Rethinking Resonance Detectability during Binary Neutron Star Inspiral

Alberto Revilla Peña[12], Ruxandra Bondarescu[12], Andrew P. Lundgren[34], Roque Marquez Rodriguez[12], Arnau Rios Huguet[12] and Jordi Miralda Escudé[124]

Institut de Ciències del Cosmos (ICCUB)[1], Departament de Física Quàntica i Astrofísica, Universitat de Barcelona (UB)[2], Institut de Física d'Altes Energies (IFAE)[3], Institució Catalana de Recerca i Estudis Avançats (ICREA)[4]

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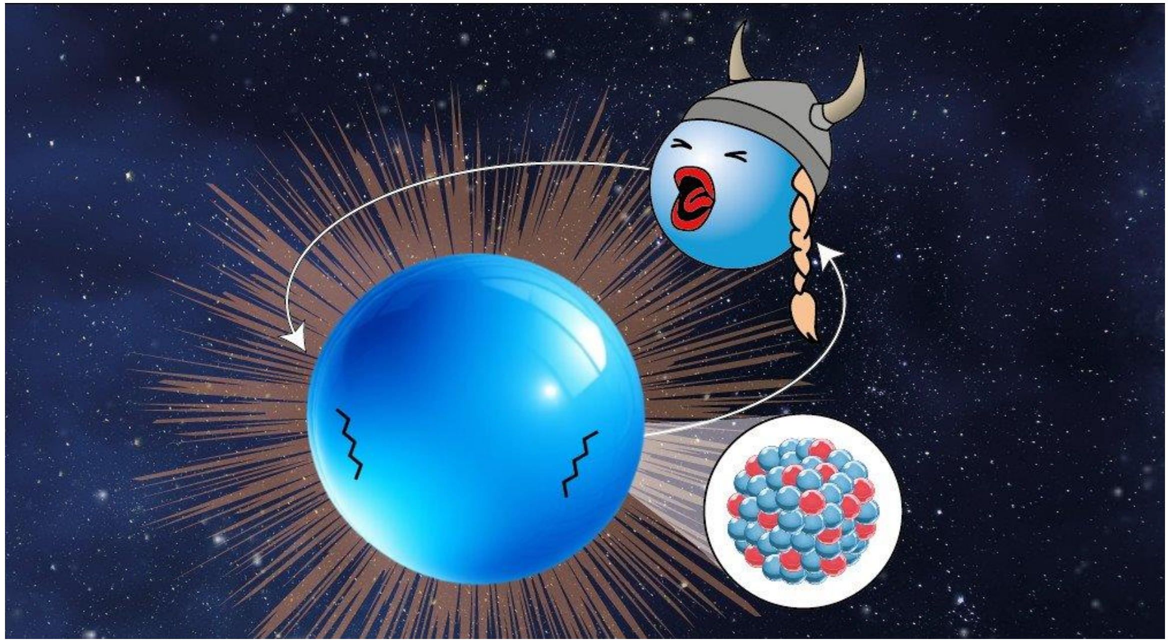
Motivation

- **Oscillatory modes** are excited when the **orbital frequency** of the binary system sweeps through the **mode's resonant frequency**.
- Leading to the **transfer of energy** from the orbit to the mode.
- Which changes the emitted GW **waveform**.
- We study the **detectability** of resonant effects in BNS signals.



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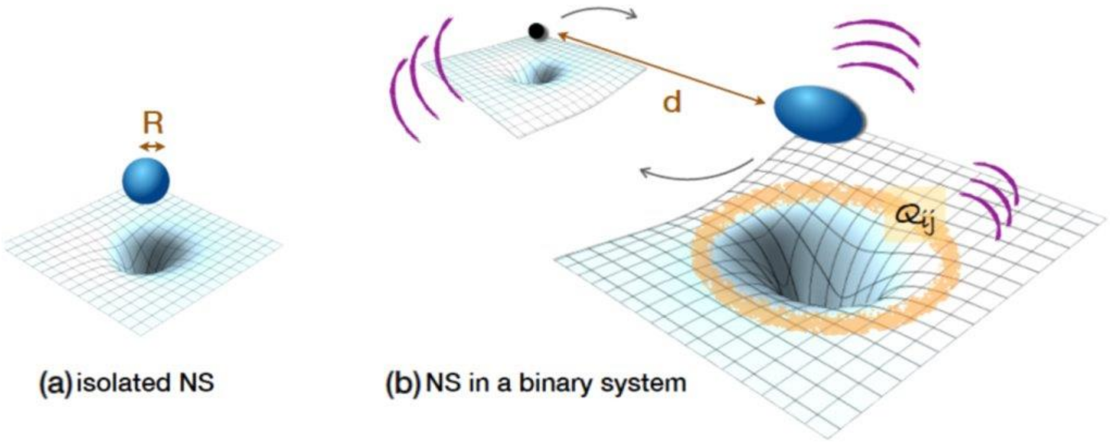
Physical scenario

- We focus on **low frequency modes** (inspiral, resonant frequency ~ 10 - 400 Hz) coinciding with the **maximum sensitivity of current LVK detectors**.
- **Default BNS set-up:**
  - o $M_1 = M_2 = 1.4\ M_o$
  - o Neglect NS spin $S_1 = S_2 = 0$
  - o Luminosity distance fixed at d_L = 100 Mpc



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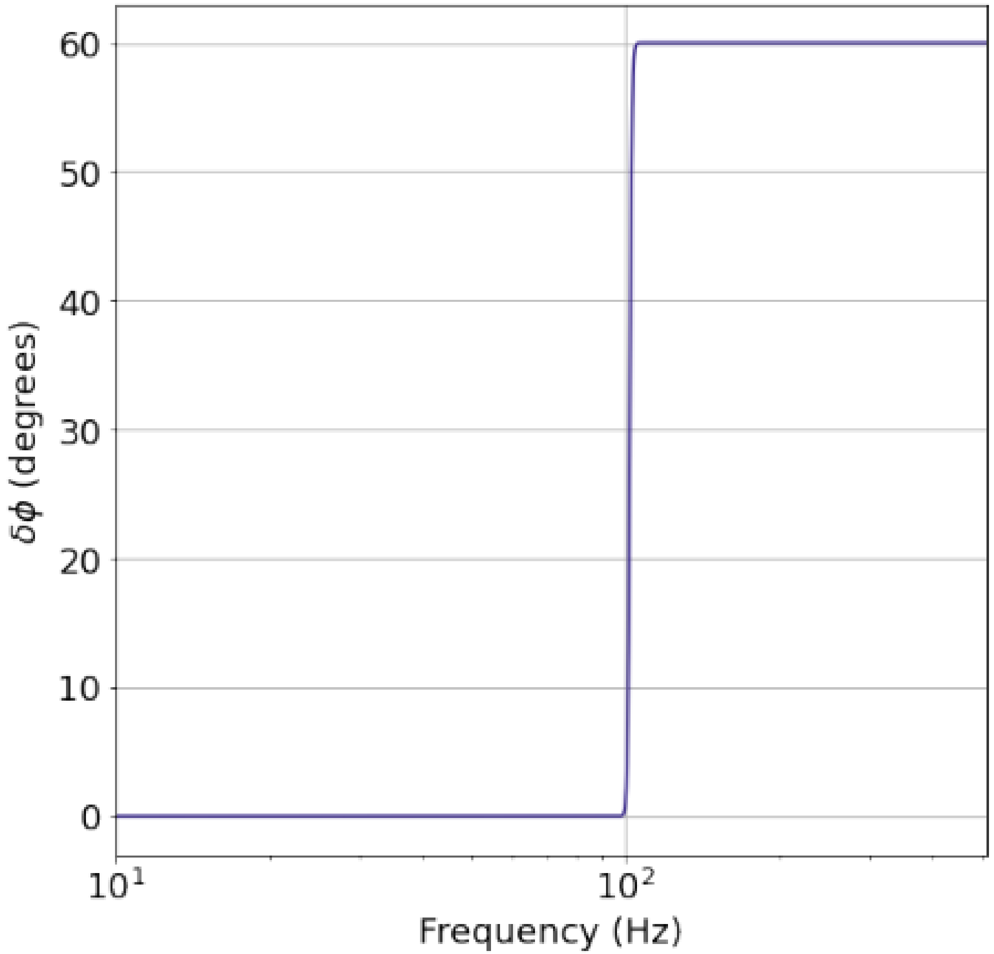
Prior work

- Search for a **phase jump** in data from merger events [Edelman et al 2022].
    - **Orbital energy lost and regained.**
    - An abrupt orbital energy regain from the mode is not a physical effect.
- The physical effect on the waveform caused by resonance is a **time shift** [Flanagan & Racine 2007].
    - **Incompatible** with the type of effect searched in [Edelman et al 2022].

Edelman et al 2022

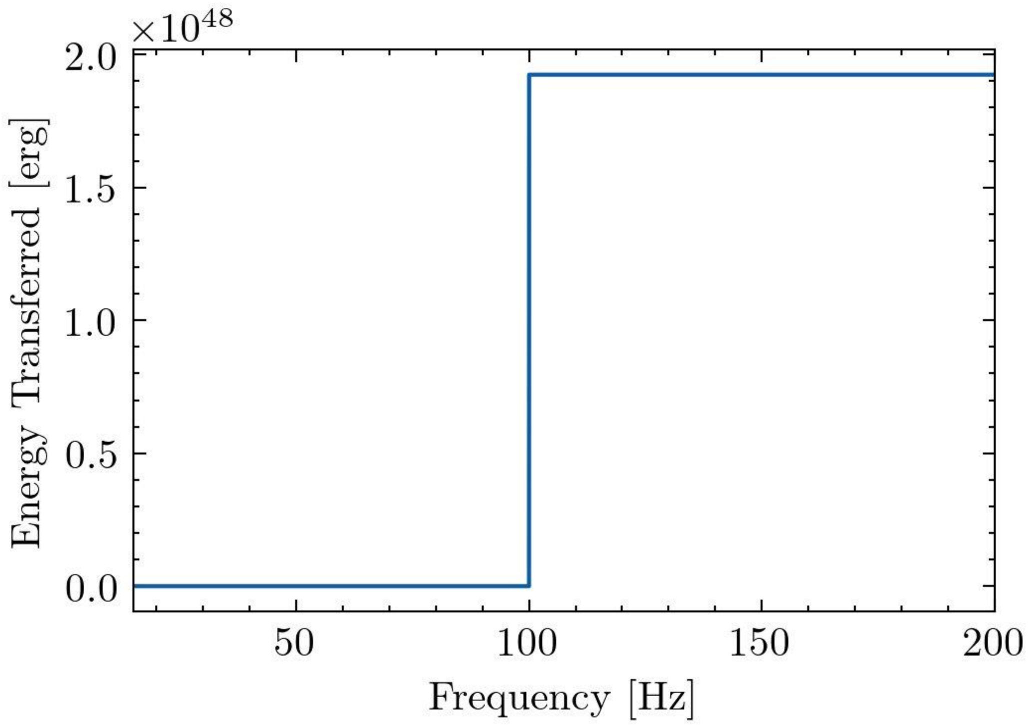


Narrow resonance approximation

- We assume a **sudden mode excitation**
  - Part of the orbital energy/ energy-flux is transferred to the modes instantaneously.
  - E stands for the **orbital energy** and ΔE for the **energy transferred to the mode**.
- In an analogous way, it is possible to account for resonance by adding an **extra energy-flux term**

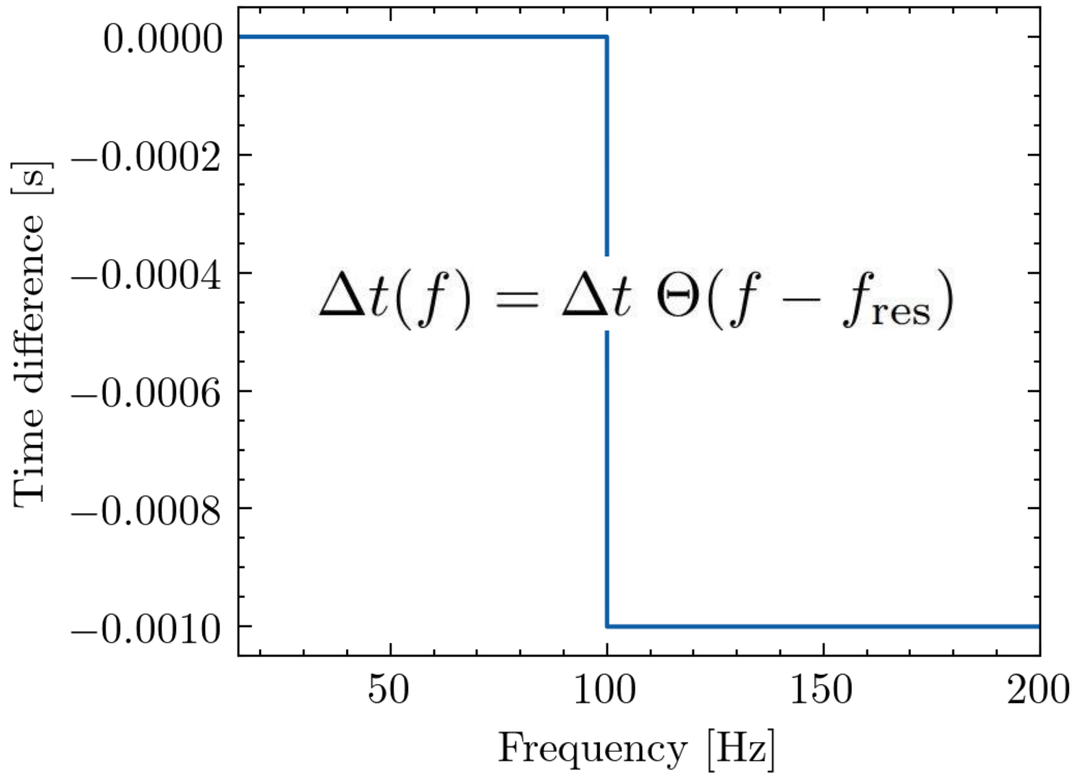
$$\Delta\mathcal{F}(f) = \Delta\mathcal{F}\ \delta(f - f_{\rm res})$$

$$E \to E + \Delta E$$

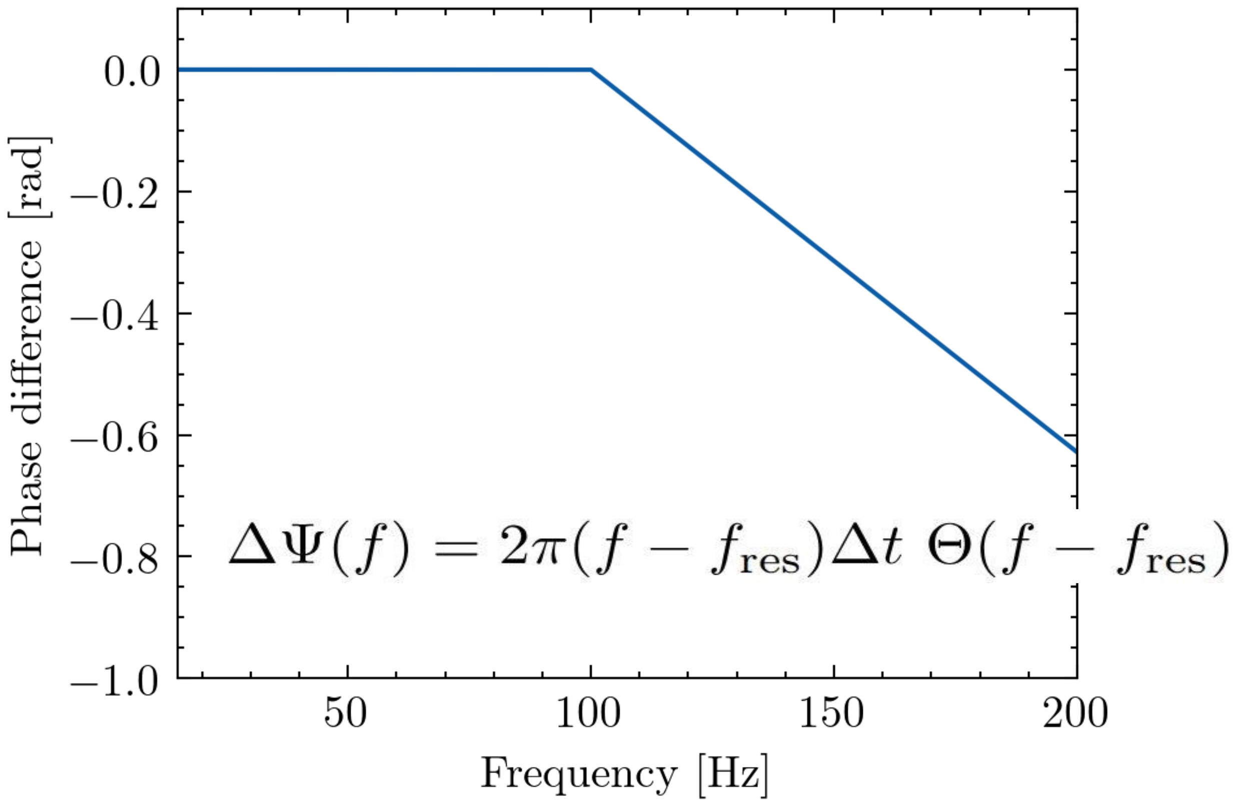


Energy balance equation $\frac{dE(v)}{dt} = -\mathcal{F}(v)$

- The **time to merger**



- The **FD waveform phase.** $\frac{d\Psi}{df} = 2\pi t(f)$



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Phase difference properties

- We analyze the **phase's dependency on frequency** by computing its second derivative.
- We write it in terms of the **extra energy flux**.

$$\frac{d^2\Delta\Psi}{df^2} = 2\pi\frac{d\Delta t}{df}$$
$$= -\frac{2\pi^2 M E'(v)}{3v^2\mathcal{F}(v)}\frac{\Delta\mathcal{F}(v)/\mathcal{F}(v)}{1+\Delta\mathcal{F}(v)/\mathcal{F}(v)}$$

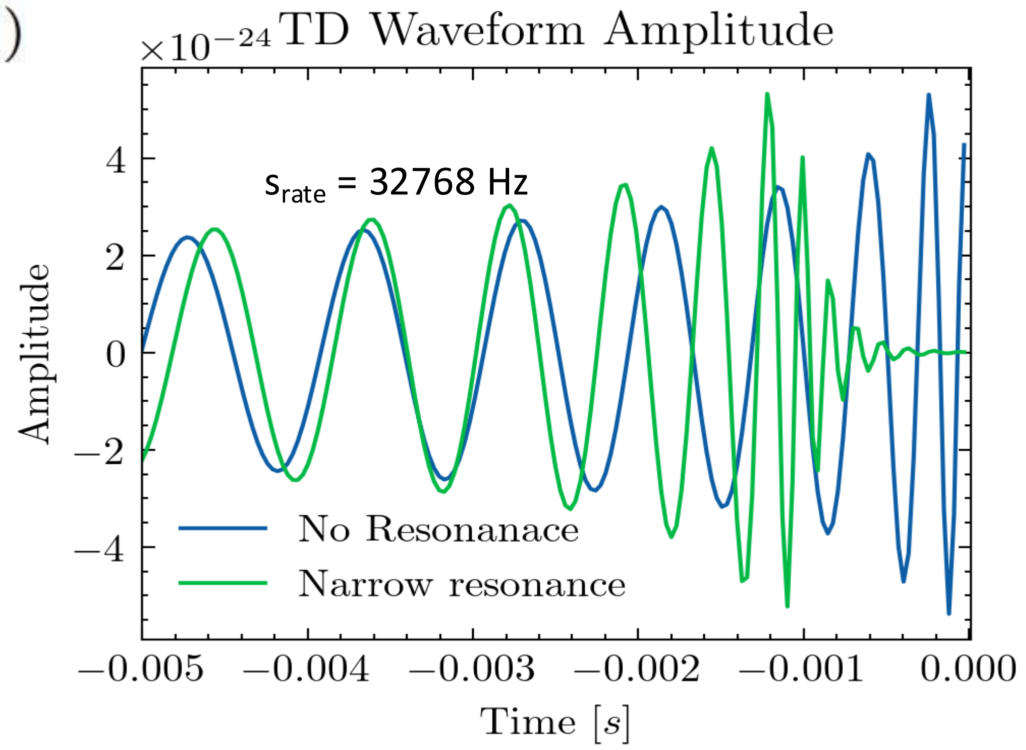
- We assume that a **fraction of the orbital energy** goes to the resonant mode and it is **not regained**.
- The **second derivative of the phase difference is positive**, neglecting small backreaction effects.

Impact of a narrow resonance in the waveform

- The **resonant effects** cause a **change in the phase** in the FD waveform and the **amplitude remains unchanged**

$$h_\Delta(f) = h_0(f)\, e^{i\Delta\Psi(f)}$$

- **Default match-filtering set-up:**
  - Approximant: 'IMRPhenomD'
  - Detector noise curve: A+
  - Minimum frequency: f_{low} = 15 Hz
  - Maximum frequency: f_{high} = 1024 Hz
  - Sampling rate: s_{rate} = 4096 Hz
  - Time length: t_{len} = 512 s



Match computation and numerical maximization

- Inner product

$$\langle a, b \rangle = 2 \int \frac{a^*(f)b(f) + a(f)b^*(f)}{S_n(f)}\, df$$

Signal-to-noise ratio (SNR)

$$\rho = \sqrt{\langle a, a \rangle}$$

- The **overlap** among two waveforms can be written as: $\mathcal{O}(a,b) = \frac{\langle a, b \rangle}{\sqrt{\langle a, a \rangle \langle b, b \rangle}}$
- **Match** among the waveforms with and without resonance effects is

$$\mathcal{M}(h_0, h_\Delta) = \max_{\delta\phi, \delta t} \mathcal{O}(h_0(f), h_\Delta(f; \delta\phi, \delta t))$$

- **PyCBC** match function (pycbc.filter.match).
- We apply a **numerical maximization** to the overlap as a function of the time and phase shift, to **improve resolution**.

Match quadratic approximation

- Analytical match expansion up to the **quadratic order** in the phase difference and time and phase offsets [Owen & Sathyaprakash 1998]

$$\mathcal{M}(h_0, h_\Delta) = 1 - \tfrac{1}{2}\left(\mathcal{J}[\Delta\Psi^2] - \mathcal{J}[\Delta\Psi]^2 - \frac{(\mathcal{J}[2\pi f\Delta\Psi] - \mathcal{J}[2\pi f]\mathcal{J}[\Delta\Psi])^2}{\mathcal{J}[(2\pi f)^2] - \mathcal{J}[2\pi f]^2}\right)$$

where the **noise moment integrals** are defined as

$$I[\alpha(f)] = 4\mathrm{Re}\int \frac{|h_0(f)|^2}{S_n(f)}\alpha(f)\ \mathrm{d}f\ ,$$

$$\mathcal{J}[\alpha(f)] = I[\alpha(f)]\ /\ I[1]\ .$$

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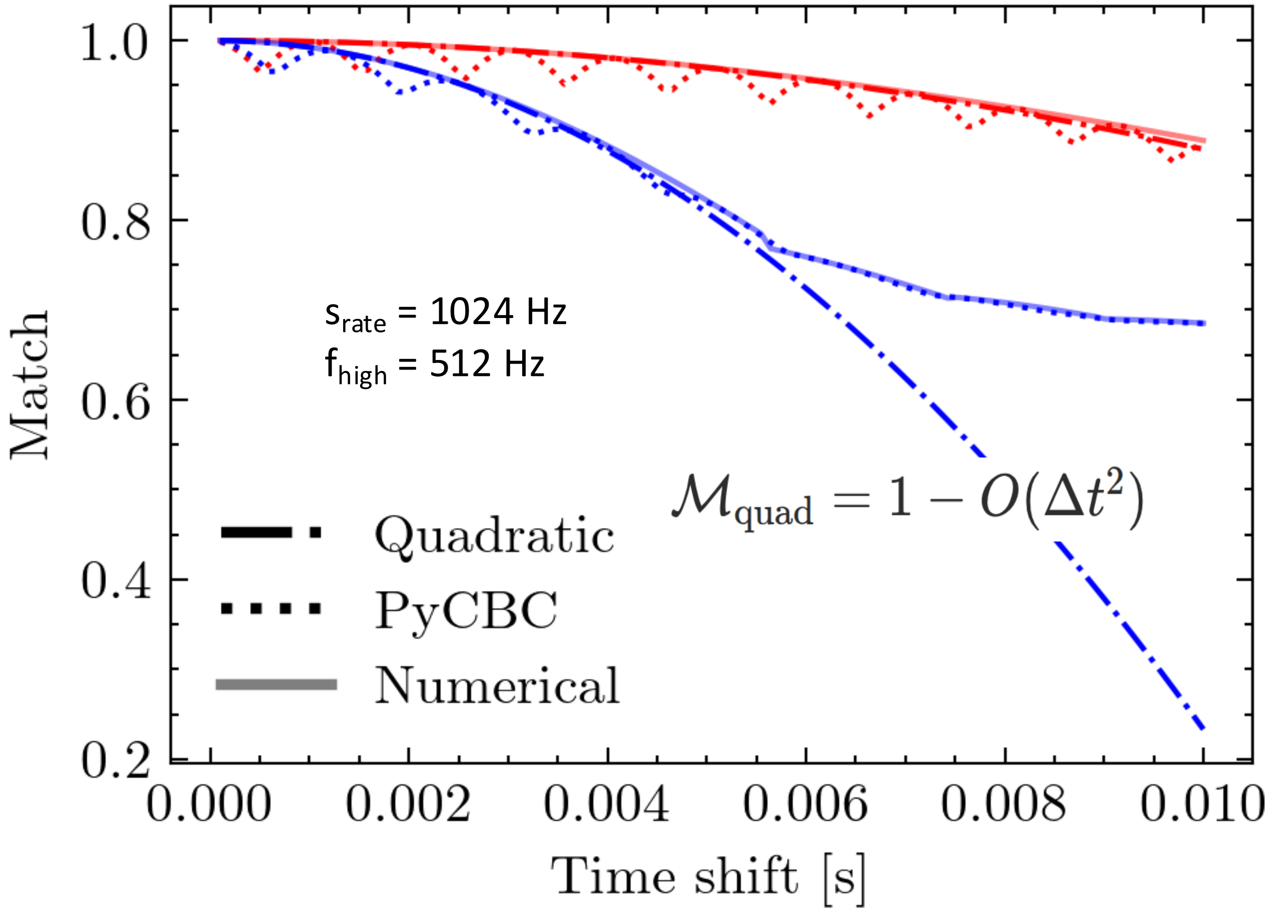
Match computation comparison

- The match approximation is **quadratic** in the time shift.
  - More accurate results for **small deviations**.
  - Works for **any** perturbations in the phase.
  - **Faster** than numerical computation and **avoids numerical errors**.
- The PyCBC match function **wiggles** with respect to the numerically maximized version.



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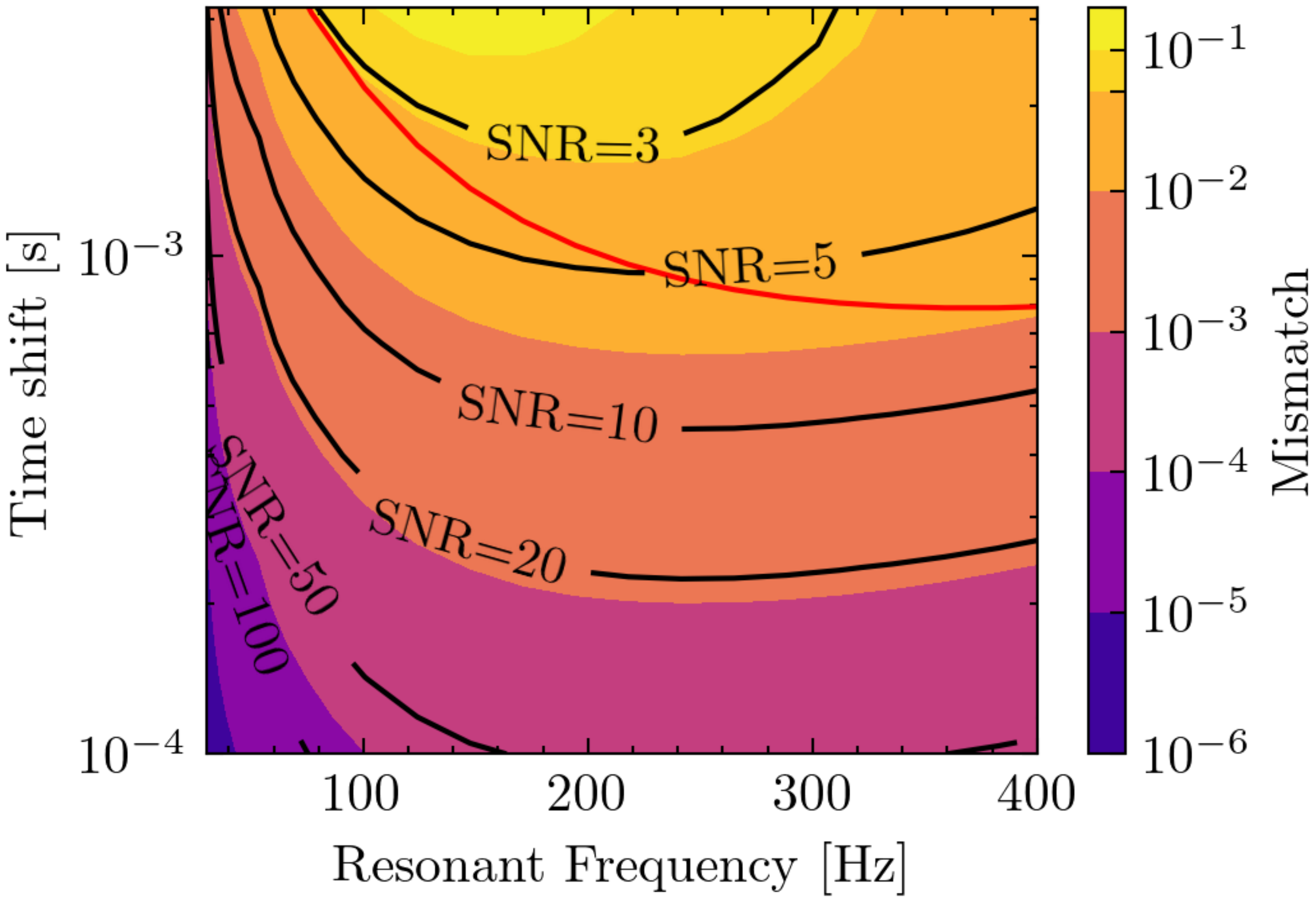
Resonance detectability analysis

- **Detectability criterion** [Lindblom et al 2008]
  - Two waveforms are considered "indistinguishable" when

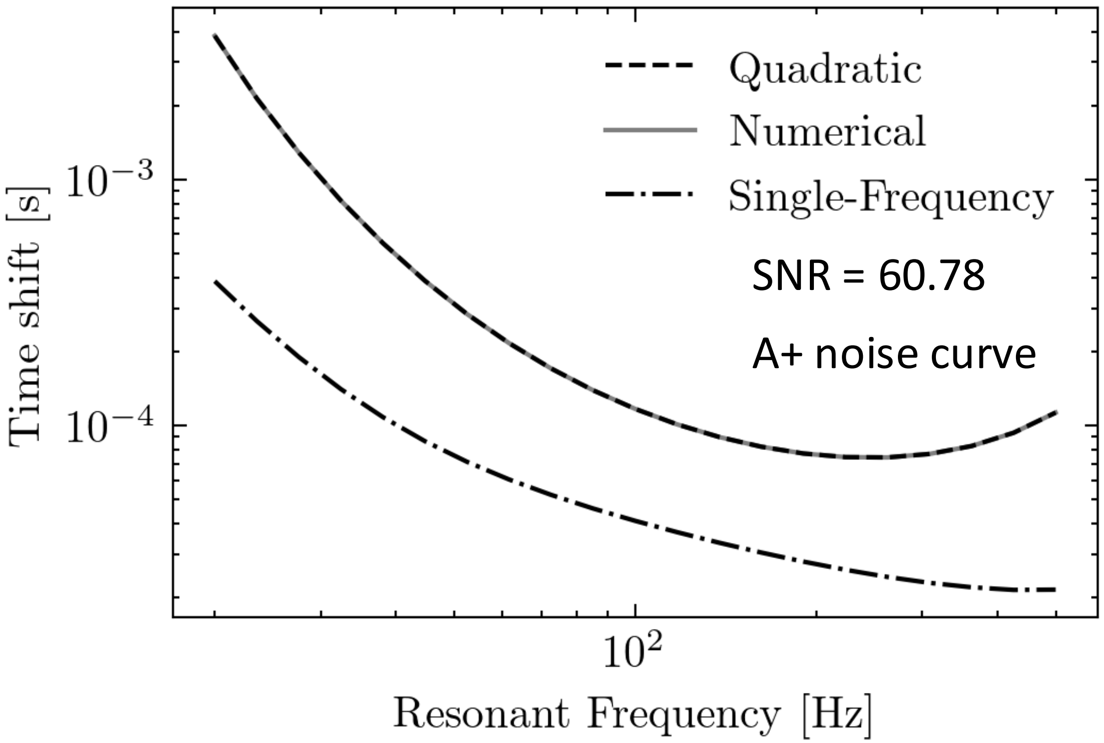
$$\langle \delta h, \delta h \rangle < 1 \,, \quad \delta h = h_\Delta - h_0$$

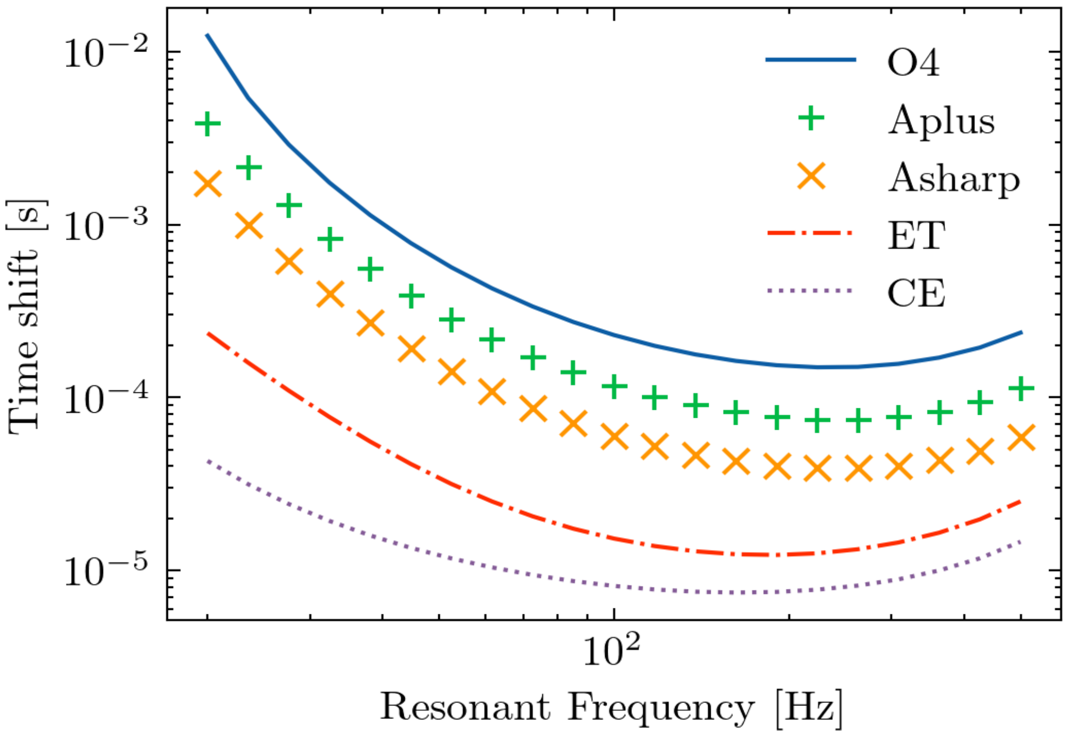
- Black curves indicate the SNR required to detect a time shift **satisfying the Lindblom criterion**

$$\mathcal{MM}(h_\Delta, h_0) = \frac{1}{2\rho^2}$$



Detectability threshold





- **Single-frequency approximation** [Read 2023] which is widely used in the literature.

$$|\Delta t(f)| < \frac{1}{2\pi f} \frac{\sqrt{S_n(f)}}{2|\tilde{h}(f)|\sqrt{f}}$$

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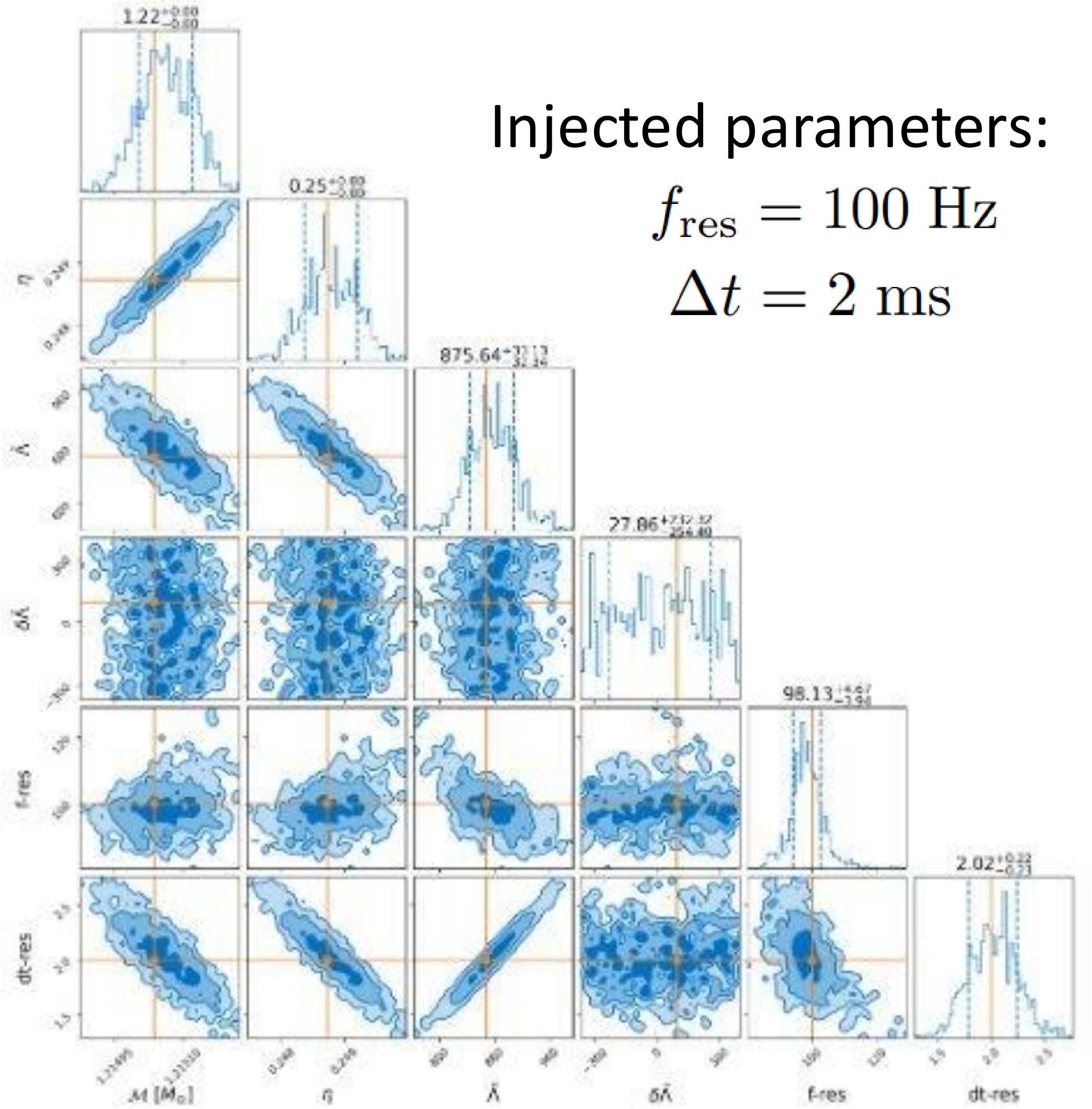
Ongoing work

- **Accurate mode computations** (ICCUB: Roque Márquez, Arnau Ríos; University of Bath: Duncan Neill)
    - Dependency on interior neutron star physics: mode coupling, EOS, etc.
- **Parameter estimation** (Software: **Bilby**)
    - **Search** for resonant effects in BNS data in Bilby.
    - Run data **injections** to recover the resonant effect from the injected signal.
    - Future: **Dingo**, machine learning, neural networks.



Injected parameters:

$$f_{\rm res} = 100 \text{ Hz}$$

$$\Delta t = 2 \text{ ms}$$

Conclusions

- For a BNS at a luminosity distance of **100 Mpc,** a mode with a resonant frequency of **100 Hz** producing a time shift of **1ms** would be detectable by **A+** for a signal with an **SNR of 8**.
- The **single-frequnecy approximation** overestimates the detectability of mode excitations by around one order of magnitude.
- We **increase the resolution** of PyCBC match built-in-function by performing a **numerical maximization of the overlap**.
- **Match quadratic approximation**:
  - **Semi-analytical, smooth, fast** and **accurate** for small resonant effects.
  - Connects **interior physics** to **detector sensitivity**.
- **Work in progress includes**:
  - Improved simulation of the modes and the energy transfer.
  - Parameter estimation: search for resonance effects on BNS sources.

Extra slides index

- Energy transfer [D. Lai 1994]
- Energy, flux fraction transfer
- Realistic resonant frequency and coupling computation

Energy transfer [D. Lai 1994]

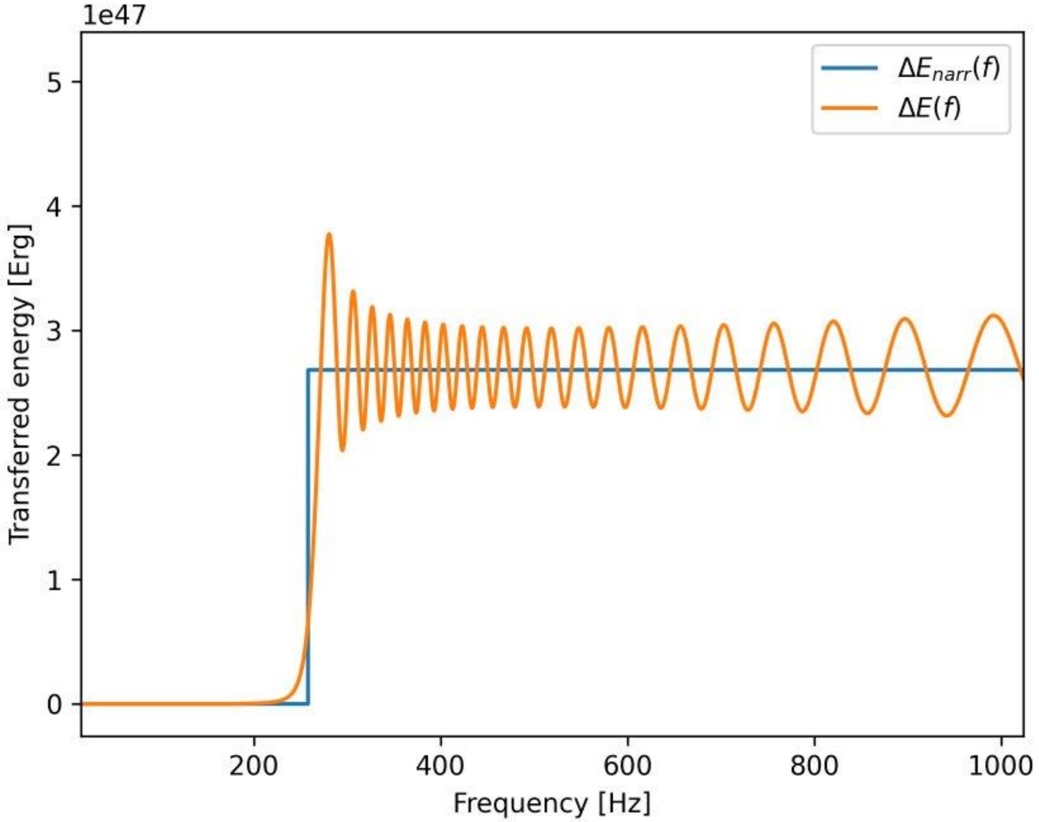
- Fluid displacement splitting the time and radial dependance.

$$\xi(t,x) = \sum_\alpha a_\alpha(t)\xi_\alpha(x)$$

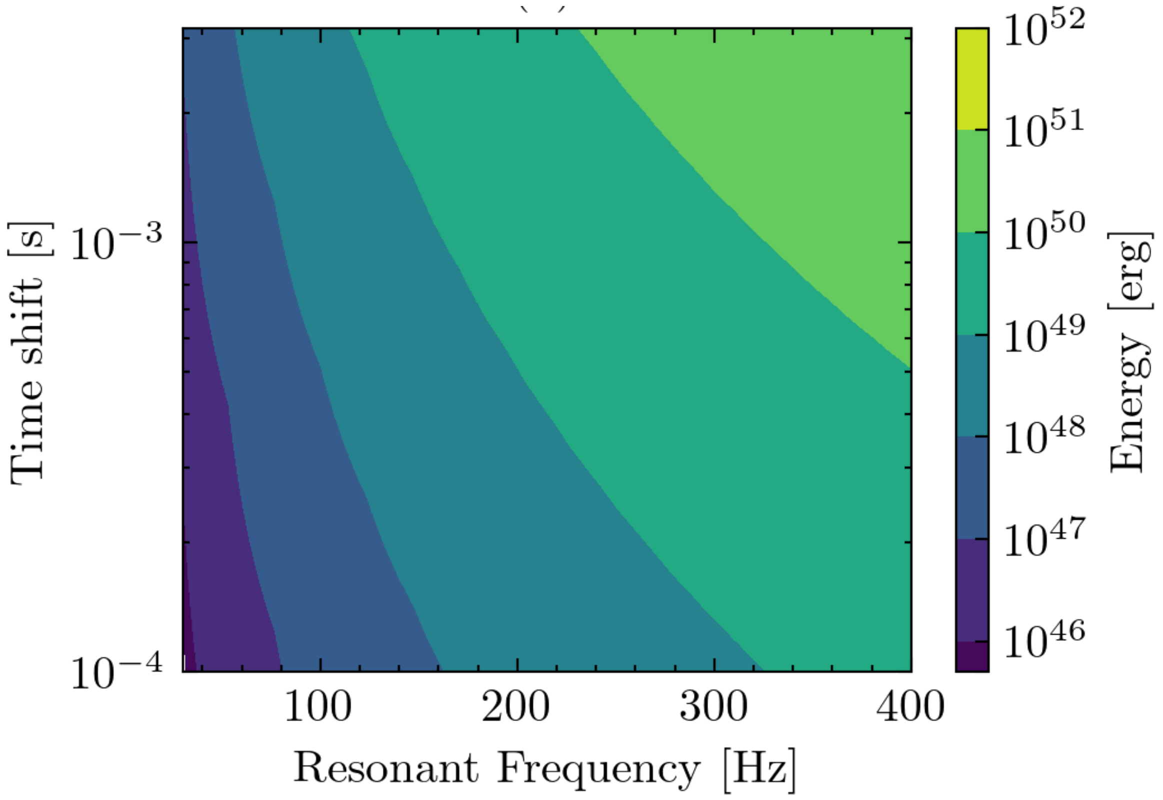
- Driven harmonic oscillation equation describes the time evolution

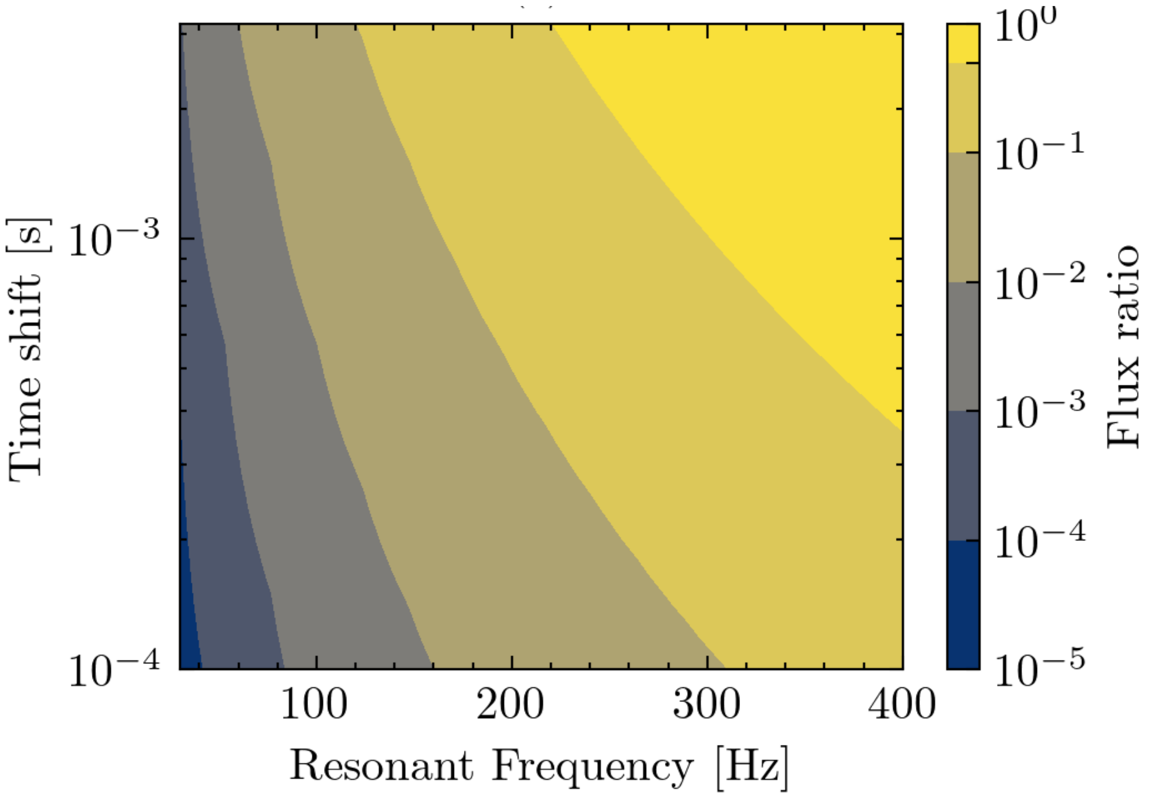
$$\left(\rho\frac{\partial^2}{\partial t^2} + \mathcal{L}\right)\xi = -\rho\nabla U \qquad \ddot{a}_\alpha + \omega_\alpha^2 a_\alpha = \frac{GM' W_{lm} Q_{nl}}{D^{l+1}} e^{-im\Phi(t)}$$

- Obtain energy and flux as a function of frequency.



Resonant energy and energy-flux fraction





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Realistic resonant frequency and coupling computation

- Fix **EOS** to obtain the relation among thermodynamical variables.
  - BSK family [Counsell et al 2024]
- Solve TOV equations to fix the **structure of the star**.
- Apply **adiabatic perturbations** to Einstein equations. Write this as an eigenvalue problem:
  - **Eigenvalue**: Resonant frequency.
  - **Eigenfunctions**: Fluid displacements.

$$\mathcal{L}\xi_\alpha = \rho\omega_\alpha^2\xi_\alpha$$

- Compute the **coupling** among the tidal field and the resonant mode.

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