Precision $\beta\beta$ Nuclear Matrix Elements

Daniel Castillo (ICCUB, UB)

Dorian Frycz (UB, ICCUB), Lotta Jokiniemi (TUDa), Javier Menéndez (ICCUB, UB), Beatriz Benavente (UB)

Institut de Ciències del Cosmos
Universitat de Barcelona

January 27, 2026

Engel et al. Rep. Prog. Phys. 80, 046301, 2017

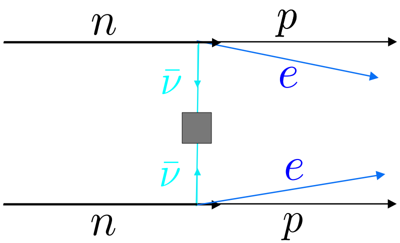
$\beta\beta$ DECAY

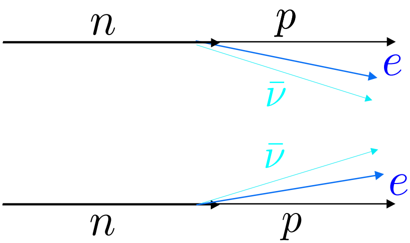
$0\nu\beta\beta : 2n \rightarrow 2p + 2e^-$

- Beyond Standard Model $\rightarrow$ Lepton Number Violation
- Majorana Fermions
- Hypothetical decay

$2\nu\beta\beta : 2n \rightarrow 2p + 2e^- + 2\bar{\nu}_e$

- Standard Model allowed $\rightarrow$ Lepton Number Conservation
- Dirac Fermions
- Already measured



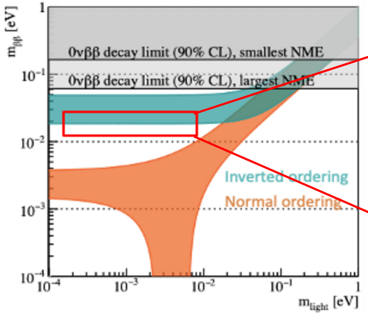


Engel et al. Rep. Prog. Phys. 80, 046301, 2017

MOTIVATION

$$(T^{\beta\beta}_{1/2})^{-1} \propto g_A^4 G_0 |M^{\beta\beta}|^2 m^2_{\beta\beta}$$

$$m_{\beta\beta} = \begin{cases} 1 & \text{if } 2\nu\beta\beta \\ \sum_{j=\text{light}} U_{ej} m_j & \text{if } 0\nu\beta\beta \end{cases}$$



Agostini et al. Rev. Mod. Phys. 95, 025002, 2023

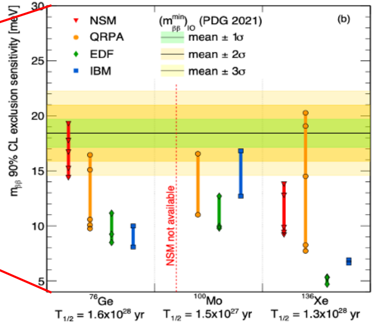
$M^{\beta\beta} \equiv$ Nuclear matrix elements

$G_0 \equiv$ Phase-space factor (PSF)

$g_A \equiv$ Axial coupling

$m_{\beta\beta} \equiv$ Effective neutrino mass

Next Decade Experiments



Agostini et al. Phys. Rev. C 104, L042501, 2021

NUCLEAR MATRIX ELEMENTS

Wavefunctions

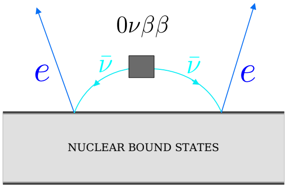
- Nuclear Shell Model (NSM) and Quasiparticle Random Phase Approximation (QRPA)
- Phenomenological Hamiltonian

Decay operators

- Chiral Effective Field Theory (χEFT) (LO+NLO+N^{2}LO...)
- $\hat{\mathcal{O}}_k \equiv$ Spin-space operator
- $\tau^- \equiv$ Ladder isospin operator

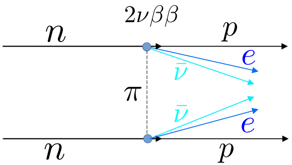
Cirigliano et al. Phys. Rev. C 97, 065501, 2018
el Morabit et al. JHEP, 06, 082, 2025

$$M^{\beta\beta} = \langle 0_f^+ | \sum_{a,b} \hat{\mathcal{O}}_k \tau_a^- \tau_b^- | 0_i^+ \rangle$$



$\longrightarrow$ N^{2}LO

NLO $\longleftarrow$

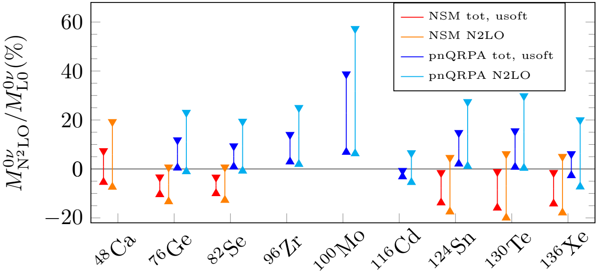


$0\nu\beta\beta$: TOTAL N^{2}LO NMEs

$$M^{0\nu} = M^{0\nu}_{\rm LO} + M^{0\nu}_{\rm N^2LO}$$
$$= M^{0\nu}_{\rm L} + M^{0\nu}_{\rm S} + M^{0\nu}_{\rm usoft} + M^{0\nu}_{\rm loops}$$

χEFT expectations: (5 – 10)%

- $M^{0\nu}_{\rm L(S)} \equiv$ Long(Short)-range NME
- $M^{0\nu}_{\rm usoft} \equiv$ ultrasoft NME (low momentum transferred)
- $M^{0\nu}_{\rm loops} \equiv$ loop terms



DC, Jokiniemi, Menéndez, Phys. Lett. B 860, 2025

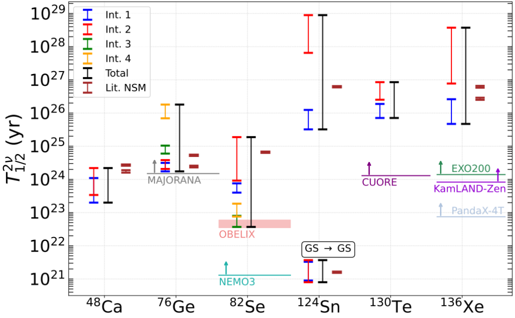
Results

- N^{2}LO contributions: Non-negligible
- Different sign between many-body methods: ultrasoft NME
- $|M^{0\nu}_{\rm N^2LO}/M^{0\nu}_{\rm LO}|$:
  - NSM $\leq$ 20%
  - QRPA $\leq$ 30%
- Central values: $|M^{0\nu}_{\rm N^2LO}/M^{0\nu}_{\rm LO}| \sim$ (5 – 15)%

$2\nu\beta\beta$: DECAY TO EXCITED STATES

$$(T^{2\nu}_{1/2}(0^+_{\rm GS} \to 0^+_1))^{-1} \propto G^{2\nu} \cdot g_A^4 \cdot (M^{2\nu})^2$$

- $M^{2\nu} \equiv$ LO NME
- $+$ Lepton energy expansion (Additional PSFs and NMEs)
- $+$ χEFT corrections up to NLO



DC, Frycz, Menéndez, Benavente, submitted Phys. Lett. B

Results

- NSM calculations
- Main uncertainty comes from different Hamiltonians: Deformation of initial and final states plays a key role (**Dorian's Talk on Monday**)
- 2 different GT operators: Bare and renormalized

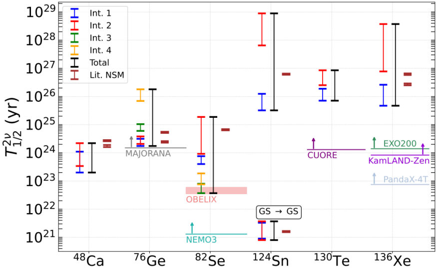
Coraggio et al. Phys. Rev. C, 100, 014316, 2019

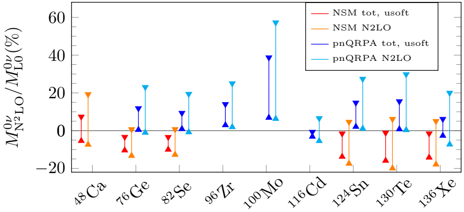
- ^{76}Ge, ^{82}Se: Experimental evidence close to theoretical predictions

SUMMARY AND OUTLOOK

Summary

- $0\nu\beta\beta$ N^{2}LO NMEs and $2\nu\beta\beta$ half-lives ($0^+_{GS} \to 0^+_1$)
- $0\nu\beta\beta$ N^{2}LO NMEs are not negligible: $|M^{0\nu}_{N^2LO}/M^{0\nu}_{LO}| \lesssim 30\%$
- $T^{2\nu}_{1/2}$ main uncertainty: Hamiltonian (deformation)





Outlook

- Use $\chi-$Hamiltonians to compute the wavefunctions
- Extend the decay operator study to two-body currents
- Experimental measurements

Thank you for your attention!

Neutrinoless $\beta\beta$ decay nuclear matrix elements complete up to N^2LO in heavy nuclei

Daniel Castillo[a,b,*], Lotta Jokiniemi[c,*], Pablo Soriano[a,b], Javier Menéndez[a,b]

[a]*Departament de Física Quàntica i Astrofísica, Universitat de Barcelona, 08028 Barcelona, Spain.*
[b] *Institut de Ciències del Cosmos, Universitat de Barcelona, 08028 Barcelona, Spain.*
[c]*TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada*

Two-neutrino $\beta\beta$ decay to excited states at next-to-leading order

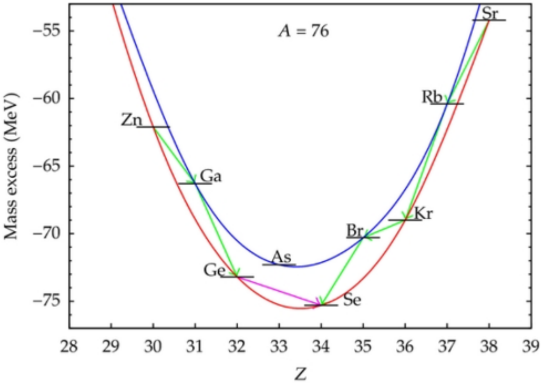
Daniel Castillo[a,b,c,*], Dorian Frycz[a,b,c,*], Beatriz Benavente[c], Javier Menéndez[a,b]

[a]*Departament de Física Quàntica i Astrofísica , Universitat de Barcelona, c. Martí i Franquès, 1, 08028 Barcelona, Spain*
[b] *Institut de Ciències del Cosmos, Universitat de Barcelona, c. Martí i Franquès, 1, 08028 Barcelona, Spain*
[c]*Facultat de Física, Universitat de Barcelona, c. Martí i Franquès, 1, 08028 Barcelona, Spain*

BACKUP SLIDES

$\beta\beta$ DECAY

- ^{76}Ge: β decay forbidden
- Energy difference between odd-odd nuclei and even-even nuclei$\rightarrow$ Pairing
- $\beta\beta$ decay allowed:
  $^{76}\text{Ge} \rightarrow {}^{82}\text{Se}$ ($Q_{\beta\beta} > 0$)
- $Q_{\beta\beta} = E_i - E_f - 2m_e$
  - $Q_{\beta\beta} \equiv Q-$value for $\beta\beta$ decay
  - $m_e \equiv$ Electron mass
  - $E_i \equiv$ Initial energy
  - $E_f \equiv$ Final energy



Giuliani, et al. Adv. High Energy Phys, 2012, 857016, 2012.

NUCLEAR SHELL MODEL

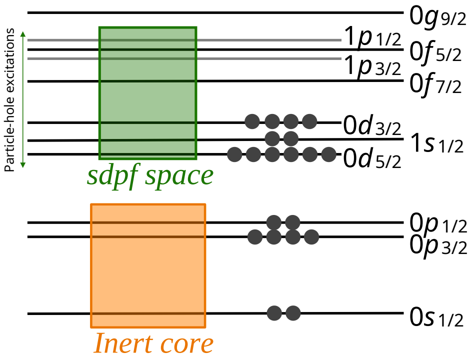
Hamiltonian

- To solve the Schrödinger equation $\rightarrow H_{\text{eff}}|0^+_{\text{GS}}\rangle = E|0^+_{\text{GS}}\rangle$
- $H_{\text{eff}} \equiv$ Phenomenological Hamiltonian. Fitted to experimental data (NN scattering) and renormalized within each valence space
- Good description of nuclear spectroscopy

Orbitals

- Empty orbitals
- Valence Space $\rightarrow H_{\text{eff}}$
- Inert Core

^{40}Ca:



Caurier et al. Rev. Mod. Phys. 77, 427, 2005

PHENOMENOLOGICAL INTERACTIONS

pf−shell interactions($A = 40 - 80$)

- KB3G: Kuo-Brown interaction Mass dependence and monopole modifications
- GXPF1A: Bonn-C potential Two-body matrix elements from $A = 47 - 66$

pfg−shell interaction($A = 56 - 100$)

- JUN45: Bonn-C interaction 133 two-body matrix elements, 4 single-particle energies with $A = 63 - 96$
- GCN2850: G-matrix, fit to 300 energy levels
- JJ4BB
- RG.PROLATE

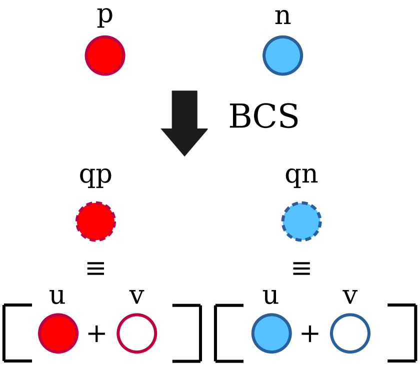
$sdgh$−shell($A = 100 - 140$)

- GCN5082
- QX5082: Bonn-C potential Binding energies of 157 low-lying yrast states from $^{102-132}Sn$

QRPA

- $|\mathrm{QRPA}\rangle = |0^+\rangle \rightarrow$ Reference state
- $n = 0 \rightarrow 2$ harmonic oscillator shells above the Fermi level
- Larger valence spaces than NSM but less complex correlations between nucleons (less paremeters to fit in the Hamiltonian)

Suhonen, Springer-Verlag, Berlin Heidelberg, 2007

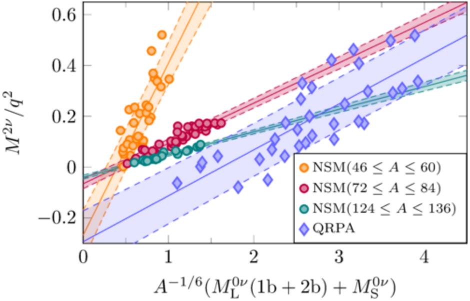


$$|\mathrm{QRPA}\rangle = \sum_{\mathrm{pn}}(\mathrm{a_n qp} + \mathrm{b_n qn})$$

Credit, Jokiniemi

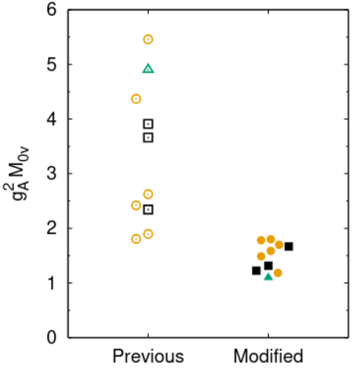
EXPERIMENTS

Good correlation: systematic calculation of different nuclei



Jokiniemi et al. Phys. Rev. C 107, 044305, 2023

^{136}Xe: Uncertainty reduction of $0\nu\beta\beta$ NME using $2\nu\beta\beta$ quenching



Civitarese et al. arXiv.2509.16605, 2025

$0\nu\beta\beta$: χEFT DIAGRAMS

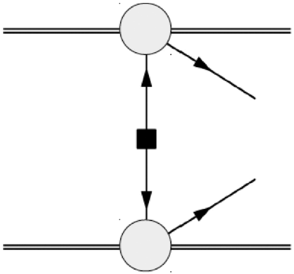
$$M^{0\nu} = M^{0\nu}_{\mathrm{L}} + M^{0\nu}_{\mathrm{S}} + M^{0\nu}_{\mathrm{usoft}} + M^{0\nu}_{\mathrm{loops}}$$

Leading Order (LO)$\rightarrow$already computed

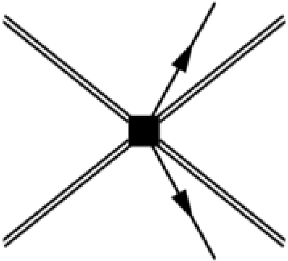
Jokiniemi et al. Phys. Lett. B, 823, 136720, 2021

$q \equiv$ Transferred momentum

- Long-range(L)
  ($q \sim$ 100MeV $+q-$dependent N^{2}LO)



- Short-range(S)
  ($q >> 100$ MeV)

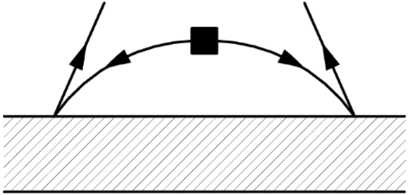


Cirigliano et al. Phys. Rev. C 97, 065501, 2018

$$M^{0\nu} = M^{0\nu}_{\mathrm{L}} + M^{0\nu}_{\mathrm{S}} + M^{0\nu}_{\mathrm{usoft}} + M^{0\nu}_{\mathrm{loops}}$$

$q \equiv$ Transferred momentum

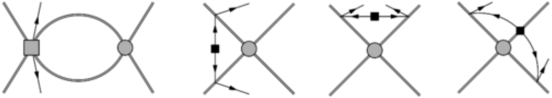
- Ultrasoft(usoft) ($q << 100$MeV)



Next-to-next leading order(N^2LO)$\rightarrow$New terms

DC, Jokiniemi, Menéndez, Phys. Lett. B 860, 2025

- Loop terms (soft(3)+ultrasoft(1))



Cirigliano et al. Phys. Rev. C 97, 065501, 2018

$0\nu\beta\beta$: LEADING ORDER

$$M^{0\nu}_{\mathrm{L}} = M^{0\nu}_{\mathrm{L,GT}} + M^{0\nu}_{\mathrm{L,F}} + M^{0\nu}_{\mathrm{L,T}}$$

Finite Size Corrections:
$q-$dependent $\mathrm{N}^2\mathrm{LO}$ $(g^2_A(q^2),\ g^2_V(q^2))$

$$M^{0\nu}_{\mathrm{S}} \propto g^{NN}_{\nu} e^{-q^2/\Lambda^2}$$

$g^{NN}_{\nu} \equiv$ Nucleon-Nucleon coupling
$\Lambda \equiv$ Gaussian cutoff

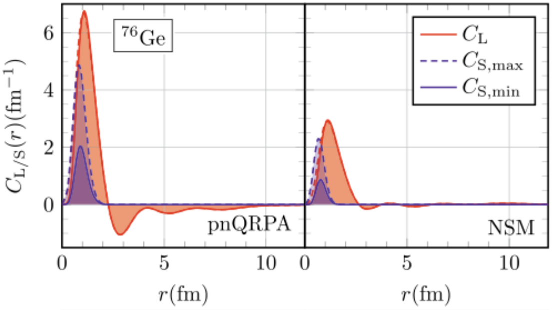
Results

- NSM: $M^{0\nu}_{\mathrm{S}}/M^{0\nu}_{\mathrm{L}} = +(15-50)\%$
- QRPA: $M^{0\nu}_{\mathrm{S}}/M^{0\nu}_{\mathrm{L}} = +(30-80)\%$

Jokiniemi et al. Phys. Lett. B 823, 136720, 2021

$$M^{0\nu}_{\mathrm{L/S}} = \int_0^\infty C_{\mathrm{L/S}}(r)dr$$

- Hard Neutrinos
- Soft Neutrinos



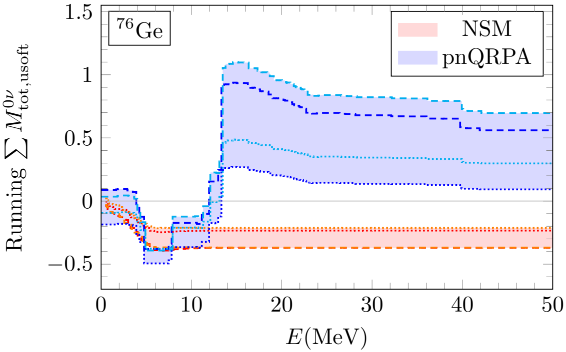
Jokiniemi et al. Phys. Lett. B 823, 136720, 2021

- Study of the intermediate states dependence of the total ultrasoft NMEs
- $M^{0\nu}_{\text{usoft}} \propto \langle 0^+_f|\text{GT}|1^+_n\rangle\langle 1^+_n|\text{GT}|0^+_i\rangle \cdot \ln(\mu_{\text{us}})$
- $\mu_{\text{us}} = \frac{m_\pi}{2} \ldots 2m_\pi \equiv$ Ultrasoft scale→Main uncertainty

Results

Different behaviour between models around 10 MeV→Different sign

$$M^{0\nu}_{\text{tot,usoft}} = M^{0\nu}_{\text{usoft}} + M^{0\nu}_{\text{tot,usoft}}$$



Jokiniemi et al. Phys. Lett. B 823, 136720, 2021

$0\nu\beta\beta$: CLOSURE VS NON-CLOSURE

Non-closure:

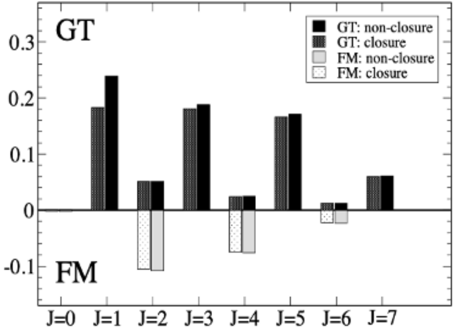
$$M^{0\nu}_{\mathrm{non-cl}} \propto \frac{\langle 0^+_f | J_\mu(x) | J^\pi_n \rangle \langle J^\pi_n | J^\mu(y) | 0^+_n \rangle}{q(q+E_n-\frac{1}{2}(E_f+E_i))} \rightarrow$$

- $q \simeq k_F \simeq 100\mathrm{MeV}$
- $E_n - \frac{1}{2}(E_i + E_f) \rightarrow 0$
- $J_\mu J^\mu = h_{\mathrm{GT}} + h_{\mathrm{F}} + h_{\mathrm{T}}$

$\rightarrow$ Closure:

$$M^{0\nu}_{\mathrm{L}} \propto \frac{\langle 0^+_f | J_\mu(x) J^\mu(y) | 0^+_n \rangle}{q^2}$$

- Main difference comes from $\mathrm{GT}|1^+_n\rangle$
- Same dependence as $M^{0\nu}_{\mathrm{usoft}}$



Sen'kov, Horoim Phys. Rev. C 88, 064312

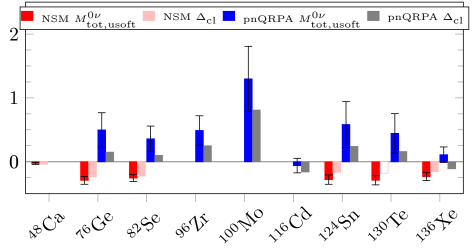
$0\nu\beta\beta$: CLOSURE VS NON-CLOSURE

- According to χEFT the ultrasoft term must be the main contribution beyond the closure approximation:
$$M^{0\nu}_{\mathrm{non-cl}} - M^{0\nu}_{\mathrm{L}} = \Delta_{\mathrm{cl}} \sim M^{0\nu}_{\mathrm{tot,usoft}}$$
Cirigliano et al. Phys. Rev. C 97, 065501, 2018

Results

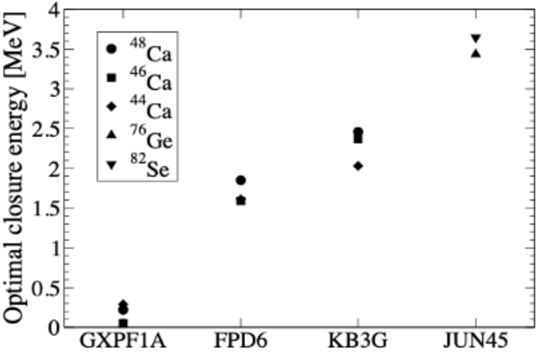
- Agreement in the sign between Δ_{cl} and $M^{0\nu}_{\mathrm{tot,usoft}}$
- NSM: Good agreement with χEFT
- QRPA: Milder agreement with χEFT

Important uncertainties due to μ_{us} dependence



DC, Jokiniemi, Menéndez, Soriano, Phys. Lett. B, 860, 2025

$0\nu\beta\beta$: CLOSURE ENERGIES



Neacsu, Horoi, Phys. Rev. C, 91, 024309, 2015

Nucleus	Interaction	$\langle E_n \rangle$ (MeV)
^{48}Ca	KB3G	2.5
^{48}Ca	GXPF1A	0.23
^{76}Ge	JUN45	3.5
^{82}Se	JUN45	3.6
^{136}Xe	GCN5082	3.7

Sarkar et al. arXiv:2406.13417v1, 2024

$$M^{0\nu}_{\text{loops,soft}} = M^{0\nu}_{AA,\text{loops}} + M^{0\nu}_{VV,\text{loops}} + M^{0\nu}_{CT,\text{loops}}$$

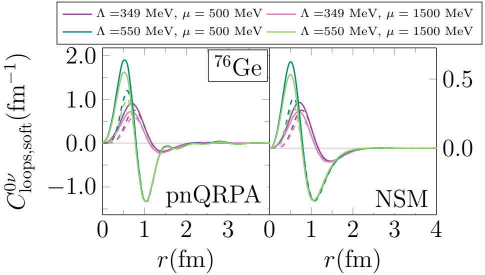
Uncertainties: H_{eff}, μ, Λ

$\mu \equiv$ Renormalization scale

$$M^{0\nu}_{\text{loops,soft}} = \int_0^\infty C^{0\nu}_{\text{loops,soft}}(r)dr$$

Results

- Important contributions in short-range distances
- Similar behaviour between models
- Main dependence $\rightarrow \Lambda$
  - $\Lambda = 349$ MeV $\Lambda = 550$ MeV
- Most reliable: $\Lambda = 349$



DC, Jokiniemi, Menéndez, Soriano, Phys. Lett. B, 860, 2025

$2\nu\beta\beta$: HALF-LIFE UP TO NLO

$$T^{2\nu}_{1/2} = g_A^4 (M^{-1}_{\mathrm{GT}})^2 \Delta_0 [G^{2\nu}_0 + \xi_{31} \frac{\Delta_2}{\Delta_0} G^{2\nu}_2 + G^{2\nu}_4 (\xi_{51} \frac{\Delta_2}{\Delta_0} + \frac{1}{3}\xi^2_{31}) + G^{2\nu}_{22} + G^{2\nu}_M]$$

Taylor expansion

- $\xi_{31} = \frac{M^{-3}_{\mathrm{GT}}}{M^{-1}_{\mathrm{GT}}}$
- $\xi_{51} = \frac{M^{-5}_{\mathrm{GT}}}{M^{-1}_{\mathrm{GT}}}$
- $M^{-2m-1}_{\mathrm{GT}} \propto \frac{\langle 0^+_f|\mathrm{GT}|1^+_n\rangle\langle 1^+_n|\mathrm{GT}|0^+_i\rangle}{(E_n - \frac{1}{2}(E_i+E_f))^{2m+1}}$

Šimkovic et al. Phys. Rev. C, 97, 034315, 2018

NLO Corrections

- $G^{2\nu}_M \equiv$ New weak magnetism contribution
- $\Delta_0,\ \Delta_2 \equiv$ one-pion exchange terms and WM insertion (same NMEs than $M^{0\nu}_{\mathrm{L}}(m_\pi)$)

el Morabit et al. JHEP, 06, 082, 2025