



# The nuclear bridge to new physics: Theoretical challenges in neutrinoless $\beta\beta$ decay

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Engel et al. Rep. Prog. Phys. 80, 046301, 2017

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# OUTLINE

1 Introduction

2  $\beta\beta$  decays

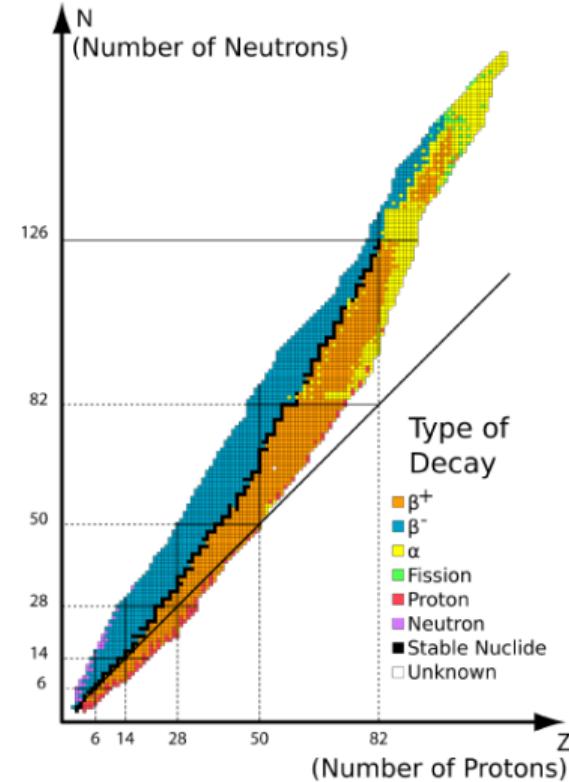
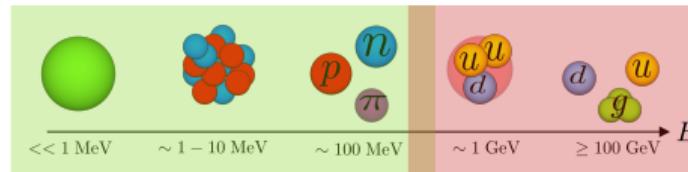
3 Nuclear Matrix Elements

4 Summary

# INTRODUCTION

# THE ATOMIC NUCLEUS

- Self-bound system of  $A$  nucleons (protons and neutrons)
- Bound by the strong force
- Weak decays and electromagnetic repulsion
- Nuclear physics  $\rightarrow$  Quantum ChromoDynamics (QCD):
  - High energy: quarks and gluons
  - Low energy: nucleons and pions



<https://chem.libretexts.org>

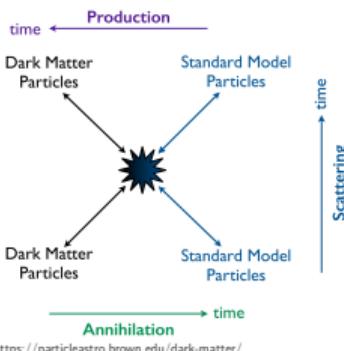
# NUCLEAR DECAYS

## Neutron stars

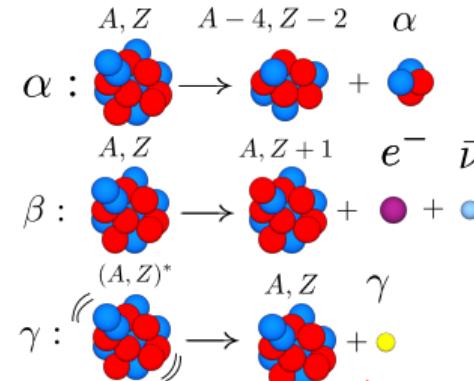


<https://www.space.com/22180-neutron-stars.html>

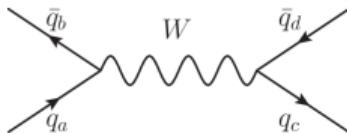
## Dark Matter



<https://particleastro.brown.edu/dark-matter/>

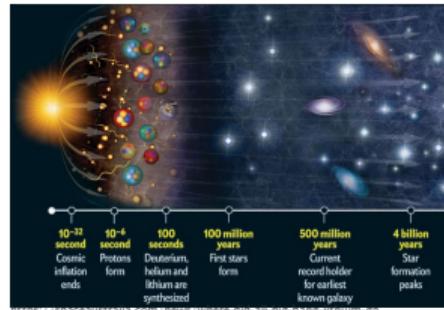


## Weak interaction's test

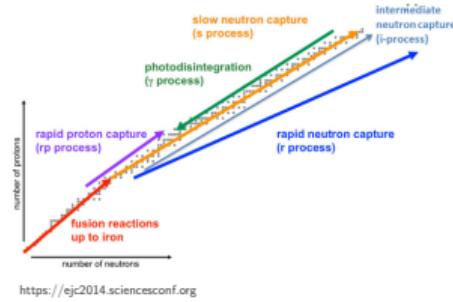


Pasztor et al. arXiv.1103.5057 ,2011

## Big Bang Nucleosynthesis



## *r*-, *s*- and *p*-processes



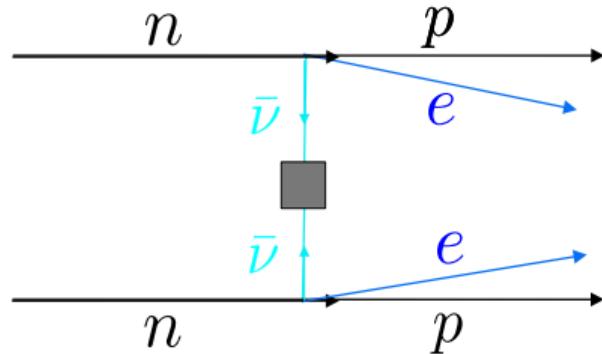
<https://ejc2014.sciencesconf.org>

# $\beta\beta$ DECAY

# $\beta\beta$ DECAY

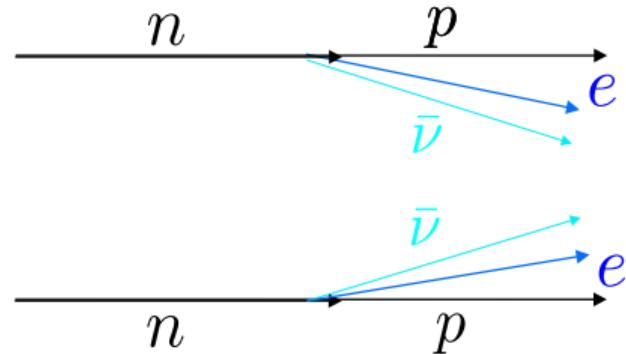
$0\nu\beta\beta : 2n \rightarrow 2p + 2e^-$

- Beyond Standard Model  $\rightarrow$  Lepton Number Violation
- Majorana Fermions
- Hypothetical decay



$2\nu\beta\beta : 2n \rightarrow 2p + 2e^- + 2\bar{\nu}_e$

- Standard Model allowed  $\rightarrow$  Lepton Number Conservation
- Dirac Fermions
- Already measured



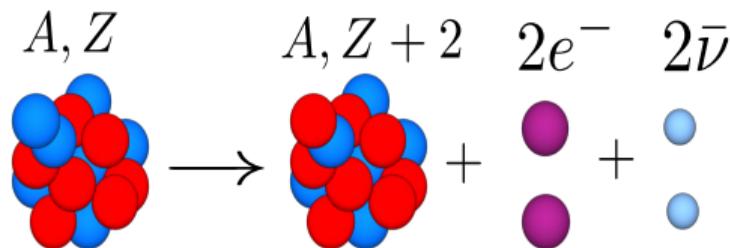
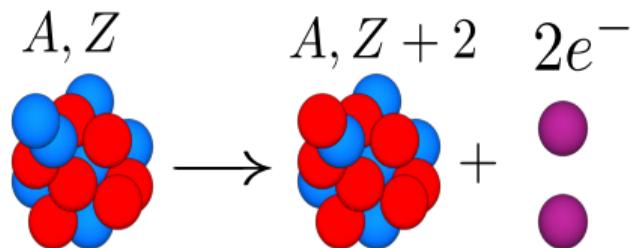
# $\beta\beta$ DECAY

$0\nu\beta\beta : 2n \rightarrow 2p + 2e^-$

- Beyond Standard Model → Lepton Number Violation
- Majorana Fermions
- Hypothetical decay

$2\nu\beta\beta : 2n \rightarrow 2p + 2e^- + 2\bar{\nu}_e$

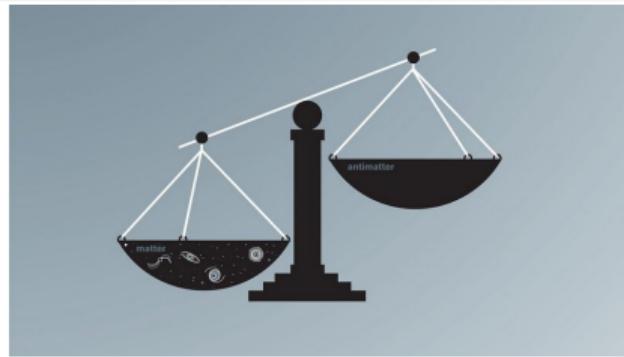
- Standard Model allowed → Lepton Number Conservation
- Dirac Fermions
- Already measured



# $0\nu\beta\beta$ : BEYOND STANDARD MODEL IMPLICATIONS

## Matter-antimatter asymmetry

- Baryogenesis connection via leptogenesis
- Constrain on CP-Violating phases
- New physics model discrimination



<https://legend-exp.org/science/neutrinoless-bb-decay/the-matter-antimatter-asymmetry>

## Neutrino physics

- Neutrino's nature
- Neutrino mass ordering
- Neutrino mass mechanism
- Exotic particles: right-handed neutrinos?



<https://amsler.web.cern.ch/NPIOP/Seesaw.pdf>

# EXPERIMENTS



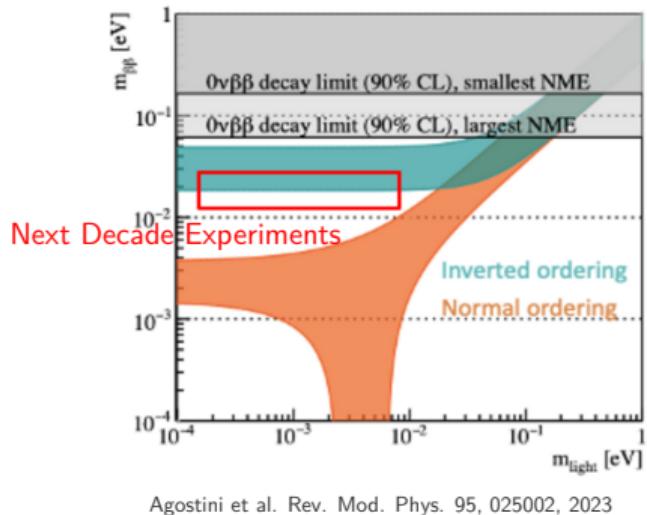
LEGEND



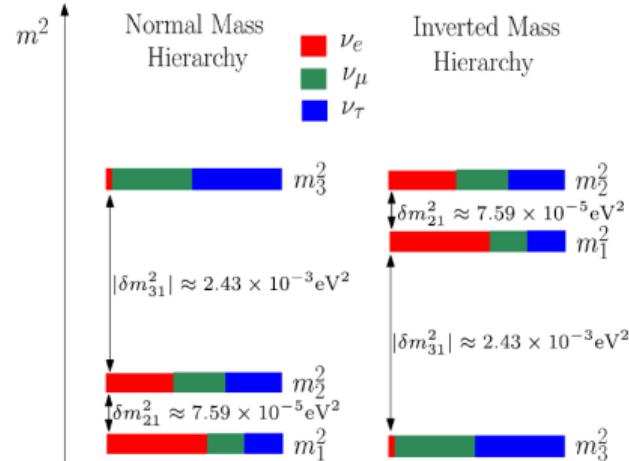
onext

# MOTIVATION

$$(T_{1/2}^{\beta\beta})^{-1} \propto g_A^4 G_0 |M^{\beta\beta}|^2 m_{\beta\beta}^2$$



$M^{\beta\beta} \equiv$  Nuclear matrix elements  
 $G_0 \equiv$  Phase-space factor (PSF)  
 $g_A \equiv$  Axial coupling to the nucleon  
 $m_{\beta\beta} \equiv$  Effective neutrino mass

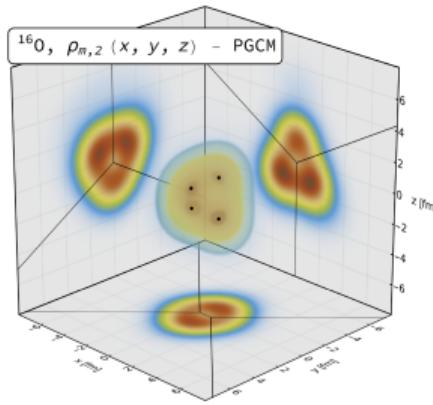


# NUCLEAR MATRIX ELEMENTS

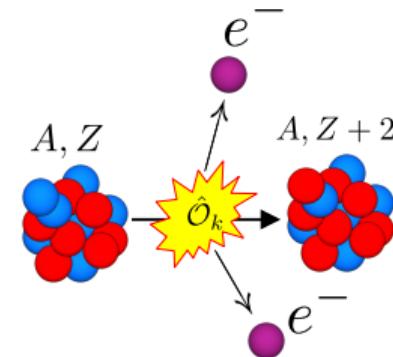
# NUCLEAR MATRIX ELEMENTS

$$M^{\beta\beta} = \langle 0_f^+ | \sum_{a,b} \hat{O}_k \tau_a^- \tau_b^- | 0_i^+ \rangle$$

Nuclear state wavefunctions



Decay operator



Giacalone et al. Phys. Rev. Lett. 135, 012302, 2025

# WAVEFUNCTIONS: THE NUCLEAR MANY-BODY PROBLEM

$$H|\psi\rangle = E|\psi\rangle$$

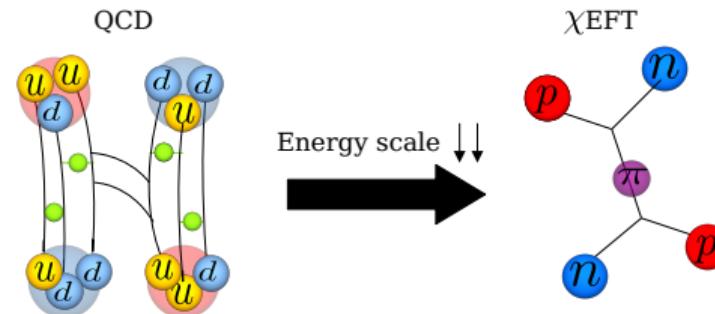
- $H \equiv$  Hamiltonian
- $E \equiv$  Energy
- $|\psi\rangle \equiv$  state wavefunction

## Hamiltonian

- Discretized QCD
- $\chi$ -Effective Field Theory ( $\chi$ EFT)
- **Phenomenological:**  
Fitted to NN data

## Many-body methods

- Lattice QCD
- NCSM, QMC, IMSRG, CC...
- NSM, QRPA, EDF...



# NUCLEAR MATRIX ELEMENTS

## Wavefunctions

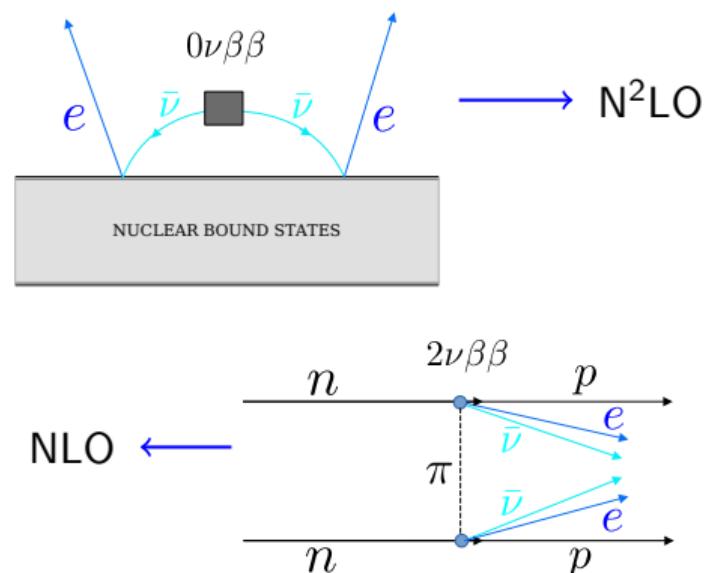
- Nuclear Shell Model (NSM) and Quasiparticle Random Phase Approximation (QRPA)
- Phenomenological Hamiltonian

## Decay operators

- Chiral Effective Field Theory ( $\chi$ EFT) (LO+NLO+N<sup>2</sup>LO...)
- $\hat{\mathcal{O}}_k \equiv$  Spin-space operator
- $\tau^- \equiv$  Ladder isospin operator

Cirigliano et al. Phys. Rev. C 97, 065501, 2018  
el Morabit et al. JHEP, 06, 082, 2025

$$M^{\beta\beta} = \langle 0_f^+ | \sum_{a,b} \hat{\mathcal{O}}_k \tau_a^- \tau_b^- | 0_i^+ \rangle$$

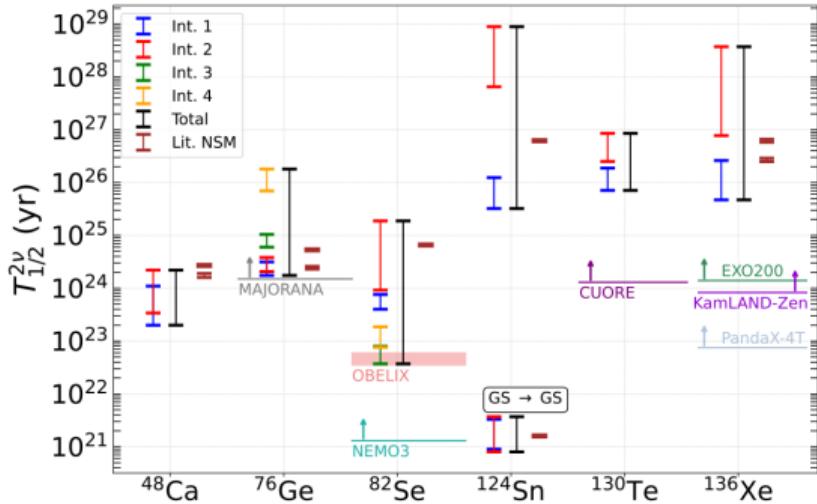


# RESULTS

# $2\nu\beta\beta$ : DECAY TO EXCITED STATES

$M^{2\nu} \equiv \text{LO NME}$

- + Additional PSFs and subleading NMEs
- +  $\chi$ EFT corrections up to NLO



## Results

- Main uncertainty comes from different Hamiltonians: Deformation of initial and final states plays a key role
- 2 different GT operators: Bare and renormalized

Coraggio et al. Phys. Rev. C, 100, 014316, 2019

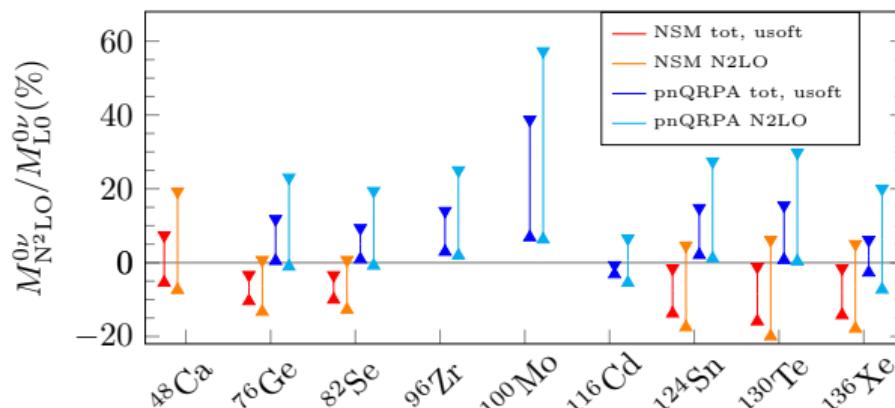
- $^{76}\text{Ge}$ : Experimental evidence close to theoretical predictions
- $^{82}\text{Se}$ : Lower bound prediction within experimental indication

# $0\nu\beta\beta$ : TOTAL N<sup>2</sup>LO NMEs

$$M^{0\nu} = M_{\text{LO}}^{0\nu} + M_{\text{N}^2\text{LO}}^{0\nu}$$
$$= M_{\text{L}}^{0\nu} + M_{\text{S}}^{0\nu} + M_{\text{usoft}}^{0\nu} + M_{\text{loops}}^{0\nu}$$

$\chi$ EFT expectations: (5 – 10)%

- $M_{\text{L(S)}}^{0\nu}$   $\equiv$  Long(Short)-range NME
- $M_{\text{usoft}}^{0\nu}$   $\equiv$  ultrasoft NME
- $M_{\text{loops}}^{0\nu}$   $\equiv$  loop terms



DC, Jokiniemi, Menéndez, Phys. Lett. B 860, 2025

## Results

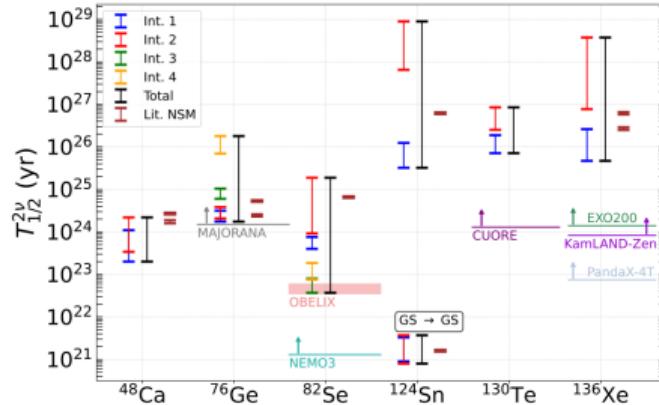
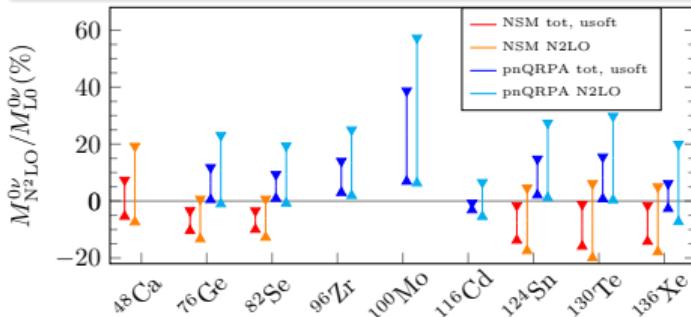
- N<sup>2</sup>LO contributions: Non-negligible
- Different sign between many-body methods: ultrasoft NME
- $|M_{\text{N}^2\text{LO}}^{0\nu} / M_{\text{LO}}^{0\nu}|$  :
  - NSM  $\leq 20\%$
  - QRPA  $\leq 30\%$
- Central values:  
 $|M_{\text{N}^2\text{LO}}^{0\nu} / M_{\text{LO}}^{0\nu}| \sim (5 - 15)\%$

# SUMMARY

# SUMMARY AND OUTLOOK

## Summary

- NMEs crucial to understand beyond standard model physics
- $0\nu\beta\beta$  N<sup>2</sup>LO NMEs and  $2\nu\beta\beta$  half-lives ( $0^+_1 \rightarrow 0^+_1$ )
- $^{76}\text{Ge}$  and  $^{82}\text{Se}$  predictions close to experimental evidence



## Outlook

- Use other methods for the wf
- Extend the decay operator study to higher order corrections
- Potential near-term measurements ( $^{76}\text{Ge}$  and  $^{82}\text{Se}$ )

# Thank you for your attention!

Neutrinoless  $\beta\beta$  decay nuclear matrix elements complete up to N<sup>2</sup>LO in heavy nuclei

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Two-neutrino  $\beta\beta$  decay to excited states at next-to-leading order

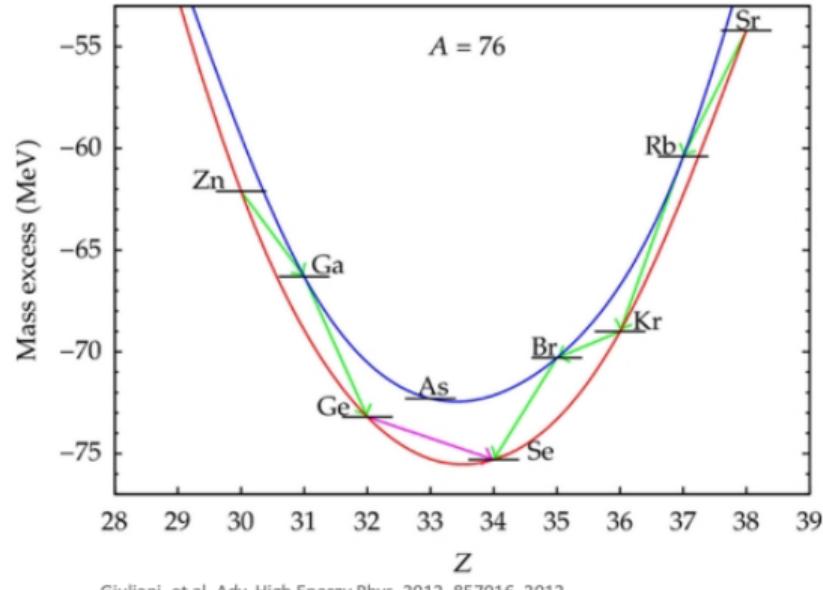
Daniel Castillo<sup>a,b,c,\*</sup>, Dorian Frycz<sup>a,b,c,\*</sup>, Beatriz Benavente<sup>c</sup>, Javier Menéndez<sup>a,b</sup>

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# BACKUP SLIDES

# $\beta\beta$ DECAY

- $^{76}\text{Ge}$ :  $\beta$  decay forbidden
- Energy difference between **odd-odd nuclei** and **even-even nuclei**  $\rightarrow$  Pairing
- $\beta\beta$  decay allowed:  
 $^{76}\text{Ge} \rightarrow ^{82}\text{Se}$  ( $Q_{\beta\beta} > 0$ )
- $Q_{\beta\beta} = E_i - E_f - 2m_e$ 
  - $Q_{\beta\beta} \equiv Q$ -value for  $\beta\beta$  decay
  - $m_e \equiv$  Electron mass
  - $E_i \equiv$  Initial energy
  - $E_f \equiv$  Final energy



Giuliani, et al. Adv. High Energy Phys., 2012, 857016, 2012.

# NUCLEAR SHELL MODEL

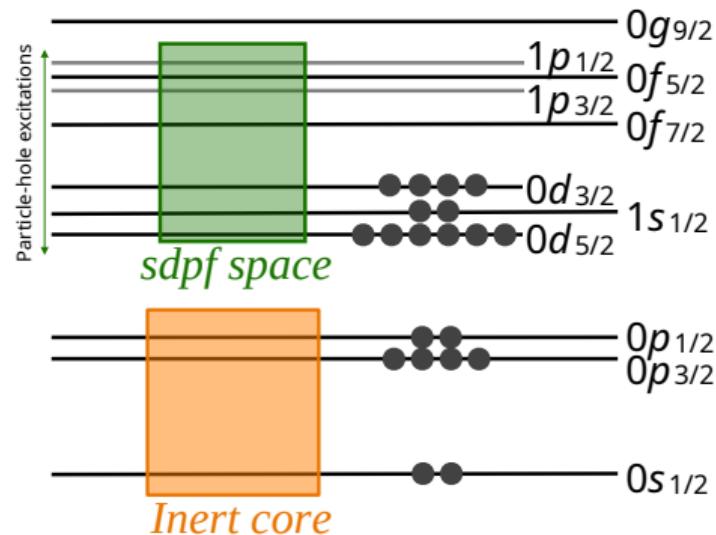
## Hamiltonian

- To solve the Schrödinger equation  
 $\rightarrow H_{\text{eff}}|0_{\text{GS}}^+\rangle = E|0_{\text{GS}}^+\rangle$
- $H_{\text{eff}} \equiv$  **Phenomenological** Hamiltonian.  
Fitted to experimental data (NN scattering) and renormalized within each **valence space**
- Good description of nuclear spectroscopy

## Orbitals

- Empty orbitals
- **Valence Space**  $\rightarrow H_{\text{eff}}$
- **Inert Core**

$^{40}\text{Ca}:$



Caurier et al. Rev. Mod. Phys. 77, 427, 2005

# PHENOMENOLOGICAL INTERACTIONS

## *pf*–shell interactions( $A = 40 - 80$ )

- KB3G: Kuo-Brown interaction Mass dependence and monopole modifications
- GXPF1A: Bonn-C potential Two-body matrix elements from  $A = 47 - 66$

## *pfg*–shell interaction( $A = 56 - 100$ )

- JUN45: Bonn-C interaction 133 two-body matrix elements, 4 single-particle energies with  $A = 63 - 96$
- GCN2850: G-matrix, fit to 300 energy levels
- JJ4BB
- RG.PROLATE

## *sdgh*–shell( $A = 100 - 140$ )

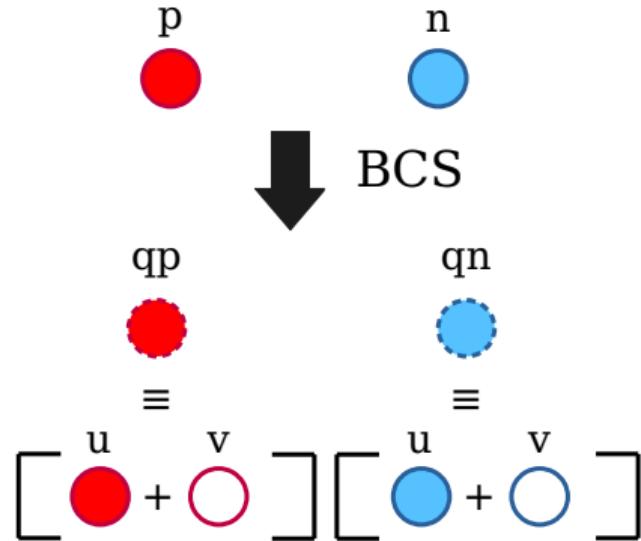
- GCN5082
- QX5082: Bonn-C potential Binding energies of 157 low-lying yrast states from  $^{102-132}\text{Sn}$

# QUASI-PARTICLE RANDOM PHASE APPROXIMATION

## QRPA

- $|\text{QRPA}\rangle = |0^+\rangle \rightarrow \text{Reference state}$
- $n = 0 \rightarrow 2$  harmonic oscillator shells above the Fermi level
- Larger valence spaces than NSM but less complex correlations between nucleons (less parameters to fit in the Hamiltonian)

Suhonen, Springer-Verlag, Berlin Heidelberg, 2007



$$|\text{QRPA}\rangle = \sum_{pn} (a_n qp + b_n qn)$$

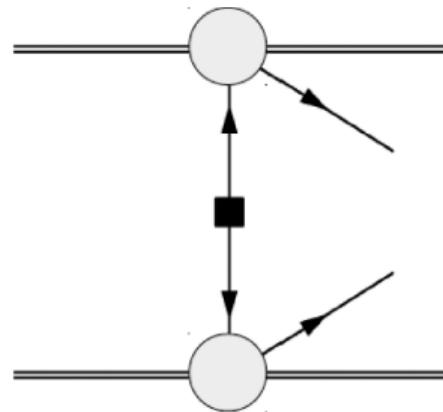
Credit, Jokiniemi

# $0\nu\beta\beta$ : $\chi$ EFT DIAGRAMS

$$M^{0\nu} = M_L^{0\nu} + M_S^{0\nu} + M_{\text{usoft}}^{0\nu} + M_{\text{loops}}^{0\nu}$$

$q \equiv$  Transferred momentum

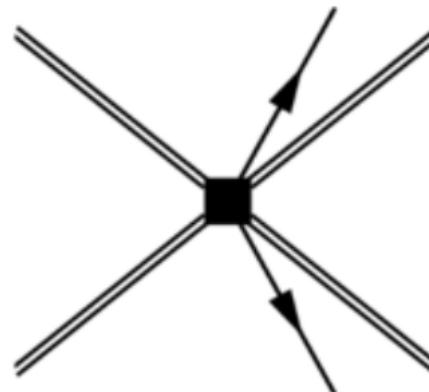
- Long-range(L)  
( $q \sim 100\text{MeV} + q$ -dependent  $N^2\text{LO}$ )



Leading Order (LO) → already computed

Jokiniemi et al. Phys. Lett. B, 823, 136720, 2021

- Short-range(S)  
( $q \gg 100 \text{ MeV}$ )



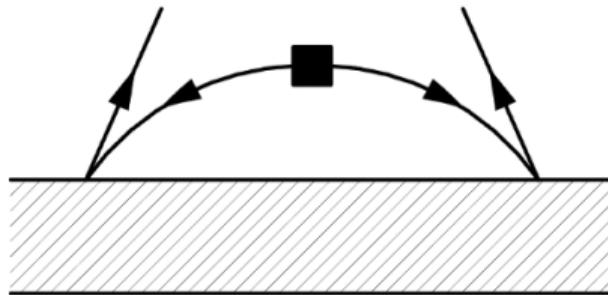
Cirigliano et al. Phys. Rev. C 97, 065501, 2018

# $0\nu\beta\beta : \chi$ EFT DIAGRAMS

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$q \equiv$  Transferred momentum

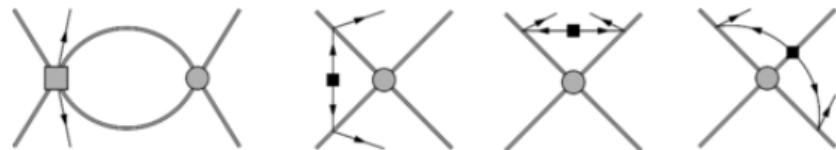
- Ultrasoft(usoft)  
( $q \ll 100\text{MeV}$  )



Next-to-next leading order(N<sup>2</sup>LO)→New terms

DC, Jokiniemi, Menéndez, Phys. Lett. B 860, 2025

- Loop terms  
(soft(3)+ultrasoft(1))



Cirigliano et al. Phys. Rev. C 97, 065501, 2018

# $0\nu\beta\beta$ : LEADING ORDER

$$M_L^{0\nu} = M_{L,GT}^{0\nu} + M_{L,F}^{0\nu} + M_{L,T}^{0\nu}$$

Finite Size Corrections:

$q$ -dependent N<sup>2</sup>LO ( $g_A^2(q^2)$ ,  $g_V^2(q^2)$ )

$$M_S^{0\nu} \propto g_\nu^{NN} e^{-q^2/\Lambda^2}$$

$g_\nu^{NN} \equiv$  Nucleon-Nucleon coupling

$\Lambda \equiv$  Gaussian cutoff

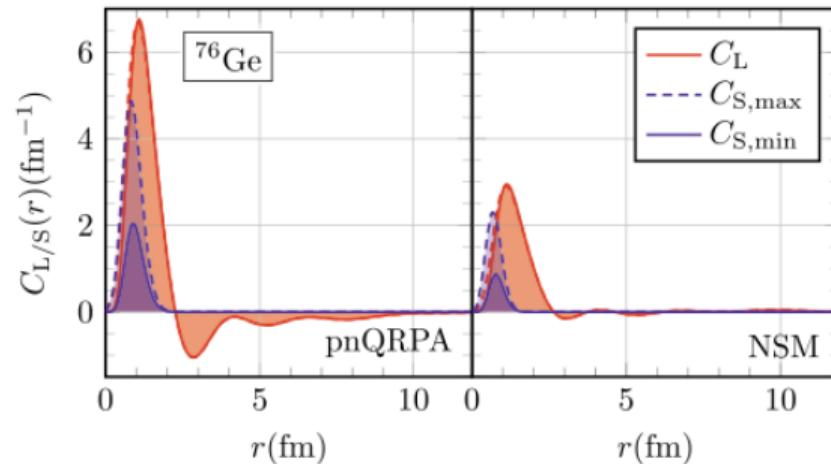
## Results

- NSM:  $M_S^{0\nu} / M_L^{0\nu} = +(15 - 50)\%$
- QRPA:  $M_S^{0\nu} / M_L^{0\nu} = +(30 - 80)\%$

Jokiniemi et al. Phys. Lett. B 823, 136720, 2021

$$M_{L/S}^{0\nu} = \int_0^\infty C_{L/S}(r) dr$$

- Hard Neutrinos
- Soft Neutrinos



Jokiniemi et al. Phys. Lett. B 823, 136720, 2021

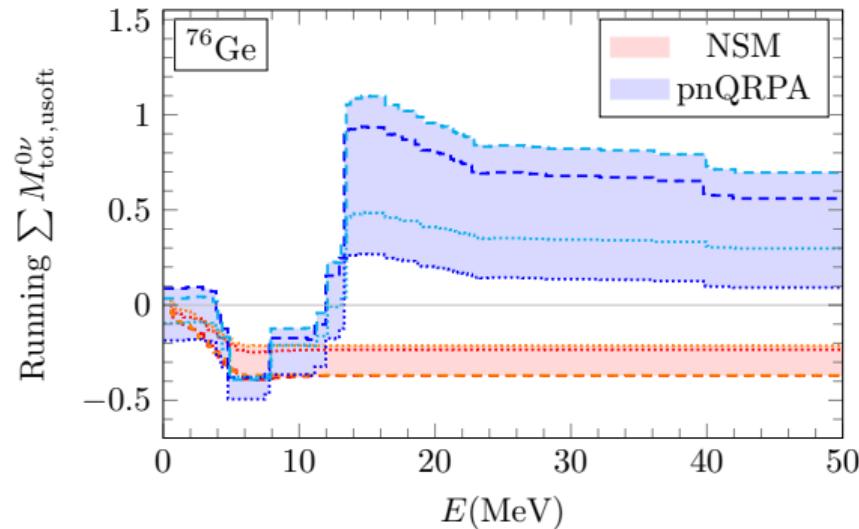
# $0\nu\beta\beta$ : ULTRASOFT NME

- Study of the **intermediate states** dependence of the **total ultrasoft NMEs**
- $M_{\text{usoft}}^{0\nu} \propto \langle 0_f^+ | \text{GT} | 1_n^+ \rangle \langle 1_n^+ | \text{GT} | 0_i^+ \rangle \cdot \ln(\mu_{\text{us}})$
- $\mu_{\text{us}} = \frac{m_\pi}{2} \dots 2m_\pi \equiv \text{Ultrasoft scale} \rightarrow \text{Main uncertainty}$

## Results

Different behaviour between models around 10 MeV → **Different sign**

$$M_{\text{tot,usoft}}^{0\nu} = M_{\text{usoft}}^{0\nu} + M_{\text{tot,usoft}}^{0\nu}$$



Jokiniemi et al. Phys. Lett. B 823, 136720, 2021

# $0\nu\beta\beta$ : CLOSURE VS NON-CLOSURE

Non-closure:

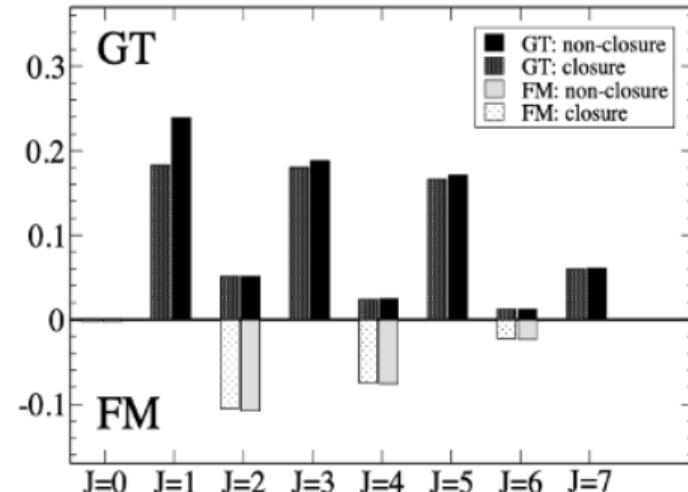
$$M_{\text{non-cl}}^{0\nu} \propto \frac{\langle 0_f^+ | J_\mu(x) | J_n^\pi \rangle \langle J_n^\pi | J^\mu(y) | 0_n^+ \rangle}{q(q+E_n - \frac{1}{2}(E_f+E_i))}$$

- $q \simeq k_F \simeq 100 \text{ MeV}$
- $E_n - \frac{1}{2}(E_i + E_f) \rightarrow 0$
- $J_\mu J^\mu = h_{\text{GT}} + h_{\text{F}} + h_{\text{T}}$

Closure:

$$M_{\text{L}}^{0\nu} \propto \frac{\langle 0_f^+ | J_\mu(x) J^\mu(y) | 0_n^+ \rangle}{q^2}$$

- Main difference comes from  $GT|1_n^+\rangle$
- Same dependence as  $M_{\text{usoft}}^{0\nu}$



Sen'kov, Horoim Phys. Rev. C 88, 064312

# $0\nu\beta\beta$ : CLOSURE VS NON-CLOSURE

- According to  $\chi$ EFT the **ultrasoft term** must be the **main contribution** beyond the **closure approximation**:

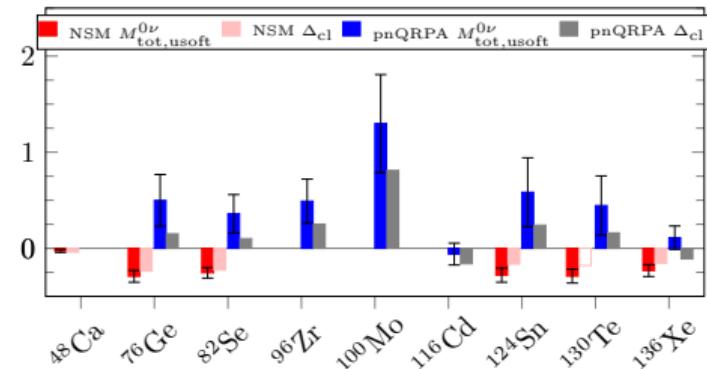
$$M_{\text{non-cl}}^{0\nu} - M_{\text{L}}^{0\nu} = \Delta_{\text{cl}} \sim M_{\text{tot,usoft}}^{0\nu}$$

Cirigliano et al. Phys. Rev. C 97, 065501, 2018

## Results

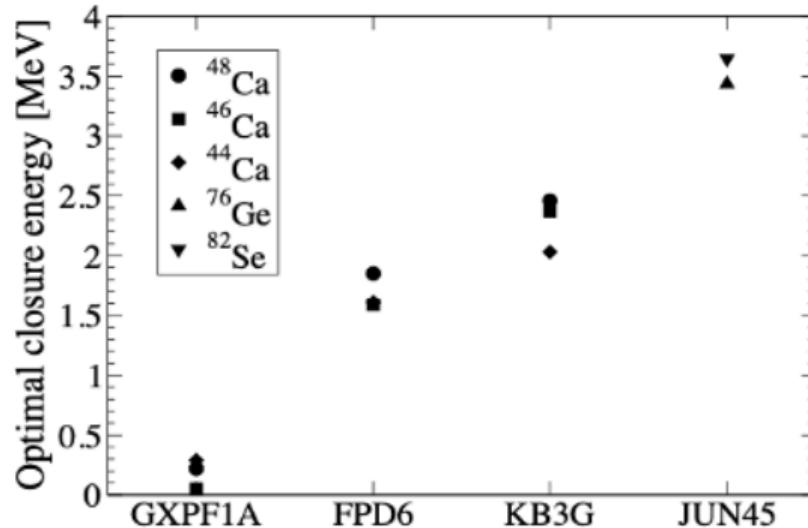
- Agreement in the sign between  $\Delta_{\text{cl}}$  and  $M_{\text{tot,usoft}}^{0\nu}$
- NSM: Good agreement with  $\chi$ EFT
- QRPA: Milder agreement with  $\chi$ EFT

Important uncertainties due to  $\mu_{\text{us}}$  dependence



DC, Jokiniemi, Menéndez, Soriano, Phys. Lett. B, 860, 2025

# $0\nu\beta\beta$ : CLOSURE ENERGIES



Neacsu, Horoi, Phys. Rev. C, 91, 024309, 2015

Nucleus	Interaction	$\langle E_n \rangle$ (MeV)
$^{48}\text{Ca}$	KB3G	2.5
$^{48}\text{Ca}$	GXPF1A	0.23
$^{76}\text{Ge}$	JUN45	3.5
$^{82}\text{Se}$	JUN45	3.6
$^{136}\text{Xe}$	GCN5082	3.7

Sarkar et al. arXiv:2406.13417v1, 2024

# $0\nu\beta\beta$ : LOOP NMEs

$$M_{\text{loops,soft}}^{0\nu} = M_{AA,\text{loops}}^{0\nu} + M_{VV,\text{loops}}^{0\nu} + M_{CT,\text{loops}}^{0\nu}$$

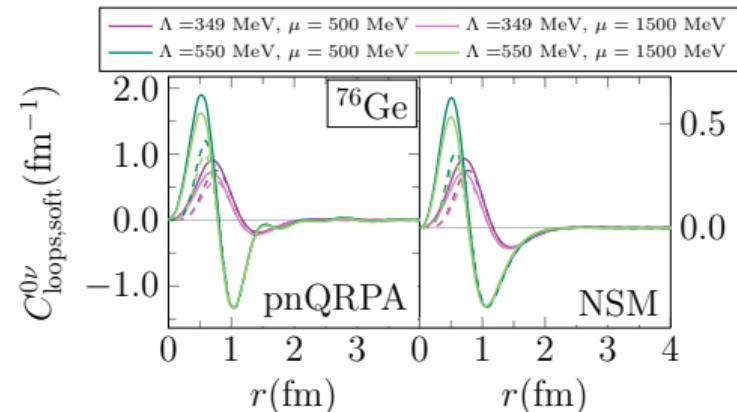
## Results

- Important contributions in short-range distances
- Similar behaviour between models
- Main dependence  $\rightarrow \Lambda$ 
  - $\Lambda = 349$  MeV  $\Lambda = 550$  MeV
- Most reliable:  $\Lambda = 349$

Uncertainties:  $H_{\text{eff}}$ ,  $\mu$ ,  $\Lambda$

$\mu \equiv$  Renormalization scale

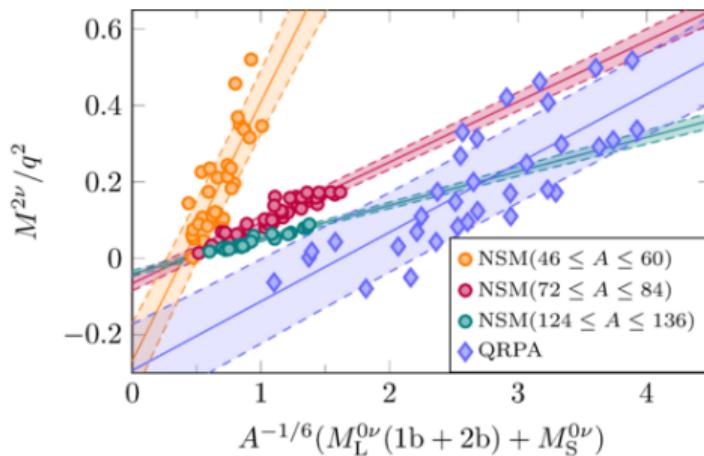
$$M_{\text{loops,soft}}^{0\nu} = \int_0^\infty C_{\text{loops,soft}}^{0\nu}(r) dr$$



DC, Jokiniemi, Menéndez, Soriano, Phys. Lett. B, 860, 2025

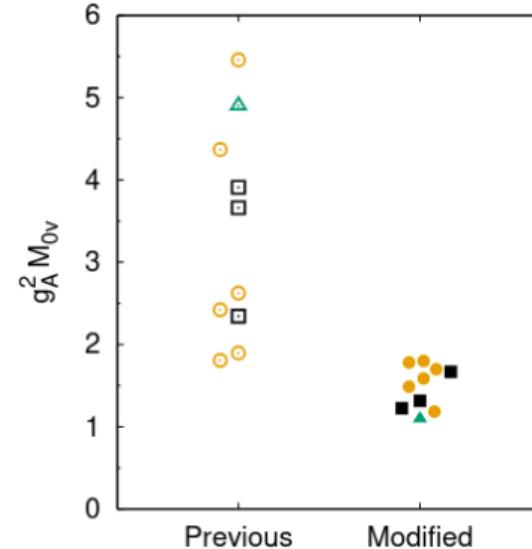
# $0\nu\beta\beta$ - $2\nu\beta\beta$ CONNECTION

Good correlation: systematic calculation of different nuclei



Jokiniemi et al. Phys. Rev. C 107, 044305, 2023

$^{136}\text{Xe}$ : Uncertainty reduction of  $0\nu\beta\beta$  NME using  $2\nu\beta\beta$  experimental data



Civitarese et al. arXiv.2509.16605, 2025

$$T_{1/2}^{2\nu} = g_A^4 (M_{\text{GT}}^{-1})^2 \Delta_0 [G_0^{2\nu} + \xi_{31} \frac{\Delta_2}{\Delta_0} G_2^{2\nu} + G_4^{2\nu} (\xi_{51} \frac{\Delta_2}{\Delta_0} + \frac{1}{3} \xi_{31}^2) + G_{22}^{2\nu} + G_M^{2\nu}]$$

## Taylor expansion

- $\xi_{31} = \frac{M_{\text{GT}}^{-3}}{M_{\text{GT}}^{-1}}$
- $\xi_{51} = \frac{M_{\text{GT}}^{-5}}{M_{\text{GT}}^{-1}}$
- $M_{\text{GT}}^{-2m-1} \propto \frac{\langle 0_f^+ | \text{GT} | 1_n^+ \rangle \langle 1_n^+ | \text{GT} | 0_i^+ \rangle}{(E_n - \frac{1}{2}(E_i + E_f))^{2m+1}}$

## NLO Corrections

- $G_M^{2\nu} \equiv$  New weak magnetism contribution
- $\Delta_0, \Delta_2 \equiv$  one-pion exchange terms and WM insertion (same NMEs than  $M_L^{0\nu}(m_\pi)$ )

Šimkovic et al. Phys. Rev. C, 97, 034315, 2018

el Morabit et al. JHEP, 06, 082, 2025