon.**

→ … and **<u>no Coulomb</u>: idealized uniform system** (focus on strong interaction, much stronger than Coulomb at nuclear scale $\sim 10^{15}$ fm). Real systems are finite and electrically neutral so no problems (divergences) in adding Coulomb.

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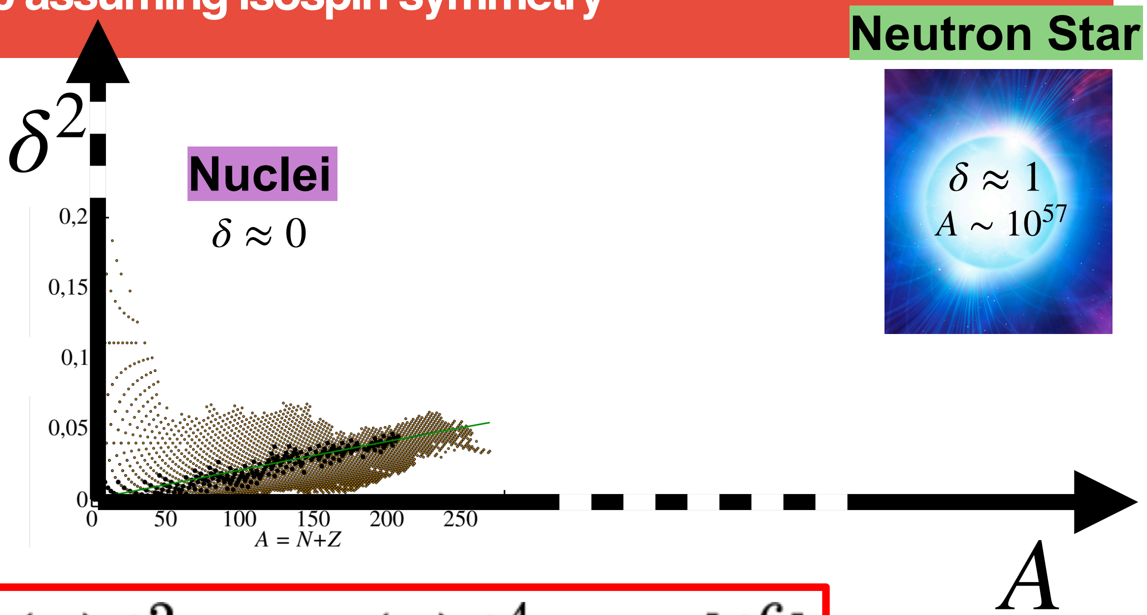
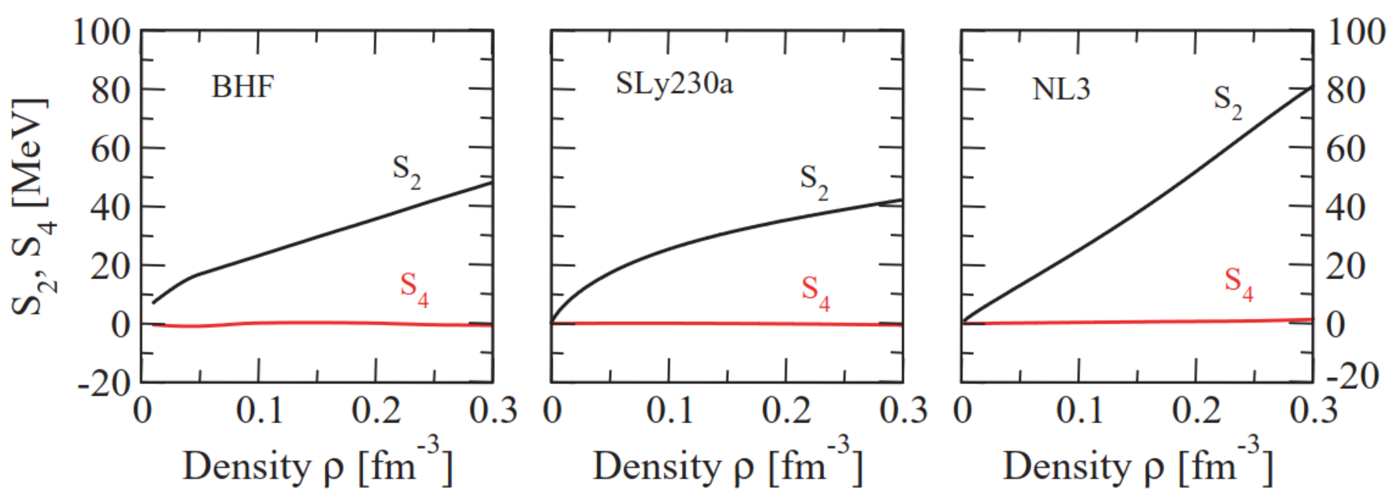
Nuclear Equation of State (EoS)

Unpolarized, uniform nuclear matter at T=0 assuming isospin symmetry

It is convenient to write the energy per nucleon (***e***) as a **function** of the **total density** **$\rho=\rho_n+\rho_p$** and the **relative difference** **$\delta=(\rho_n-\rho_p)/\rho$** for **$\delta \to 0$**:



$$e(\rho,\delta) = e(\rho,0) + S_2(\rho)\delta^2 + S_4(\rho)\delta^4 + \mathcal{O}[\delta^6]$$



Isaac Vidaña, Constança Providência, Artur Polls, and Arnau Rios
Phys. Rev. C **80**, 045806 – Published 23 October 2009

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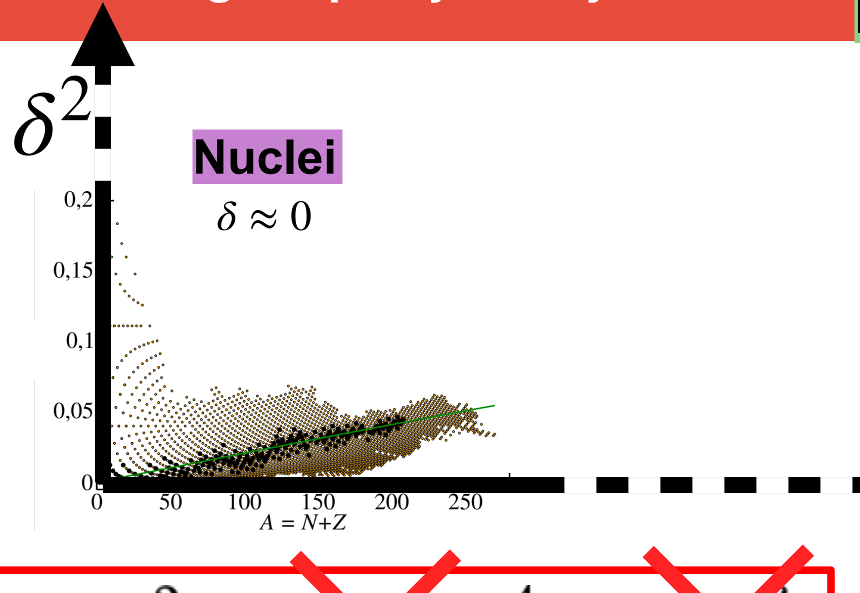
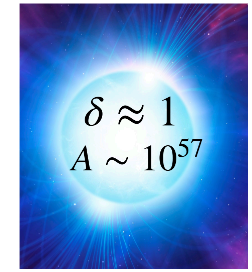


Neutron Star



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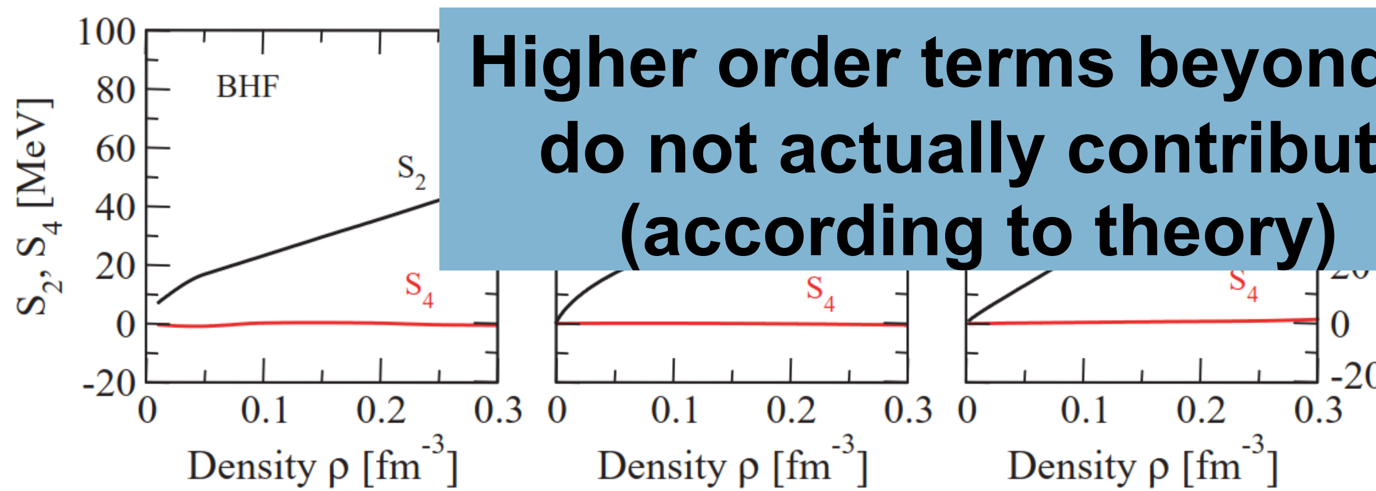

Higher order terms beyond δ^2 do not actually contribute (according to theory)

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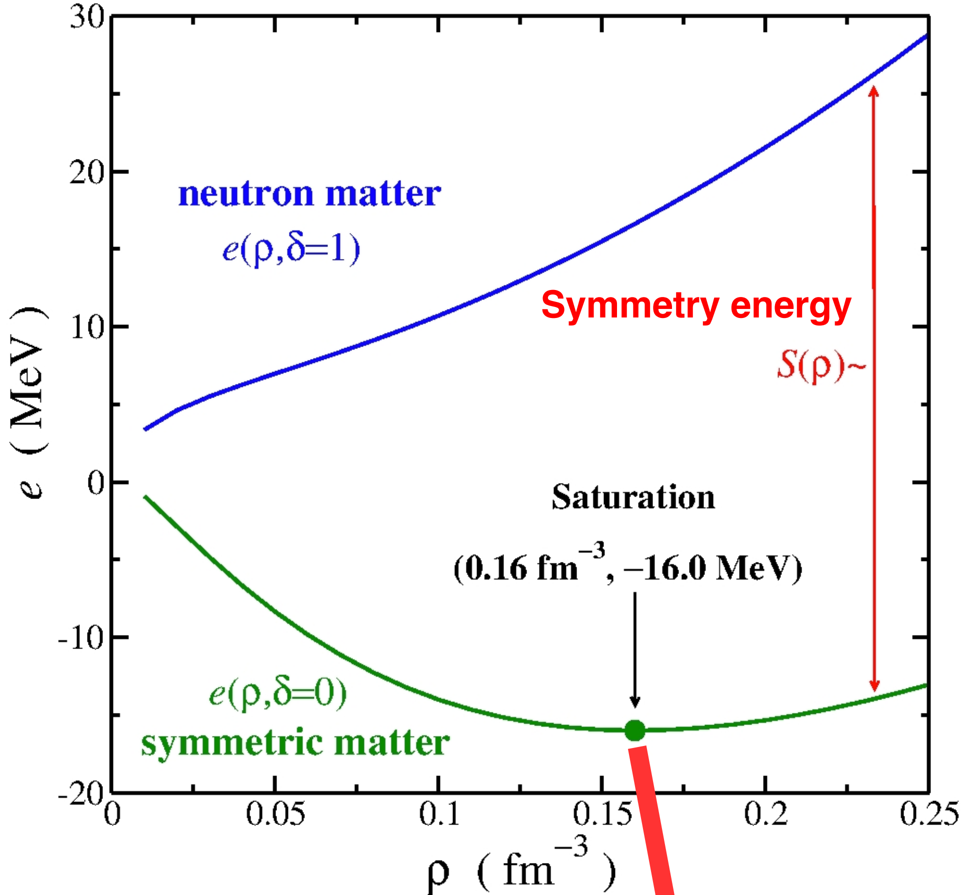


It is customary to **Taylor expand** e(ρ,0) and S(ρ) around **nuclear saturation density** ρ_0 ~ 0.16 fm^{-3}

$$e(\rho,0) = e(\rho_0,0) + \frac{1}{2}K_0x^2 + \mathcal{O}[\rho^3] \text{ where } x = \frac{\rho-\rho_0}{3\rho_0}$$

$$S(\rho) = J + Lx + \frac{1}{2}K_{\text{sym}}x^2 + \mathcal{O}[\rho^3,\delta^4]$$

K → how **compressible** is matter @ ρ_0

J → **penalty energy** for systems with $\rho_n \neq \rho_p$ @ $\rho_n + \rho_p = \rho_0$

L → **pressure** for systems with $\rho_n \neq \rho_p$ @ ρ_0

$$P(\rho=\rho_0,\delta=0) = 0 \ \text{MeV fm}^{-3}$$

How we determine the properties of the nuclear equation of state ?

(Some examples)

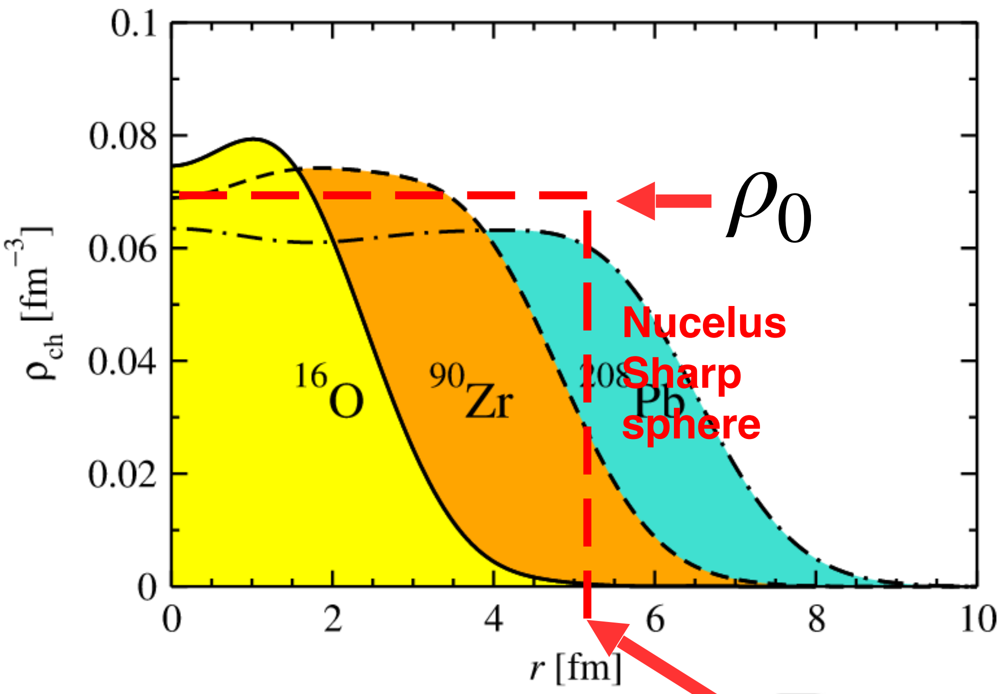
From electric charge radius of a nucleus:

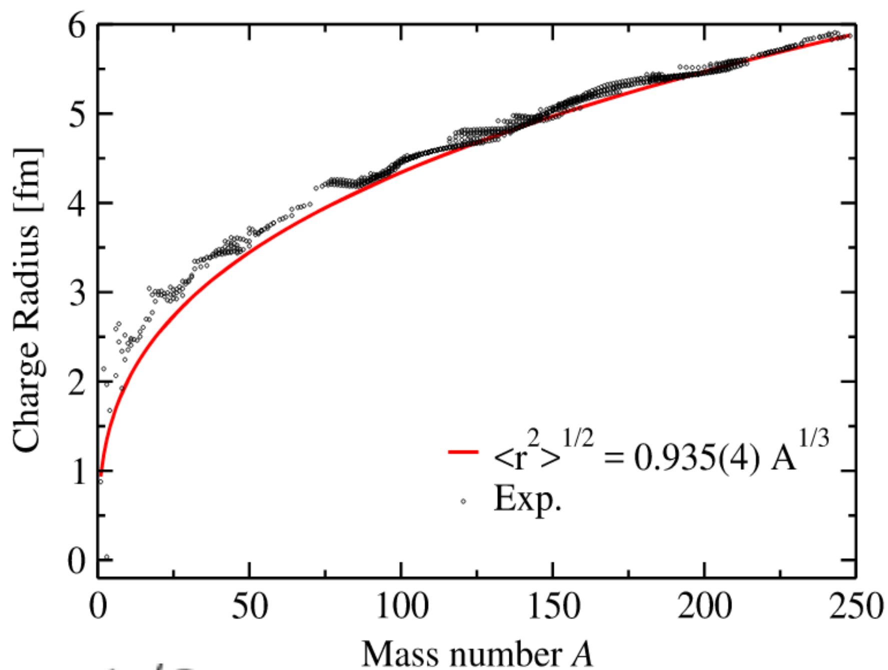
saturation density $\rho_0 \approx 0.16\ \mathrm{fm}^{-3}$

→ **Range of the nuclear interaction** ($\hbar c\ /\ m_\pi \sim$ 1 - 2 fm) typically **shorter** than the **size** of the **nucleus.** Hence, neutrons and protons just “see” their closest neighbours.

→ **Experimental charge (Z) density** in the interior of very **different nuclei** is rather constant at around **0.06-0.08 fm^{-3}**.

→ **Saturation mechanism** (equilibrium) that originates from the **short-range nature of the nuclear force,** much stronger than the Coulomb repulsion at the nuclear scale.



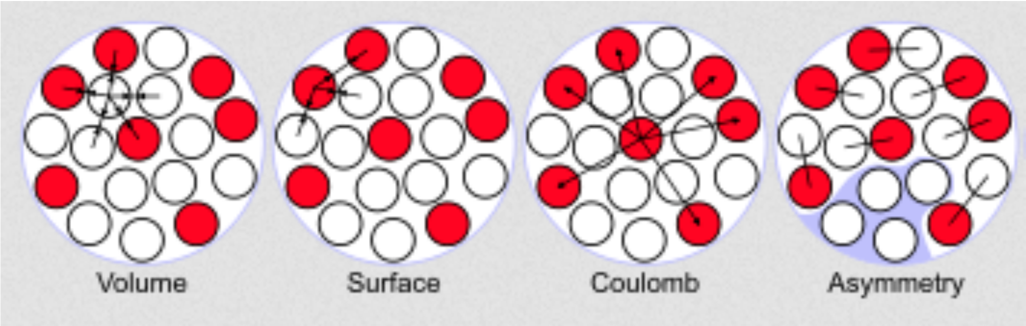


$$R \approx r_0 A^{1/3}$$

From binding energy of a nucleus:

saturation energy $e(\rho_0,0) = a_v(A$ very large$) \approx 16$ MeV

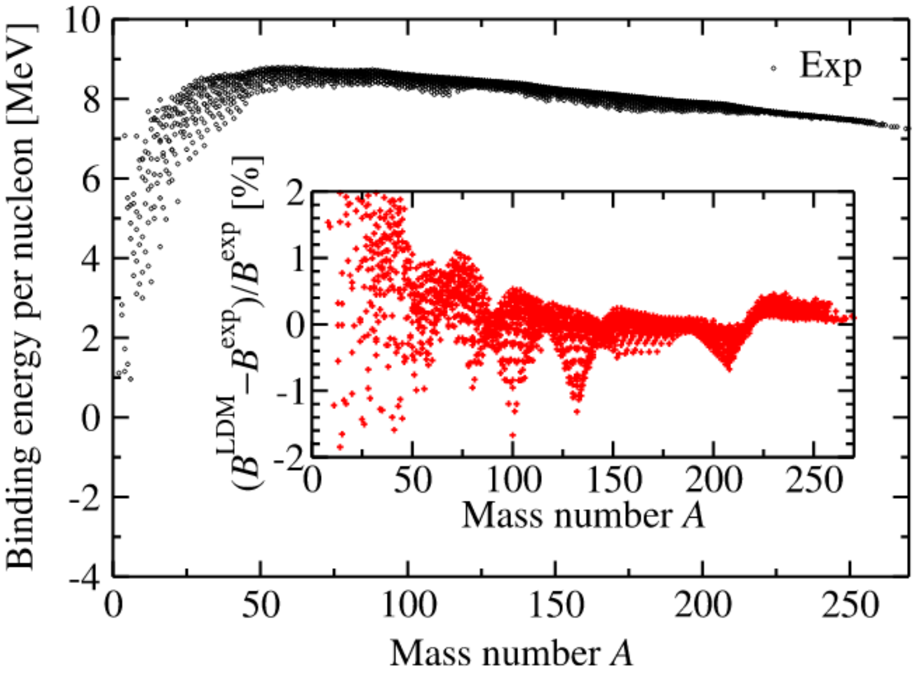
→ Nucleus seen as an incompressible liquid (ideal) drop: sharp sphere of radius $R = r_0 A^{1/3}$



R^3 R^2 R^{-1}

$$M(A,Z) = m_p Z + m_n(A-Z) - B(A,Z)$$

$$\frac{B(A,Z)}{A} \xrightarrow{A\to\infty} e_0 \approx 16 \text{ MeV}$$



Important!!

→ A **small change** in the **saturation density** will **impact** the **size** of the **nucleus. Charge radii** are determined to an average accuracy of 0.016 fm (Angeli 2013).

For example, if one aims at determining the **r_{ch} = 5.5012±0.0013 fm** in **^{208}Pb** one must be **very precise** in the determination of **ρ_0**:

$$\frac{\delta\rho_0}{\rho_0} = -3\frac{\delta R}{R} \rightarrow \frac{\delta\rho_0}{\rho_0} \lesssim 0.1\%$$

Note: typical average theoretical deviation of accurate nuclear models ~ 0.02 fm → **$\delta\rho_0/\rho_0$** is determined up to about a **1% accuracy** (That is, third digit in $\rho_0 \approx 0.16$ fm^{-3}**!!**).

→ In a similar way, a **small change** in the **saturation energy** (about $e_0 \approx -16$ MeV) will **impact** on the **nuclear mass.**

For example, if one aims at determining the **B = 1636.4296±0.0012 MeV** in **^{208}Pb** one must be **very precise** in the determination of **e_0**:

$$\frac{\delta B}{B} = \frac{\delta e_0}{e_0} \rightarrow \frac{\delta e_0}{e_0} \lesssim 10^{-6}$$

Note: typical average theoretical deviation of accurate nuclear models ~ 1-2 MeV → **$\delta e_0/\rho e_0$** is determined up to about a **0.1% accuracy** (That is, second decimal digit in $e_0 \approx -16.0$ MeV**!!**).

Neutron and proton radii difference

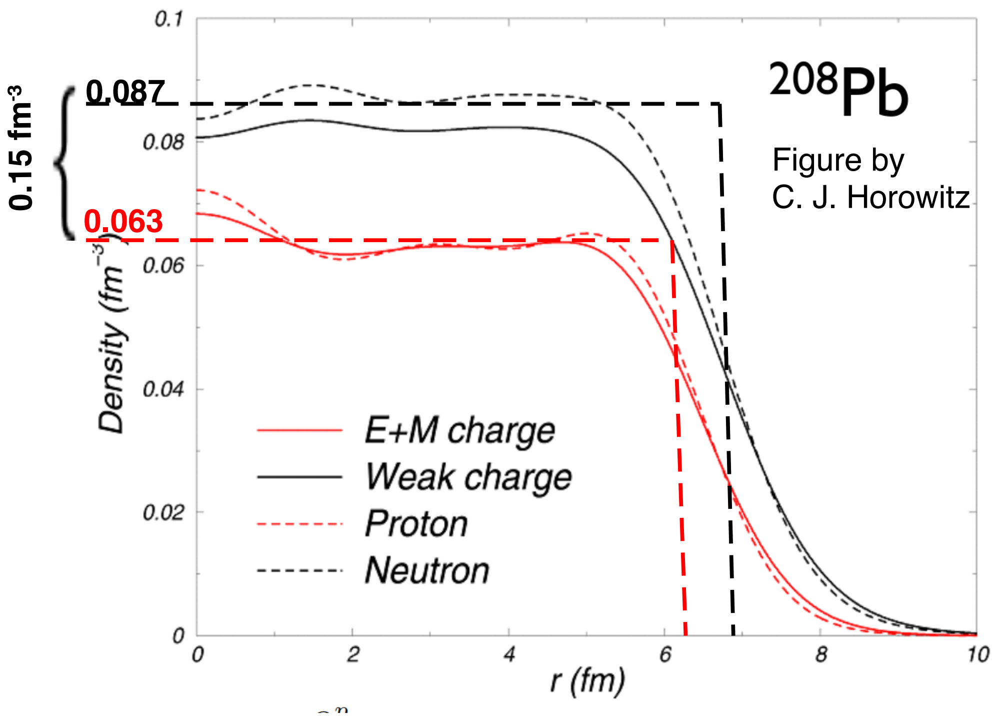
symmetry energy slope L

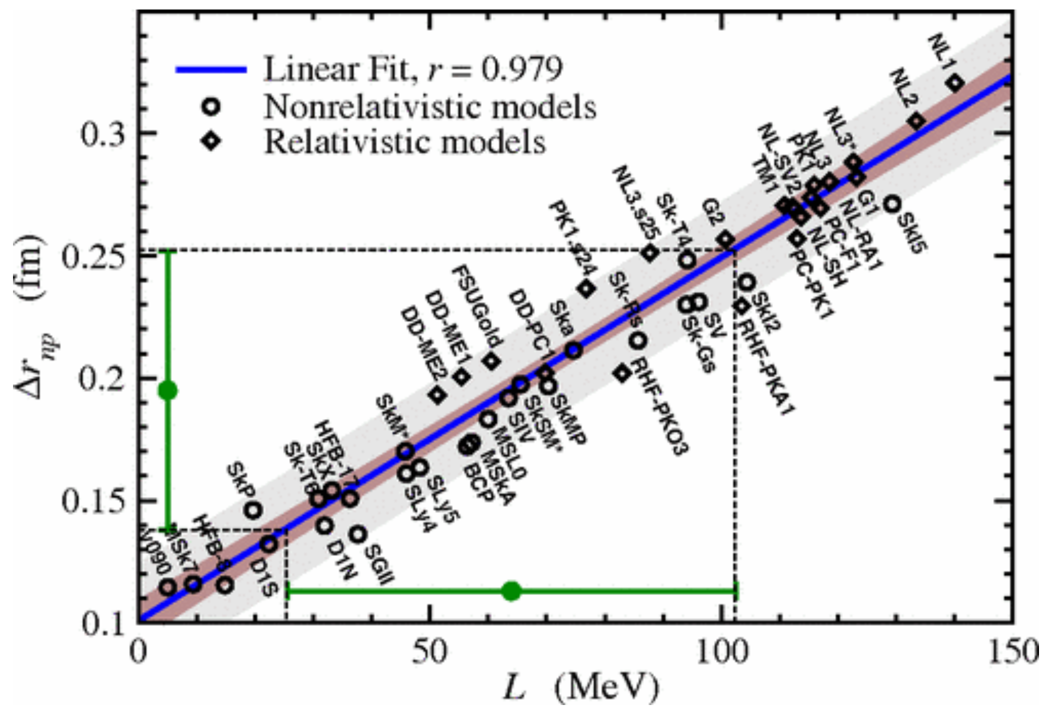
Elastic electron scattering → electromagnetic size nucleus ↔ ρ_p ($Q^p_{ch} = 1; Q^n_{ch} = 0$)

Parity violating electron scattering → weak size nucleus ↔ ρ_n ($Q^p_W \approx 0; Q^n_W \approx 1$)

If $N \neq Z$ we expect $\mathbf{R}_n$ could be different from $\mathbf{R}_p$

$$\Delta r_{np} \equiv \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$$

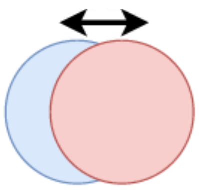




What happens if we perturb the ground state densities of a nucleus? Restoring force drive the dynamics

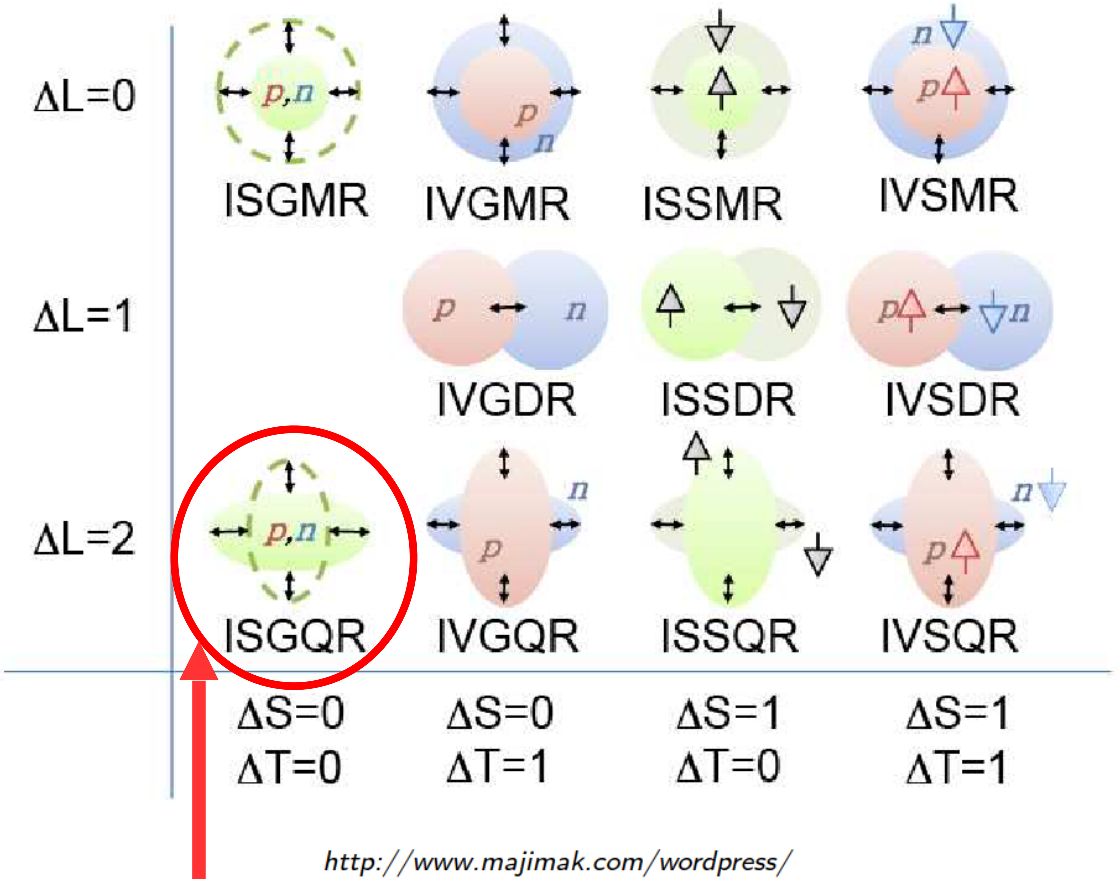
Produce a **small displacement (dl)** between **neutron and proton densities** (drops)

$$\rho = \rho_0 + \delta\rho_0 \approx \rho_0 + \vec{dl} \cdot \vec{\nabla}\rho_0$$



(Linear response theory)

Under different types of perturbations, **nuclei use to show resonant behaviors** where all nucleons oscillate coherently and the nucleus as a whole vibrate at an specific resonant energy → known as **Giant Resonances**

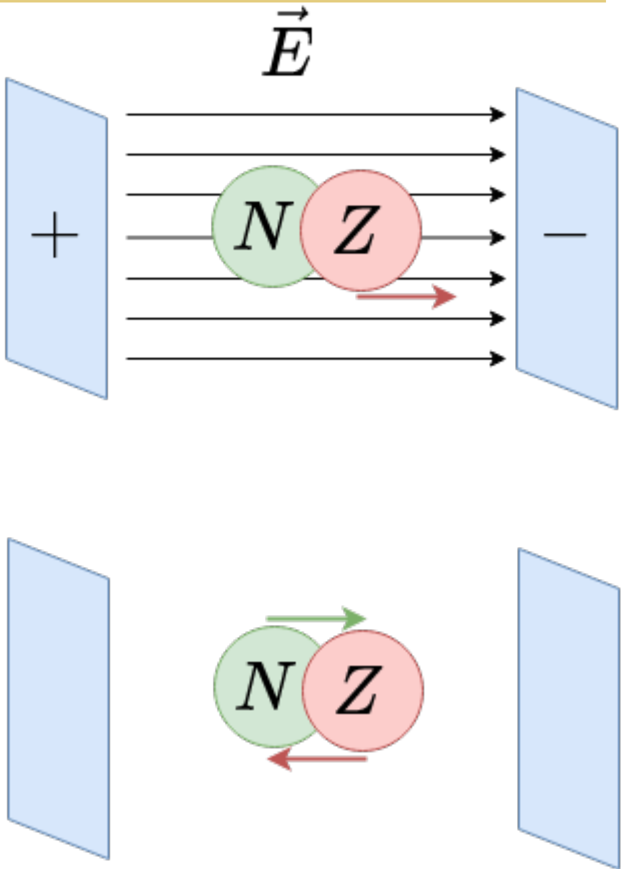


(By the way: **dominant type oscillation mode of a Neutron star in a Neutron star merger)**

Dipole polarizability
(Giant Dipole Resonance)

As in Electromagnetism course in the Physics degree, the **electric polarizability** measures **tendency** of the nuclear **charge distribution** to be **distorted**

$$\alpha = \frac{\text{electric dipole moment}}{\text{external electric field applied}}$$



How easy is to separate neutrons from protons? Symmetry energy will tell (Harmonic Oscillator model)

$$e(\rho, \delta) = e(\rho, 0) + S(\rho)\delta^2$$

$$E_x \sim \sqrt{\frac{\partial^2 e(\rho, \delta)}{\partial \delta^2}} \sim \sqrt{S(\rho)}$$

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Tidal deformability in a neutron star

↕

quadrupole polarizability

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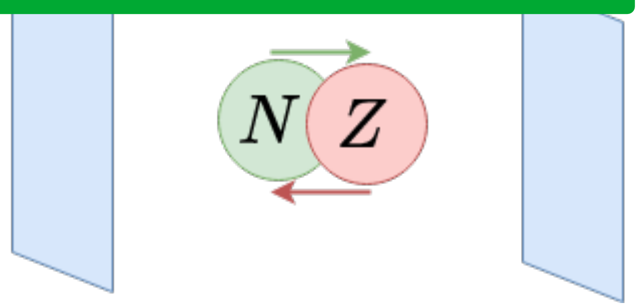


Giant Monopole Resonance (GMR)

Monopole resonance can be imagined macroscopically as an **isotropic change in the volume** of the nucleus (do not depend on the orbital angular momentum or spin): **breathing mode**

$$F = \sum_{i=1}^{A} r_i^2$$

Isotropic harmonic perturbation!

One can define the **excitation frequency** from the moments (m) of the **response function**:

$$m_1 = \frac{2\hbar^2}{m}\langle r^2 \rangle \qquad \frac{1}{m_{-1}} = 2\frac{\partial^2 \langle \mathcal{H} \rangle}{\partial \langle r^2 \rangle^2}$$

We can measure how incompressible is a nucleus!!!

Therefore,

$$(E_x^{\mathrm{ISGMR}})^2 = \frac{m_1}{m_{-1}} = 4\frac{\hbar^2}{m}\langle r^2 \rangle \frac{\partial^2 E}{\partial \langle r^2 \rangle^2} = 4A\frac{\hbar^2}{m\langle r^2 \rangle}\langle r^2 \rangle^2 \frac{\partial^2 (E/A)}{\partial \langle r^2 \rangle^2} \equiv K_A \frac{\hbar^2}{m\langle r^2 \rangle}$$

Extrapolate from finite A to very large A → incompressibility of the EoS

Neutron Star Mass

Nuclear models that account for different nuclear properties on **Earth** predict a large **variety** of **Neutron Star Mass-Radius** relations → **Observation of a 2M$_{sun}$ has constrained nuclear models.**

Tolman–Oppenheimer–Volkoff equation (sph. sym.):

$$\frac{dM(r)}{dr} = 4\pi r^2 \mathcal{E}(r);$$

$$\frac{dP}{dr} = -G\frac{\mathcal{E}(r)M(r)}{r^2}\left[1+\frac{P(r)}{\mathcal{E}(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}$$

$\mathcal{E}(r) \rightarrow$ degeneracy pressure from neutrons $\rightarrow M_{\max} = 0.7 M_{\text{sun}}$

Nuclear Physics input is fundamental

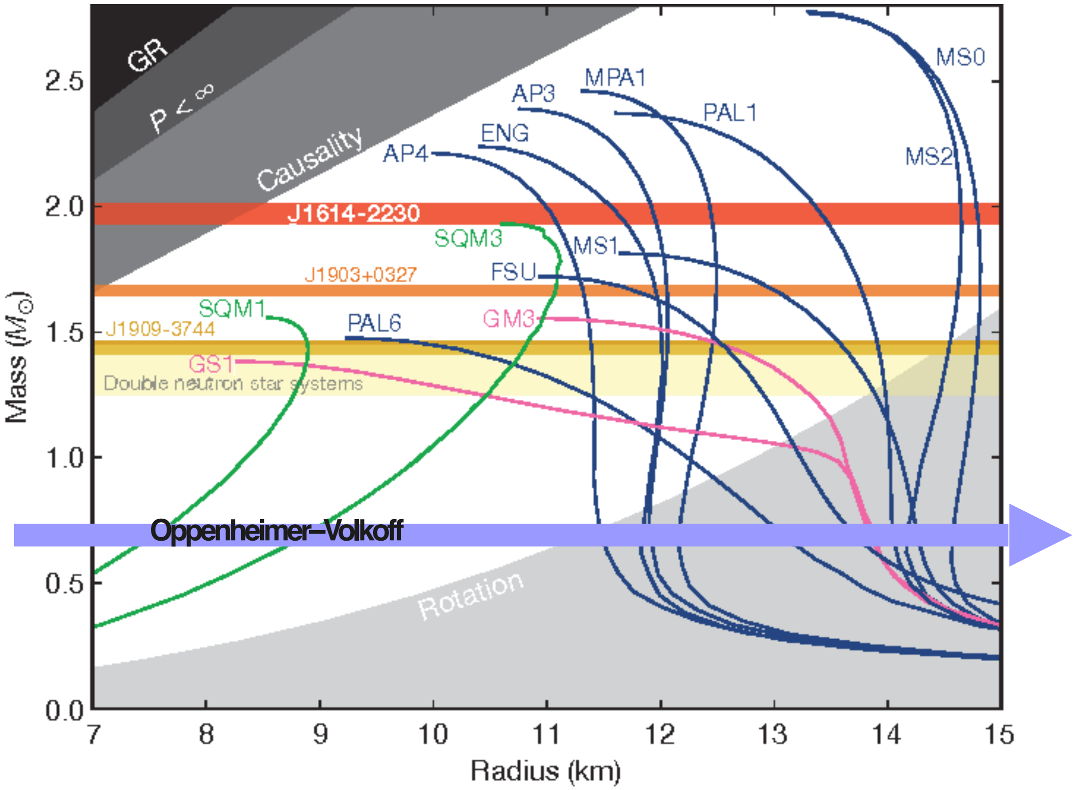
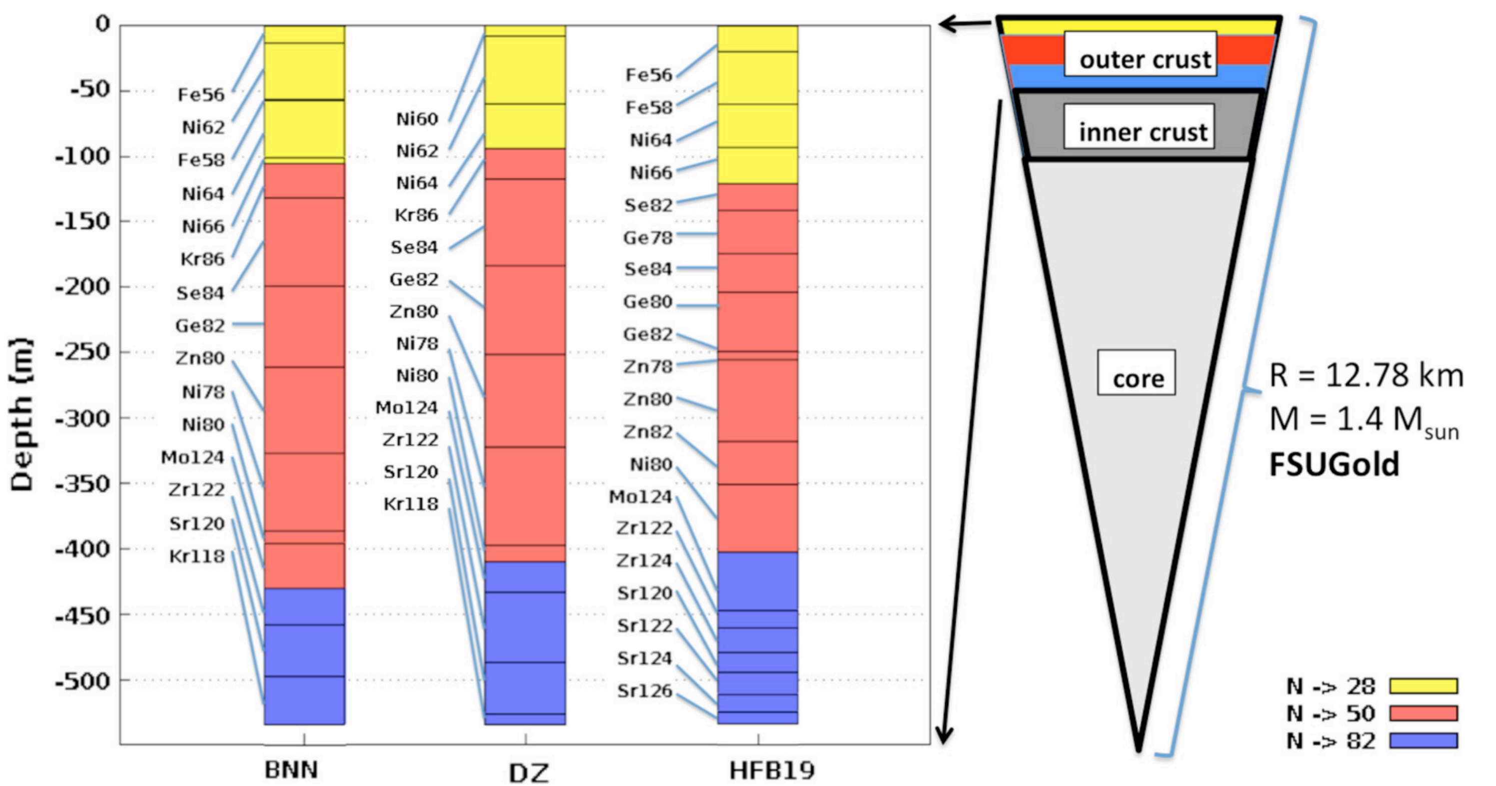


Figure 3 | Neutron star mass–radius diagram. The plot shows non-rotating

A two-solar-mass neutron star measured using Shapiro delay - P.B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts & J. W. T. Hessels - Nature volume 467, 1081–1083(2010)

Crust composition of a Neutron Star

The **crust** of a **NS** is made of very **exotic neutron rich nuclei,** stable only due to the extreme conditions (large densities). **Different nuclear models predict different compositions**

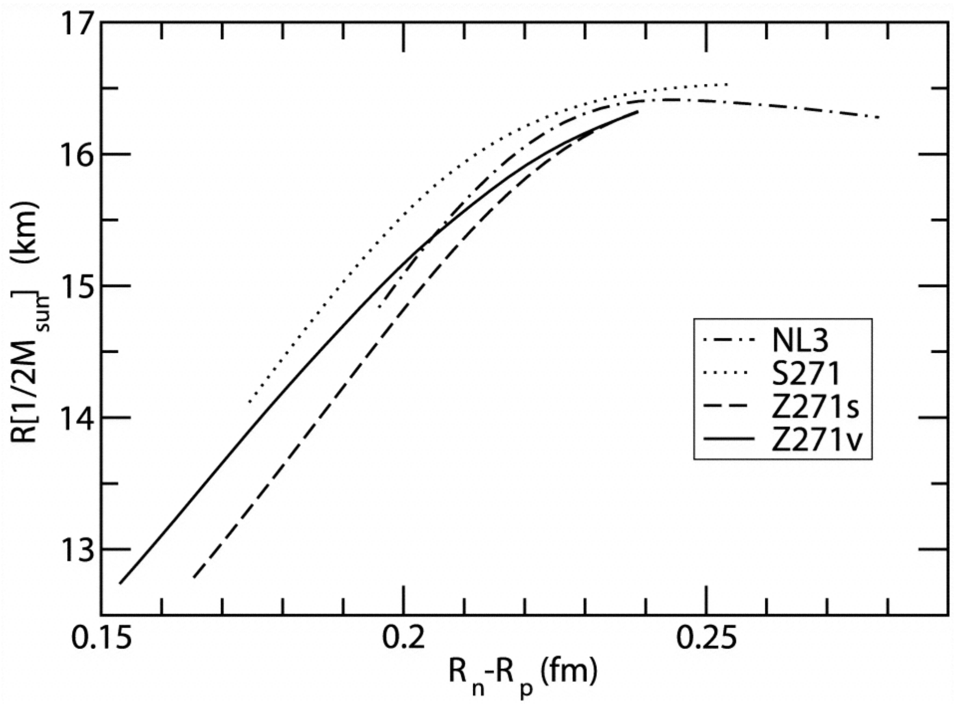


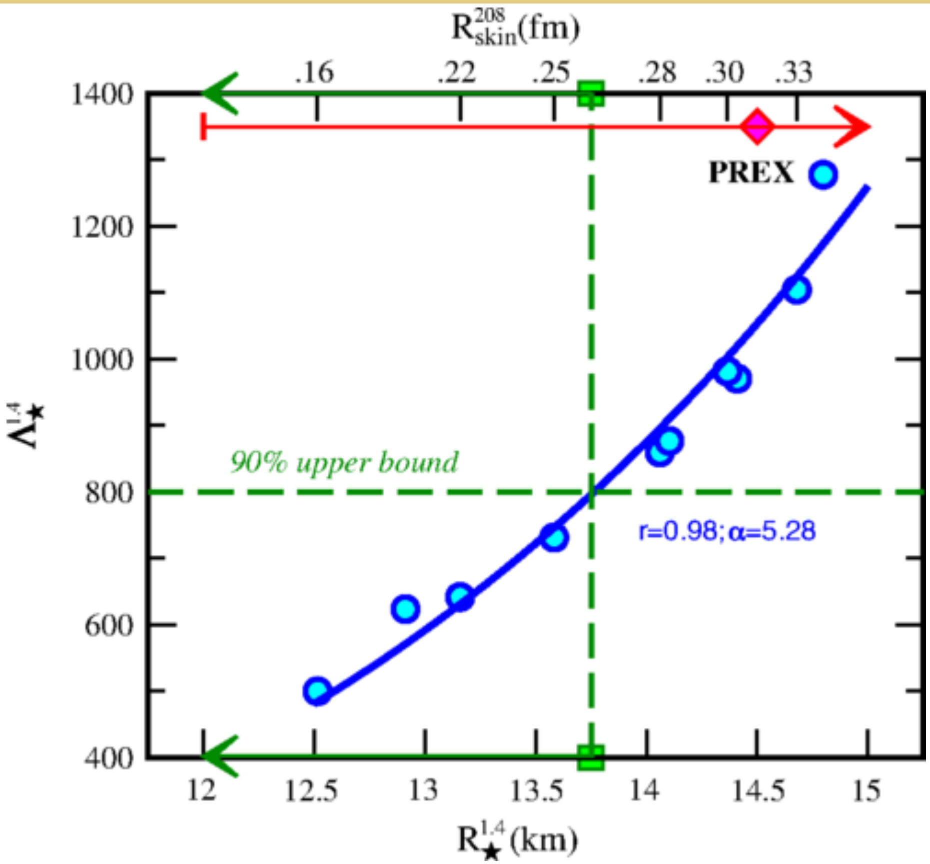
Phys. Rev. C **93**, 014311 – **Published 20 January, 2016**

The faster the symmetry energy $S(\rho)$ grows with density (L) the more exotic the composition

Neutron Skin in Nuclei, Radius & Tidal Def. of a NS

Both, the **neutron skin thickness** ($\Delta r_{np}=r_n-r_p$) in neutron rich nuclei and the **radius** of a **neutron star** are related to the **neutron pressure** in uniform matter. The former around ρ_0 (L) while the latter in a broad range of densities.





→ Only for <u>unrealistically</u> **small neutron stars**, that is, for small central densities ($\rho_c \sim \rho_0$): nuclear models predict a **linear** relation between **R** and **Δr_{np}...**

Low-Mass Neutron Stars and the Equation of State of Dense Matter - J. Carriere, C. J. Horowitz, and J. Piekarewicz - The Astrophysical Journal, 593 (2003) 463

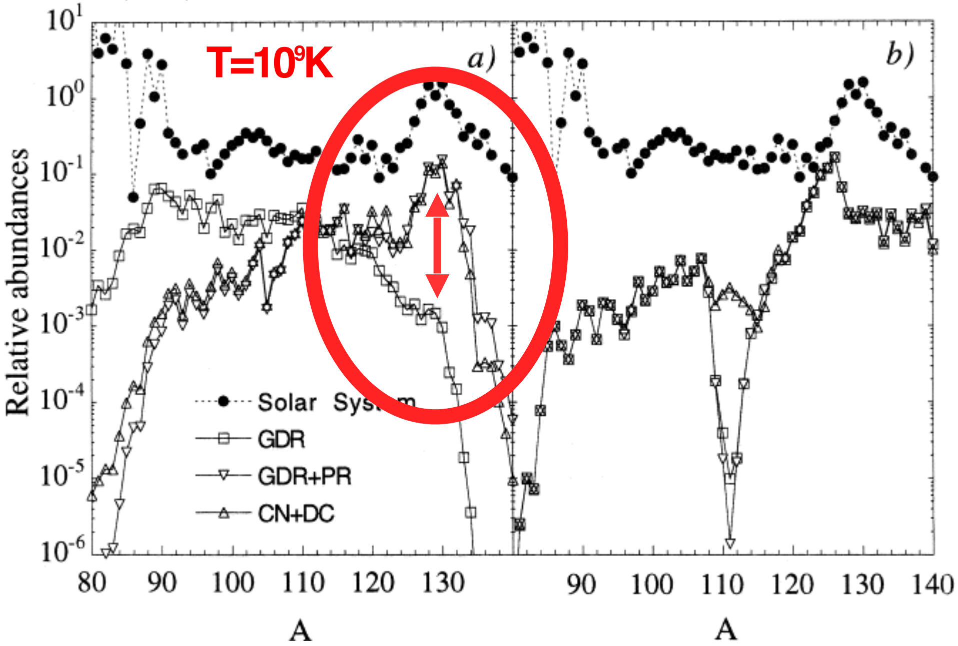
Neutron Skins and Neutron Stars in the Multimessenger Era. J. Fattoyev, J. Piekarewicz, and C. J. Horowitz Phys. Rev. Lett. 120, 172702 (2018)

Low energy dipole response and nucleosynthesis

The **largest** the **neutron pressure** (~ L), the more **neutrons** are ***"pushed out"*** → spatial ***decorrelation*** of some of those neutrons with the nucleons in the core produces **larger low lying dipole responses.**

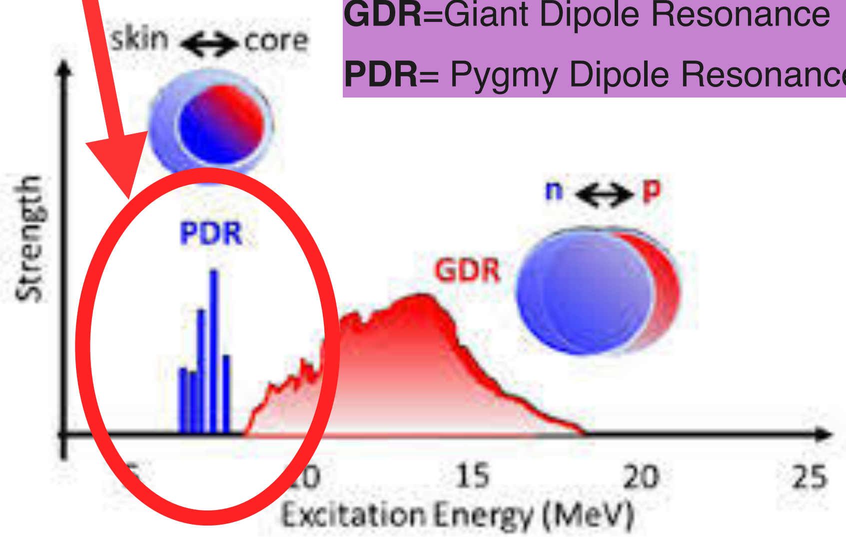
Radiative neutron captures by neutron-rich nuclei and the r-process nucleosynthesis

S. Goriely, Phys.Lett.B 436 (1998) 10-18



GDR=Giant Dipole Resonance

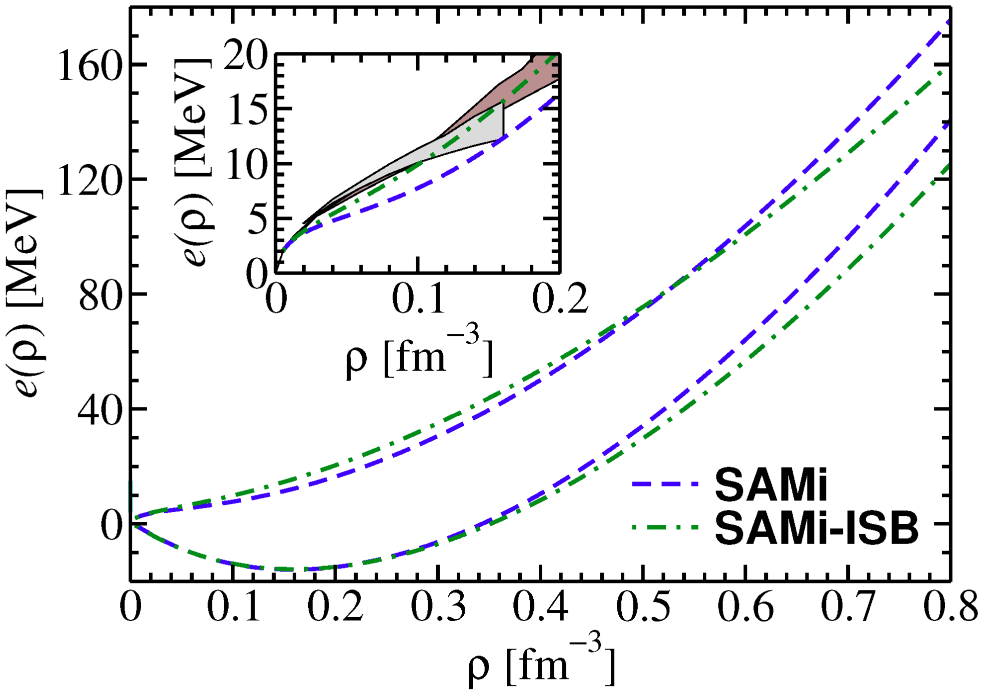
PDR= Pygmy Dipole Resonance

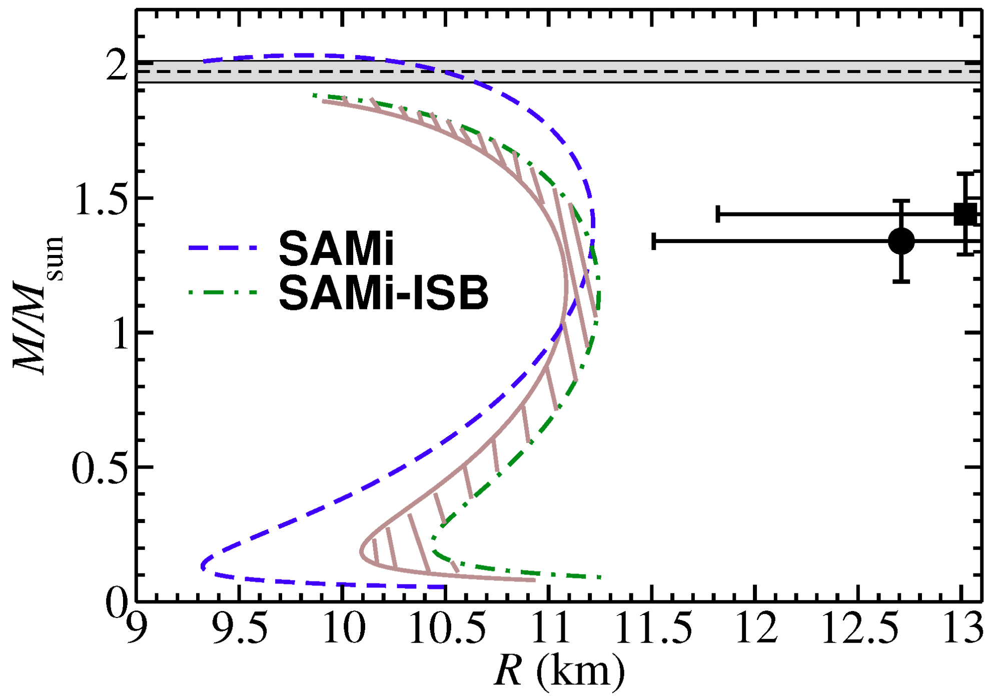


Low energy dipole strength in neutron-rich nuclei influences the **neutron capture cross section** and, thus, the **r-process nucleosynthesis**

Isospin symmetry Breaking effects on Neutron Stars?

Speculation: **isospin symmetry breaking** effects compatible with **nuclear phenomenology** may show some **effects** on the **mass** and **radius** of a **Neutron Star**.





G Selva, X Roca-Maza
and G Colò
Symmetry 2021, 13, 144.

Summary from Lab: with qualitative indication of accuracy needed to describe experiment (note that absolute values might be subject to systematics)

→ $\rho_0 \in$ [0.154,0.159] fm^{-3} → relative accuracy 2%

→ needed to describe experiment (Rch) ≤0.1%

→ $e_0 \in$ [15.6,16.2] MeV→ relative accuracy 4%

→ needed to describe experiment (B) ≤0.0001%

→ $K_0 \in$ [200,260] MeV→ relative accuracy 25%

→ needed to describe experiment (E_x^{GMR}) ≤7%

→ J ∈ [30,35] MeV → relative accuracy (α) 15%

→ needed to describe experiment ≤15%

→ L ∈ [20,120] MeV → relative accuracy (α) 150%

→ needed to describe experiment ≤50%

→ ...