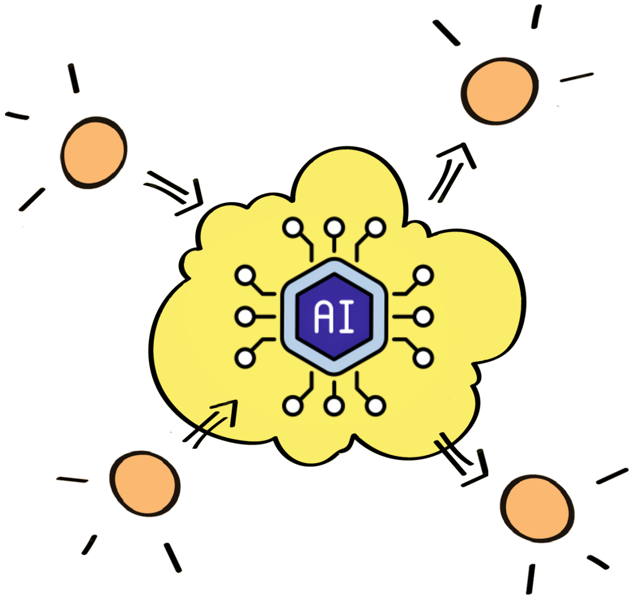
Machine Learning approaches to Amplitude Analysis



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Hadron and Nuclear Physics group workshop (HADNUCMAT)

University of Barcelona, January 26-28, 2026

Universitat de Barcelona

ICCUB

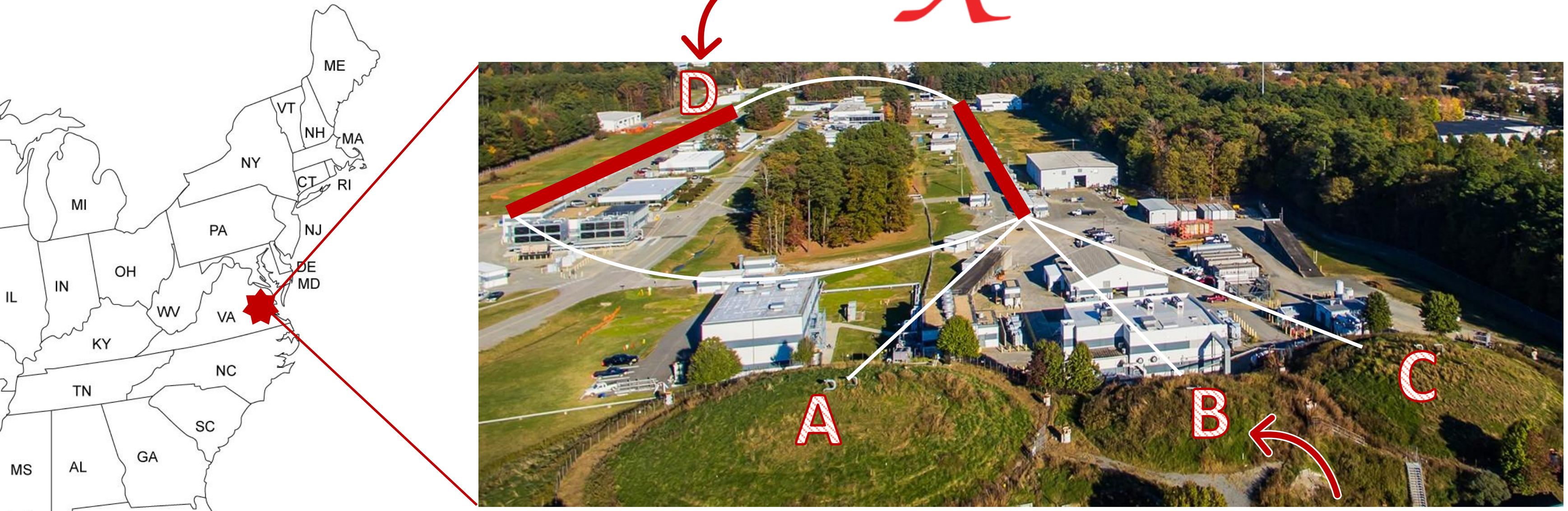
Excelencia María de Maeztu 04/2025-03/2031

Beatriu de Pinós

A(i)DAPT AI for Data Analysis and PreservaTion

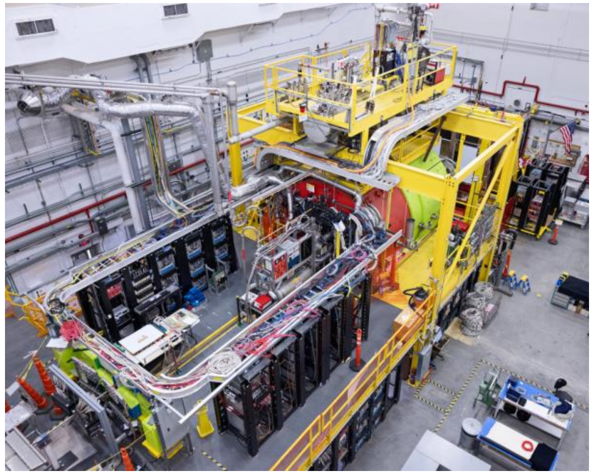
JPAC

Nuclear and Particle physics Experiments: our probe to Hadron Spectroscopy and QCD

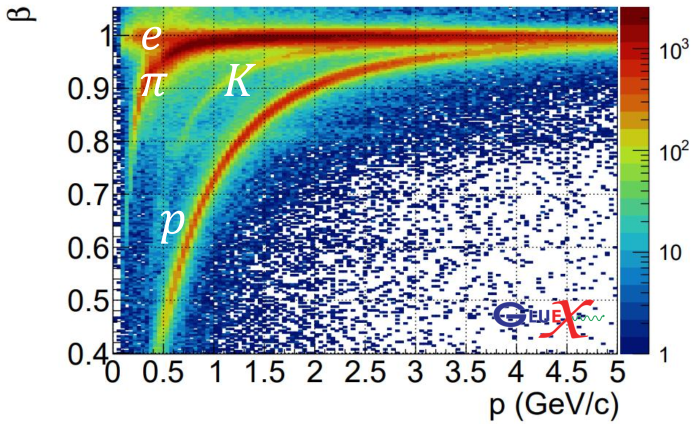


Jefferson Lab
CEBAF accelerator

GlueX experiment



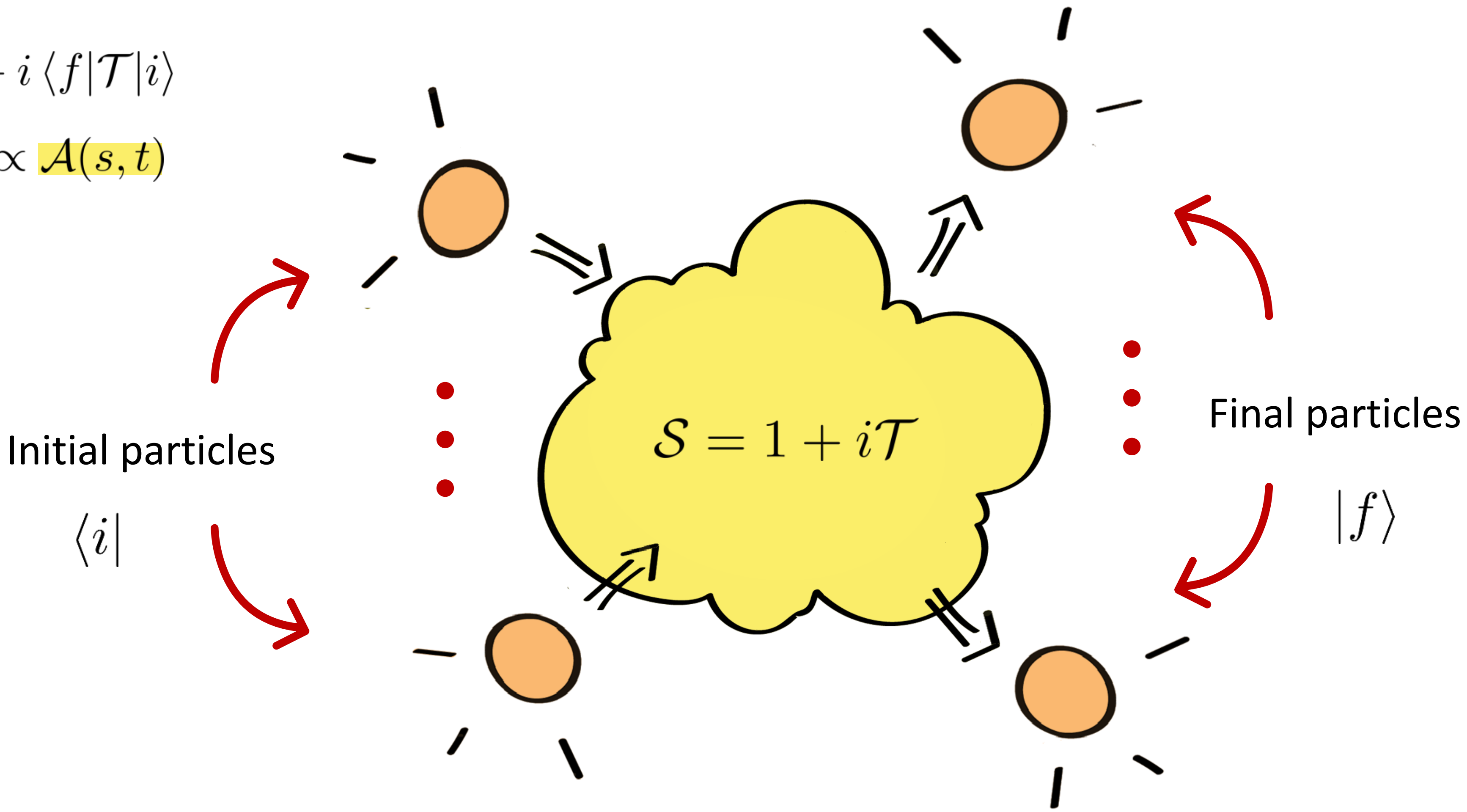
Particle identification



Theorist's perspective: Scattering Amplitude

$$\langle f|\mathcal{S}|i\rangle = \delta_{fi} + i\,\langle f|\mathcal{T}|i\rangle$$

$$\langle p_1' p_2'|\mathcal{T}|p_1 p_2\rangle \propto \mathcal{A}(s,t)$$



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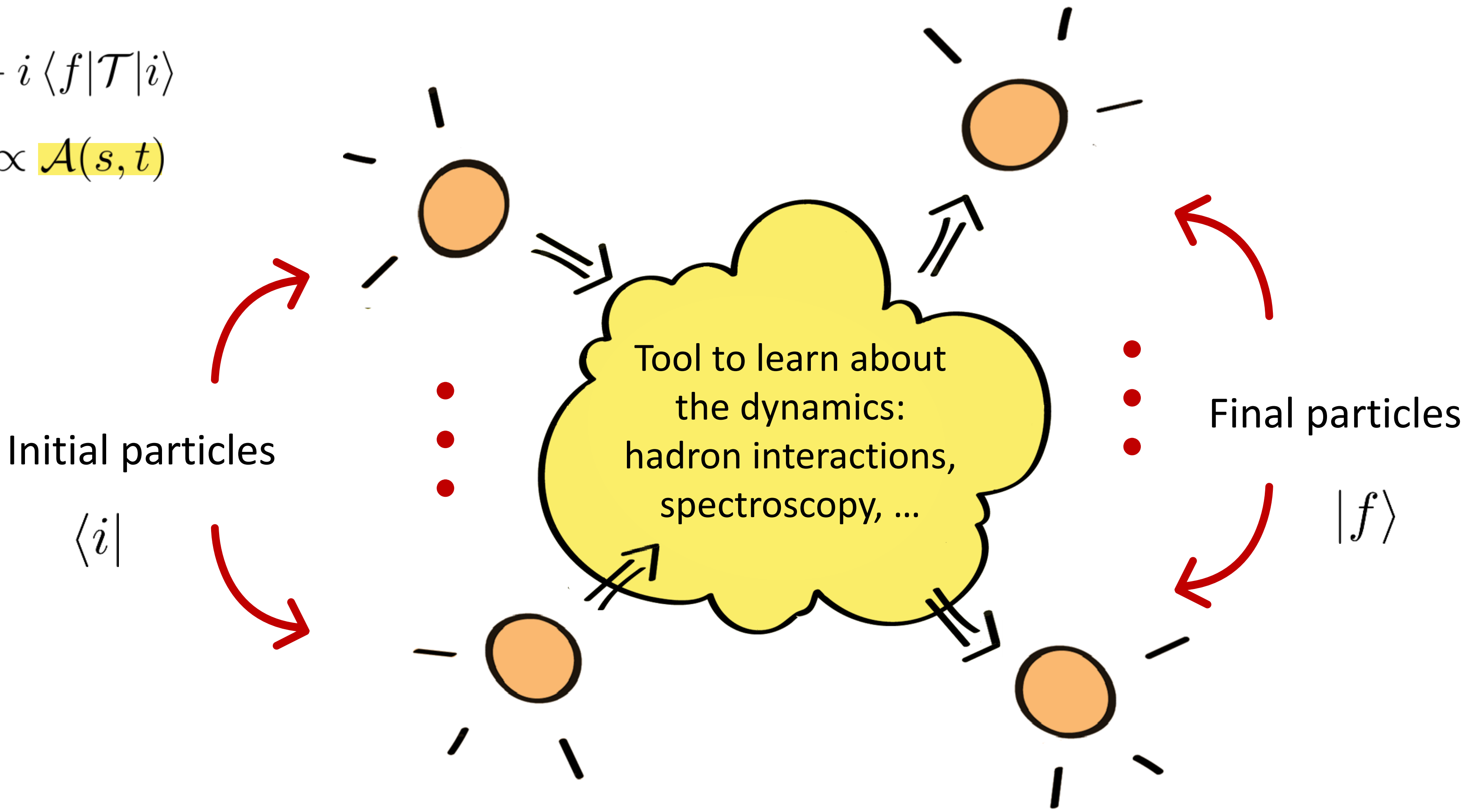

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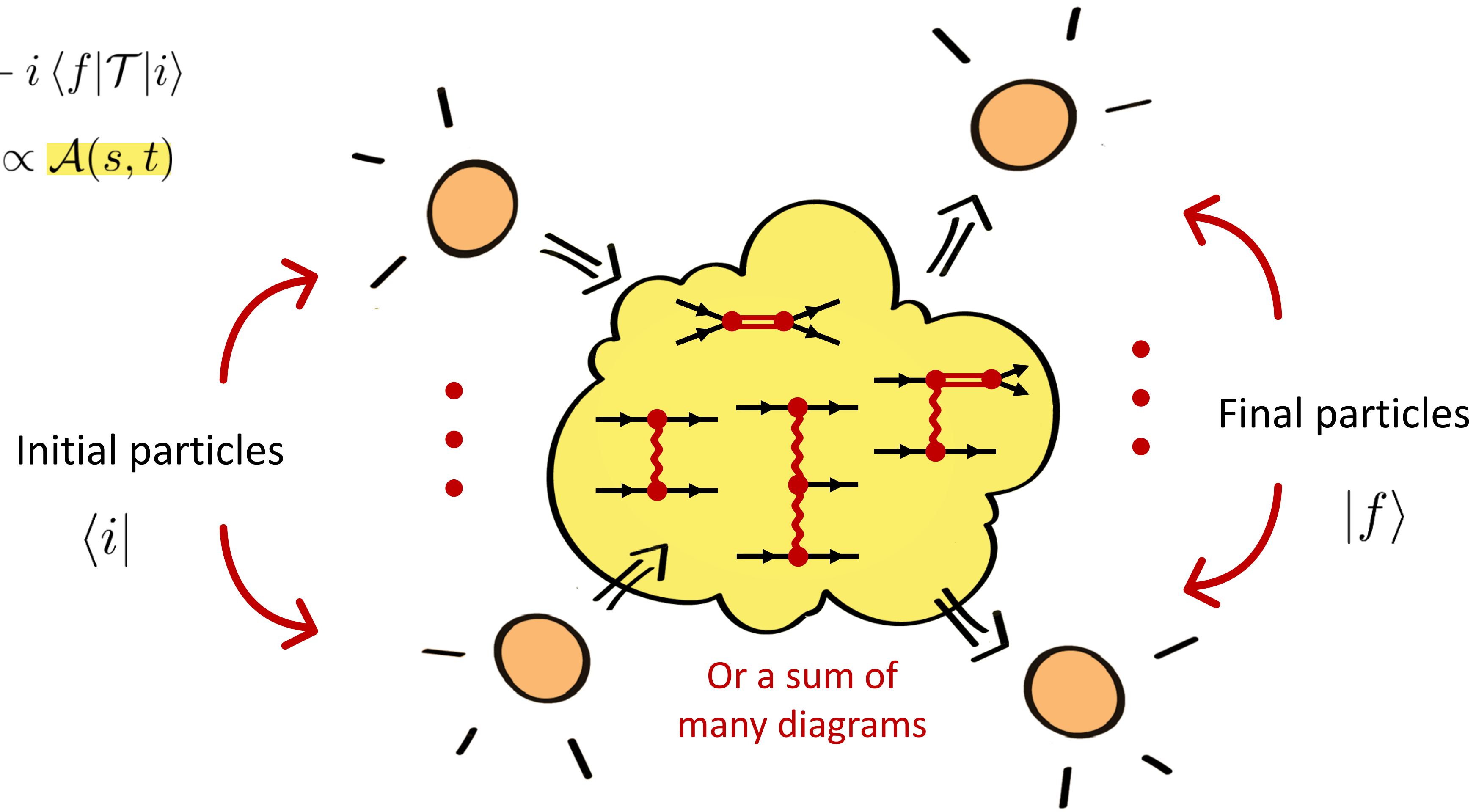


What does the blob really represent?

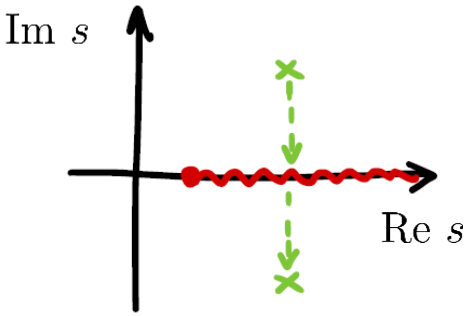
See talk by Vincent Mathieu

Properties of scattering amplitudes

Principles of scattering theory

- **Unitarity:** probability conservation (all possible outcomes must add up to 100%)
- **Analyticity:** causality (outcome cannot happen before interaction, smooth dependence on energy)
- **Crossing symmetry:** swapping particles and anti-particles (the mathematical description remains the same)

Discontinuity over the unitarity cut:



$$2\,\mathrm{Im}\,\mathcal{A}_{i\to f} \propto \sum_n \mathcal{A}_{i\to n}\mathcal{A}^{\dagger}_{n\to f}$$

Properties of scattering amplitudes

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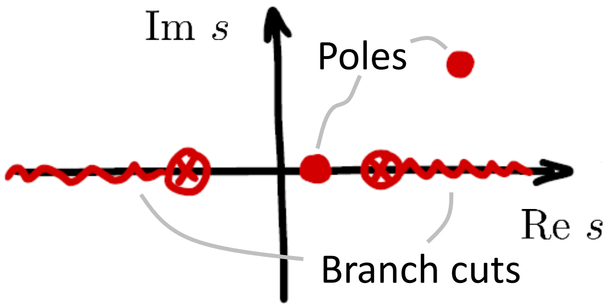

Poles correspond to physical hadronic states

Branch points at the opening to N-particle scattering thresholds

Properties of scattering amplitudes

Principles of scattering theory

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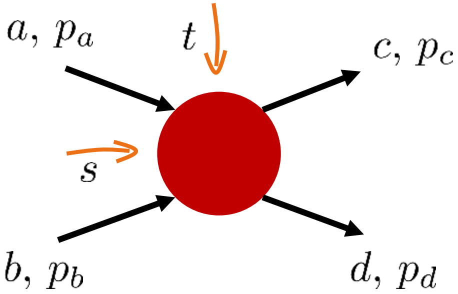
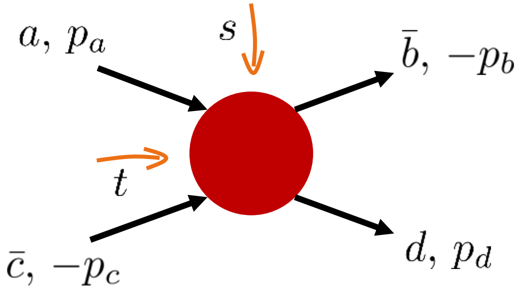
$$A_{ab\to cd}(s,t,u) = A_{a\bar{c}\to\bar{b}d}(t,s,u)$$

$a + b \to c + d$



$a + \bar{c} \to \bar{b} + d$

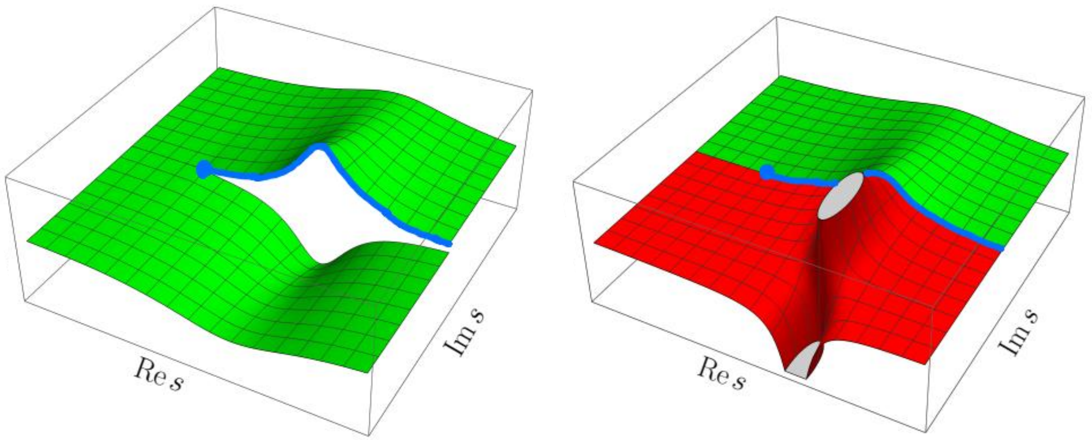


Challenges in hadron spectroscopy

Experiments measure **physical observables** (e.g. cross section) in the **real axis** (physical region)

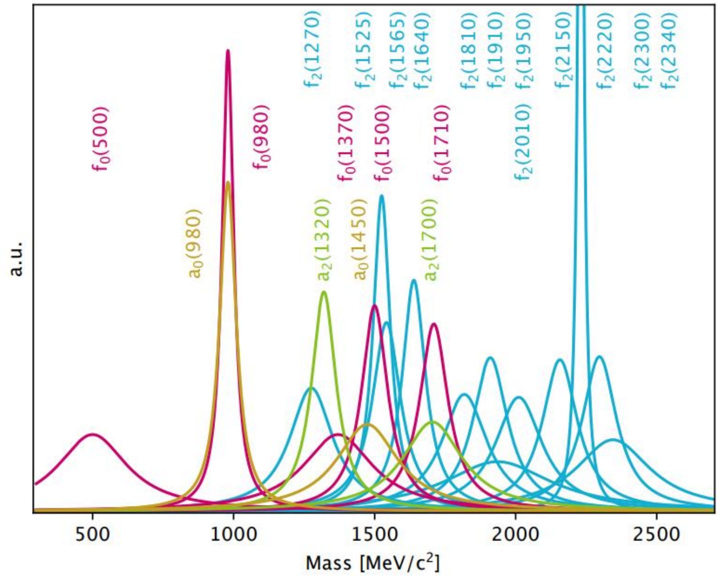
Most hadrons are **resonances** (poles of the scattering amplitude in the complex-energy plane)

Sometimes identified through “bumps” in the cross section



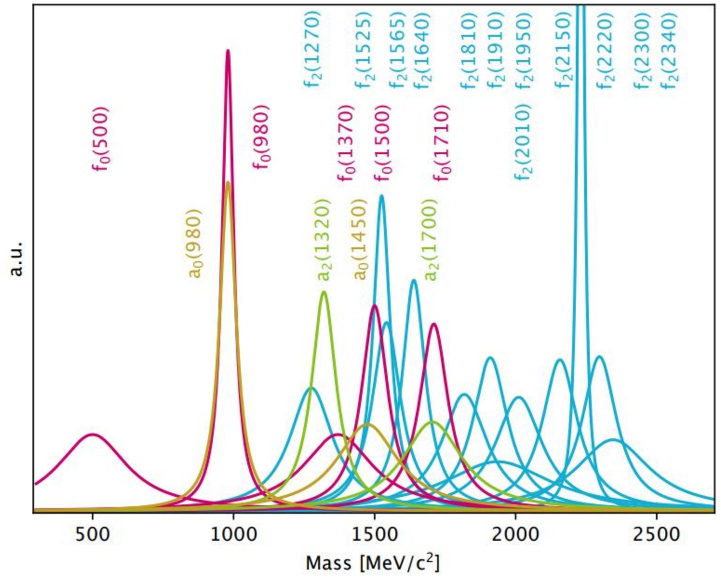
Pole parameters $\sqrt{s_R} = M_R - i\frac{\Gamma_R}{2}$

Challenges in hadron spectroscopy



- (Wide) **overlapping states**
- Partial wave analysis (PWA) techniques are complicated
- Pole positions from amplitude analyses rely on **theoretical assumptions**
- Assigning quantum numbers and internal structure (e.g. molecule vs tetraquark) is often **model dependent**

Challenges in hadron spectroscopy



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- Experimental data are inherently distorted by **detector effects** ▻ unfolded before extracting meaningful physics
- Traditional observables may not be adequate in **multi-dimensional space** (multi-particle final states)
- **Large datasets** ▻ difficult to manipulate/preserve

The A(i)DAPT collaborative effort

A(i)DAPT
AI for Data Analysis and PreservaTion

- Experimentalists
- Theorists
- ML experts

M. Battaglieri, Y. Li, A. Pilloni,
N. Sato, A. Szczepaniak,
G. Montana, G. Foti, L. Bibrzycki,
T. Alghamdi, T. Vittorini,
D. Glazier, D. Lersch, T. Reed,
M. Spreafico, and others

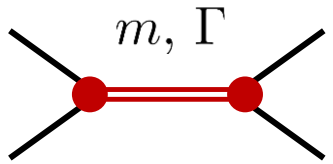
The A(i)DAPT collaborative effort

A(i)DAPT

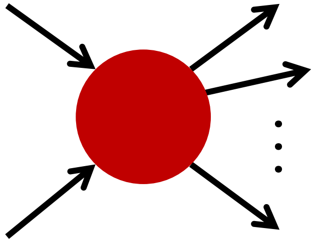
AI for Data Analysis and PreservaTion

Develop **AI tools** to:

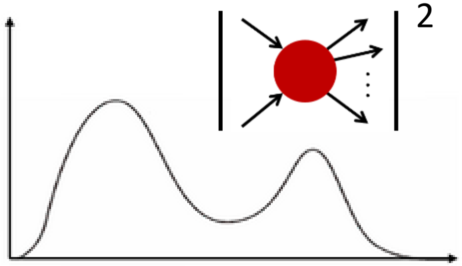
- Accurately fit data in multi-dimensional space
- Unfold detector effects
- Move from cross sections to scattering amplitudes



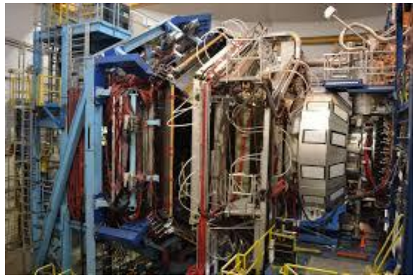
Properties of resonances



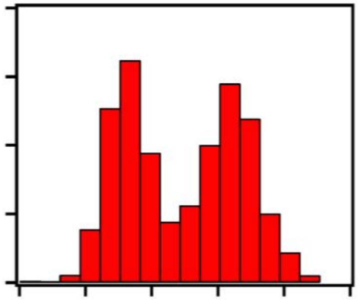
Scattering amplitude



Probability distribution



Detectors



Observed events

What happens in the experiment

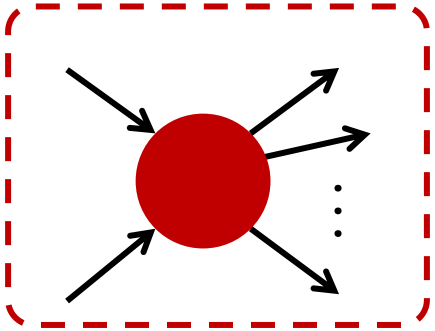
From cross section to scattering amplitude

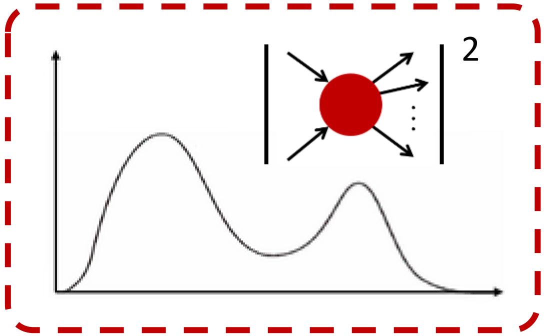
Scattering amplitude $\mathcal{A}$

- underlying dynamics
- complex quantity: magnitude + phase

Cross section σ

- experimental observable
- related to $|\mathcal{A}|^2$, missing information about the phase





From cross section to scattering amplitude

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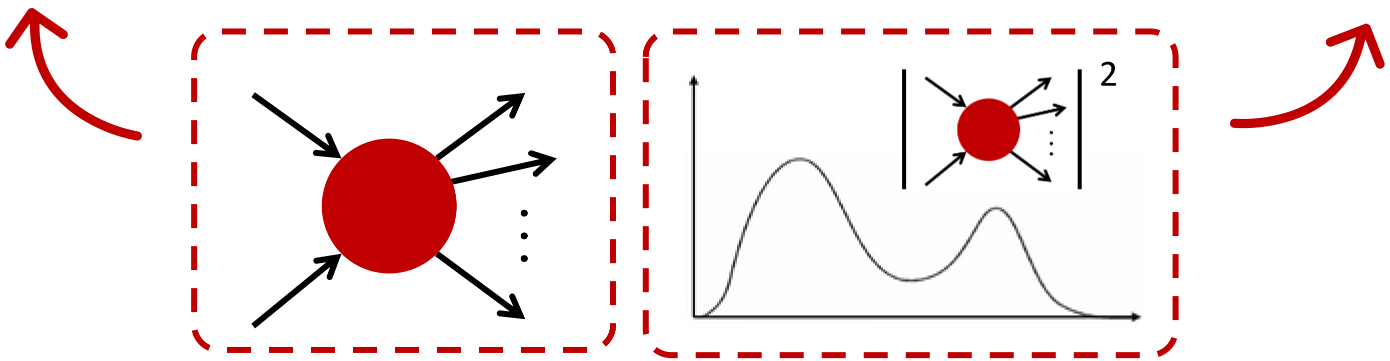
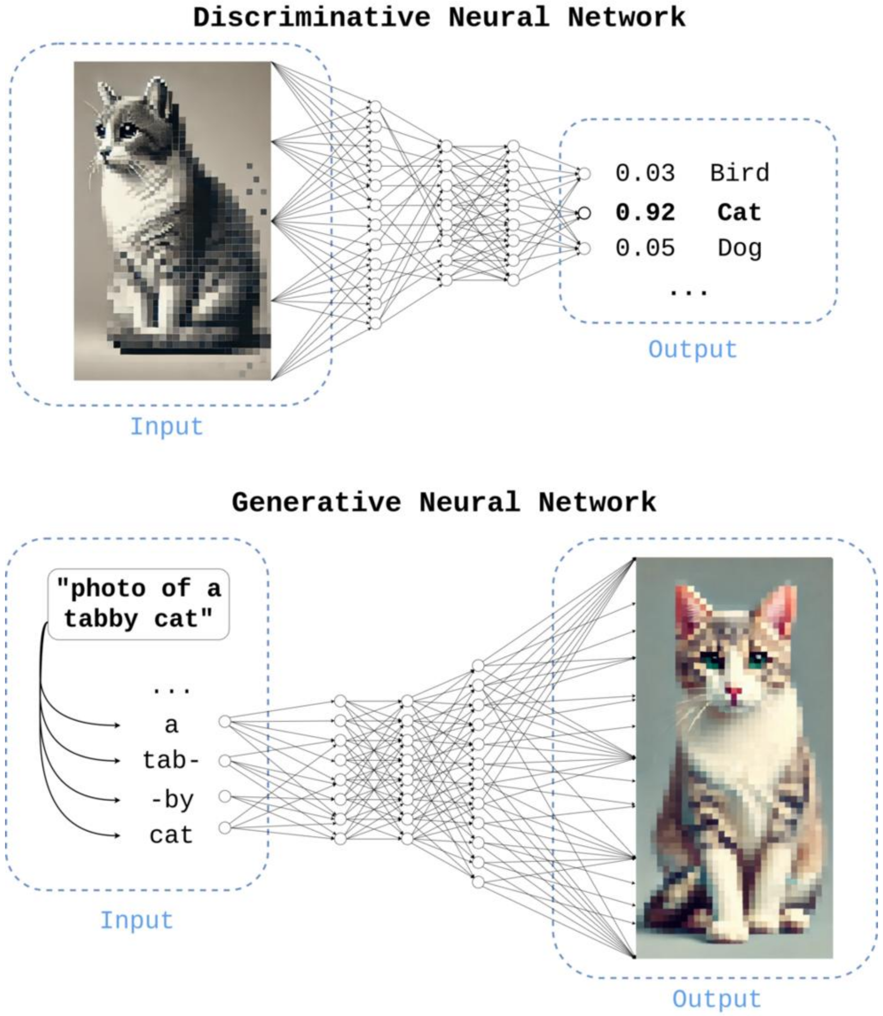
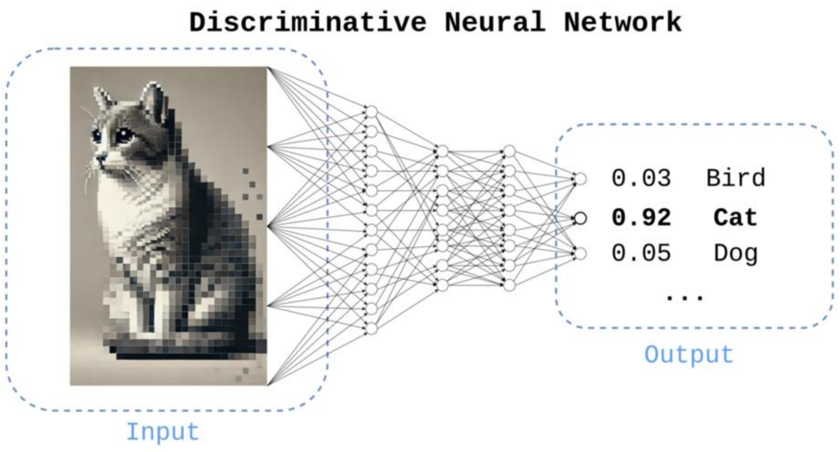


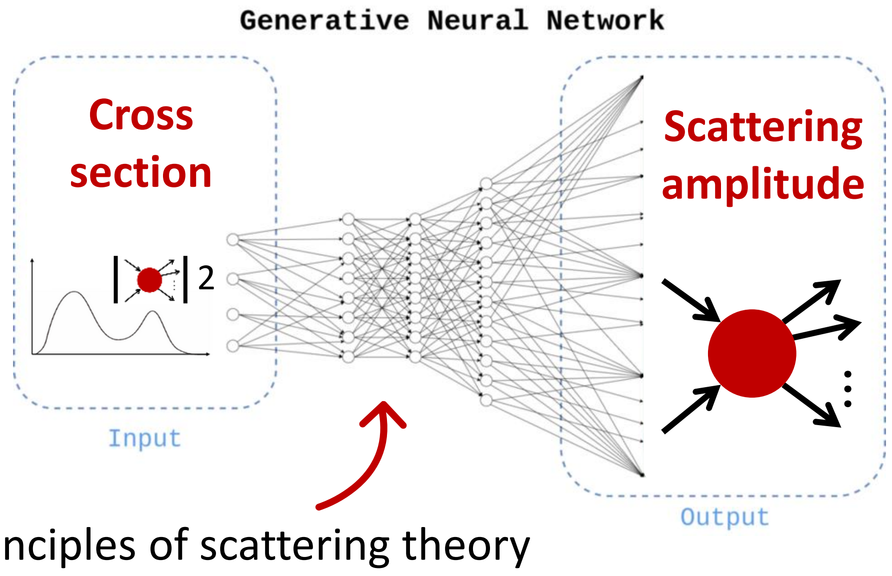
Use physics-informed generative AI

From cross section to scattering amplitude with physics-informed generative AI



From cross section to scattering amplitude with physics-informed generative AI





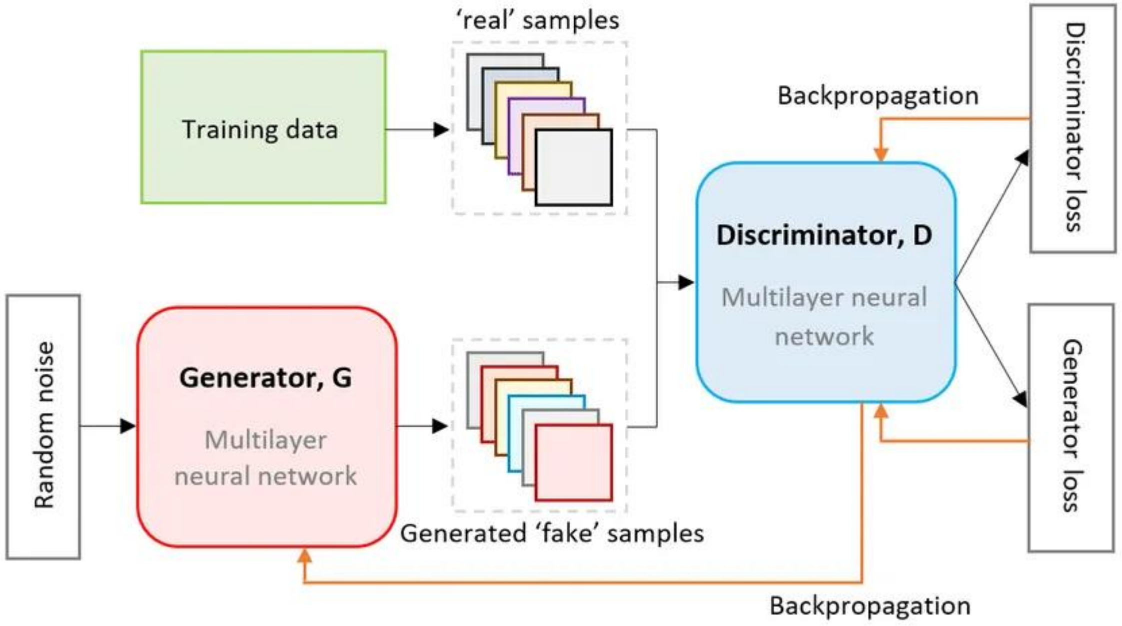
From cross section to scattering amplitude with physics-informed generative AI

Goal: physically consistent amplitude reconstruction

- Single underlying physics (no model uncertainty)
- Noisy replicas ≈ pseudo-experiments
- Strong physical constraints (unitarity)

Use ML as a regulator of the inverse problem

Generative Adversarial Network (GAN)



$\pi^+\pi^-$ scattering

Simplest case: elastic $2 \rightarrow 2$, spinless particles ▻ 2 variables (s, t)

- Dominated by $f_0(500)$ and $\rho(770)$ resonances

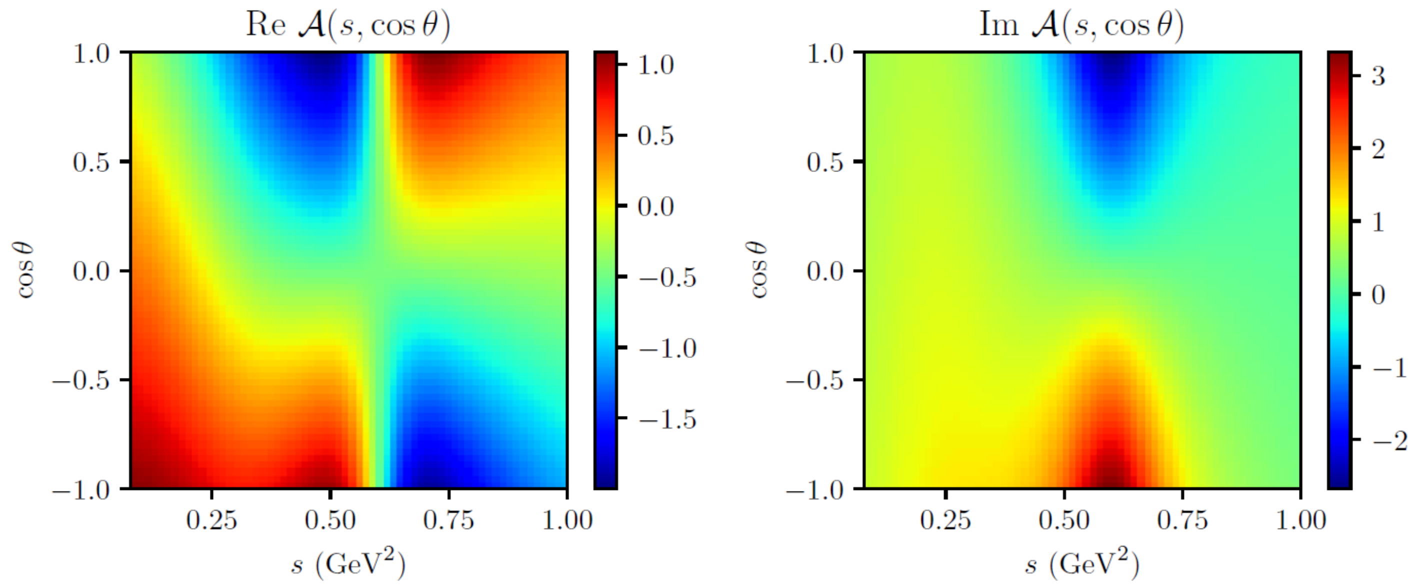
Model of the amplitude with 2 partial waves:

$$\mathcal{A}(s, \cos\theta) = \sum_{\ell=0}^{n} (2\ell+1) f_\ell(s) P_\ell(\cos\theta)$$

$$\mathcal{A}(s, \cos\theta) = f_0(s) + 3 f_1(s) \cos\theta$$

Breit-Wigner type

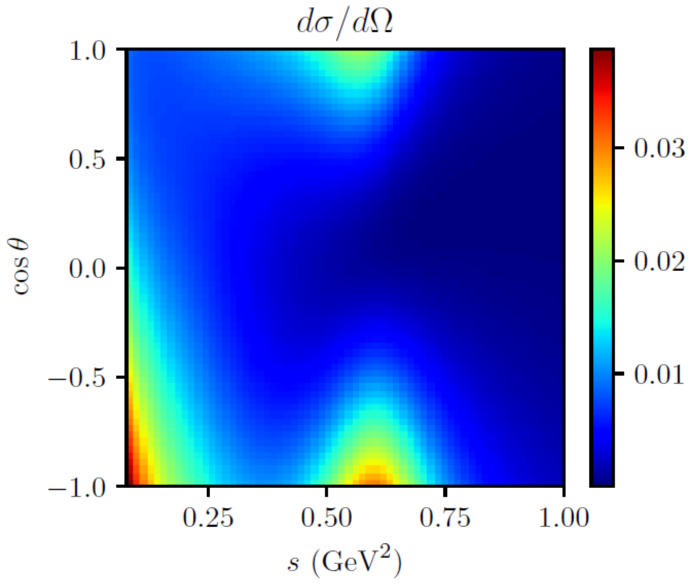
$$f_\ell = \frac{m_\ell \Gamma_\ell}{m_\ell^2 - s - i m_\ell \Gamma_\ell}$$



$\pi^+\pi^-$ scattering

Differential cross section

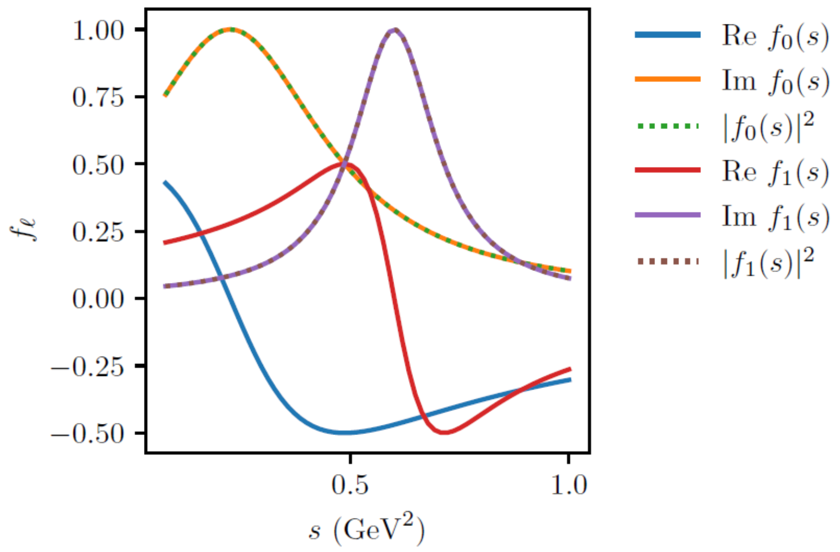
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{s}|\mathcal{A}(s,\cos\theta)|^2$$



Unitarity relation

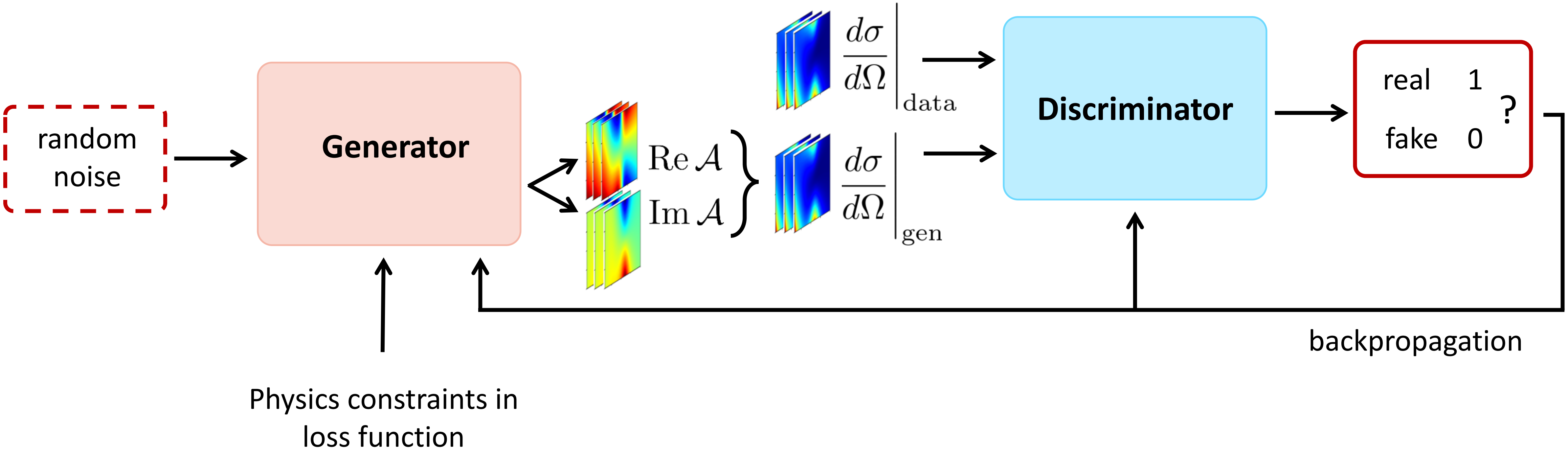
- For the partial waves $f_\ell(s) = \frac{1}{2}\int_{-1}^{+1} dz P_\ell(z)\mathcal{A}(s,z)$

$$\text{Im } f_\ell(s) = |f_\ell(s)|^2$$



Amplitudes and cross sections as images (discretized functions)

GAN with physics constraints

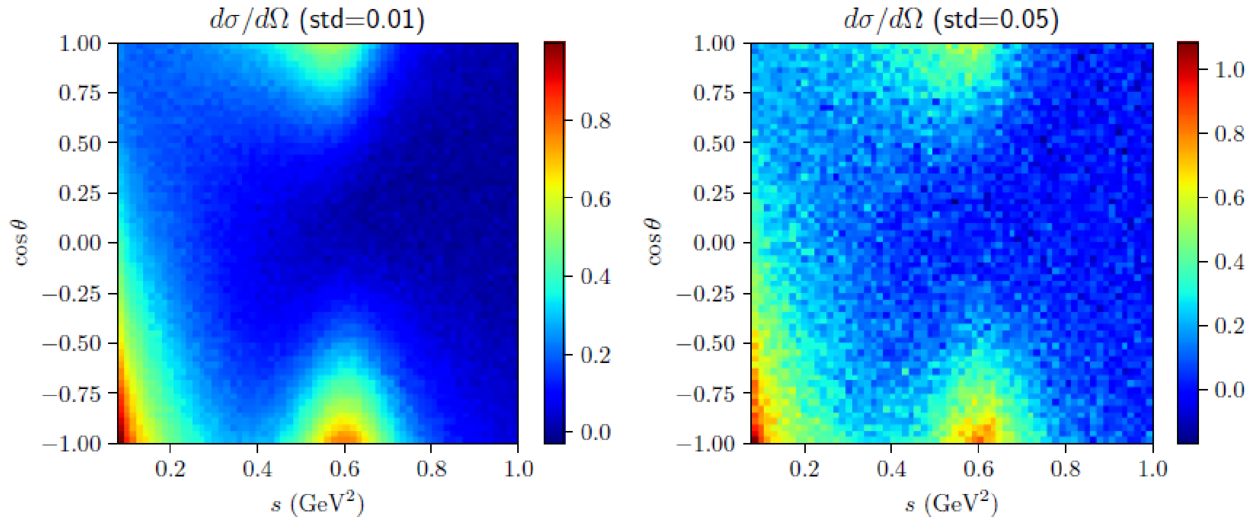


$$\mathcal{L} = \mathcal{L}_{\mathrm{GAN}} + \lambda_{\mathrm{u}}\mathcal{L}_{\mathrm{u}} + \lambda_{\delta}\mathcal{L}_{\delta}$$

GAN with physics constraints

Training data: differential cross section from the model

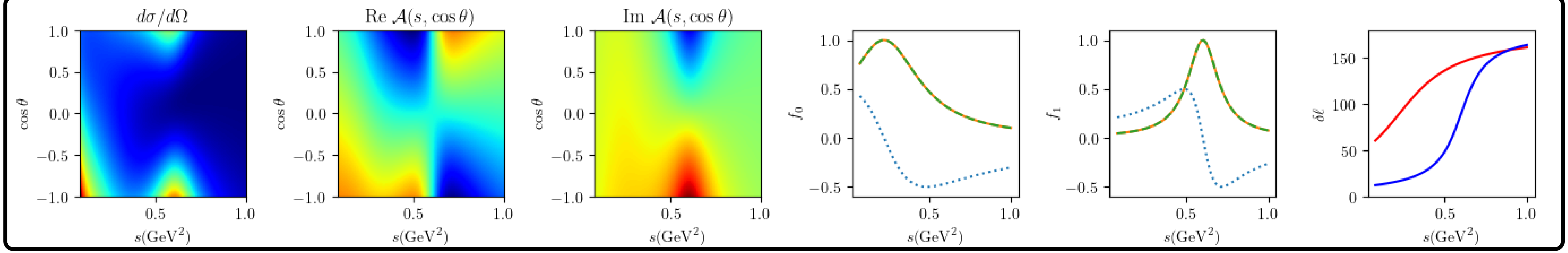
- discretized $64 \times 64,\ s \in [(2m_\pi)^2, 1\ \mathrm{GeV}^2],\ \cos\theta \in [-1, 1]$
- normalized
- noisy replicas ⊳ pseudo-data

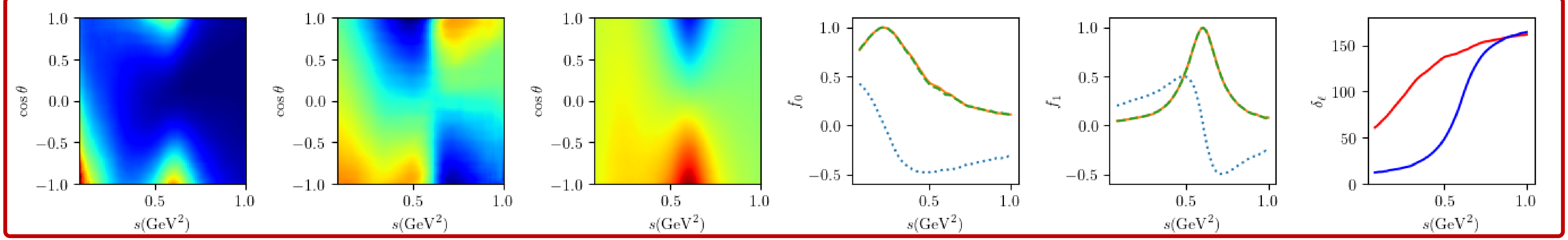


GAN with physics constraints

With std=0.01, convergence to the ground truth for ∼30% of trainings

model (ground truth)





GAN generated (“fake”)

With std=0.05, hard to converge (unitarity is not satisfied exactly)

Next steps

- Generate $\mathrm{Im}\,\mathcal{A}$ and use unitarity to reconstruct $\mathrm{Re}\,\mathcal{A}$ instead of a constraint in the loss function
- Uncertainty quantification
- Explore other generative models

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After the proof of concept with $\pi^+\pi^-$ scattering…

- Coupled channels $\pi^+\pi^-$, K^+K^-
- Spin
- $2 \rightarrow 3$ processes
- Meson photoproduction

Back-up slides

Conditional diffusion model

Training data:

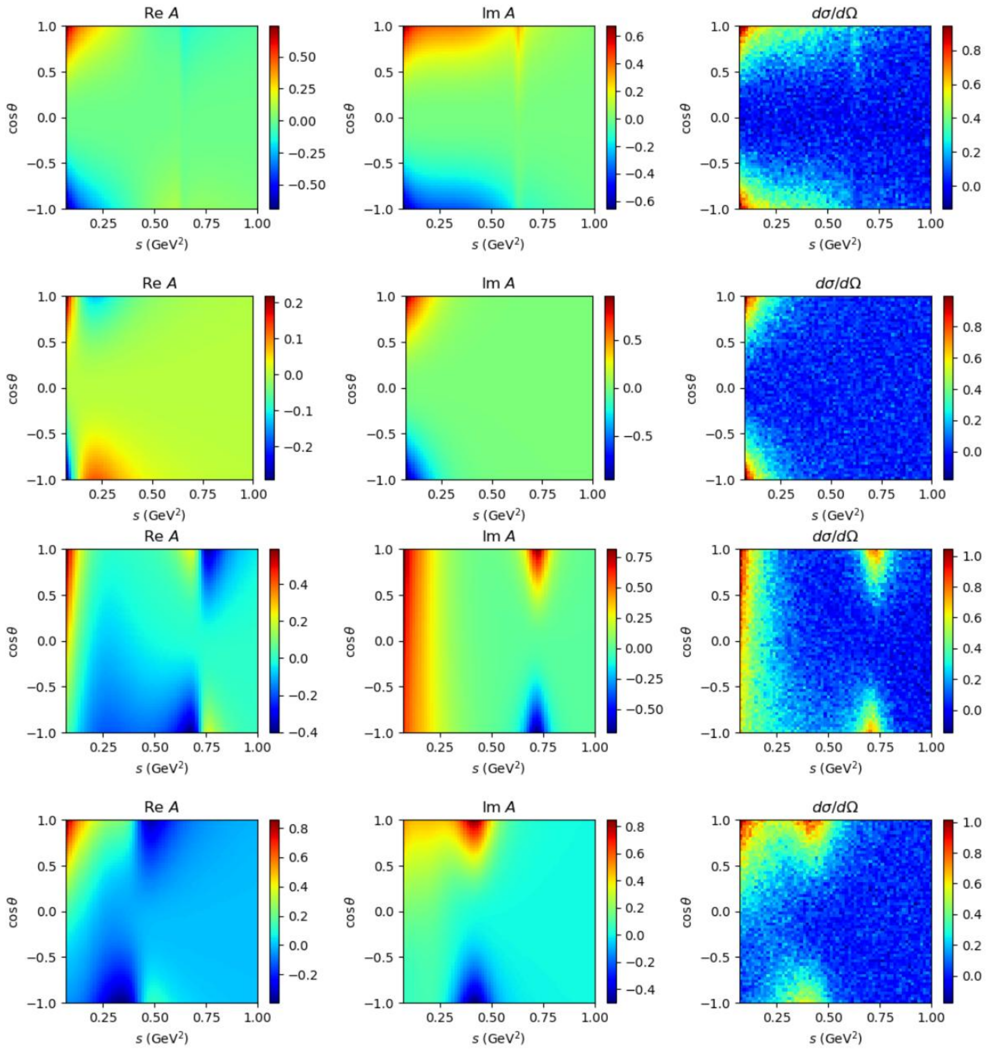
- ⊳ sample amplitudes and cross sections from 2BW model with parameters

$$m_0,\, m_1 \in [0.1, 1.0]\ \mathrm{GeV}$$
$$\Gamma_0,\, \Gamma_1 \in [0.01, 0.7]\ \mathrm{GeV}$$

- ⊳ normalized

- ⊳ with additional Gaussian noise to cross section (mimic experimental data)

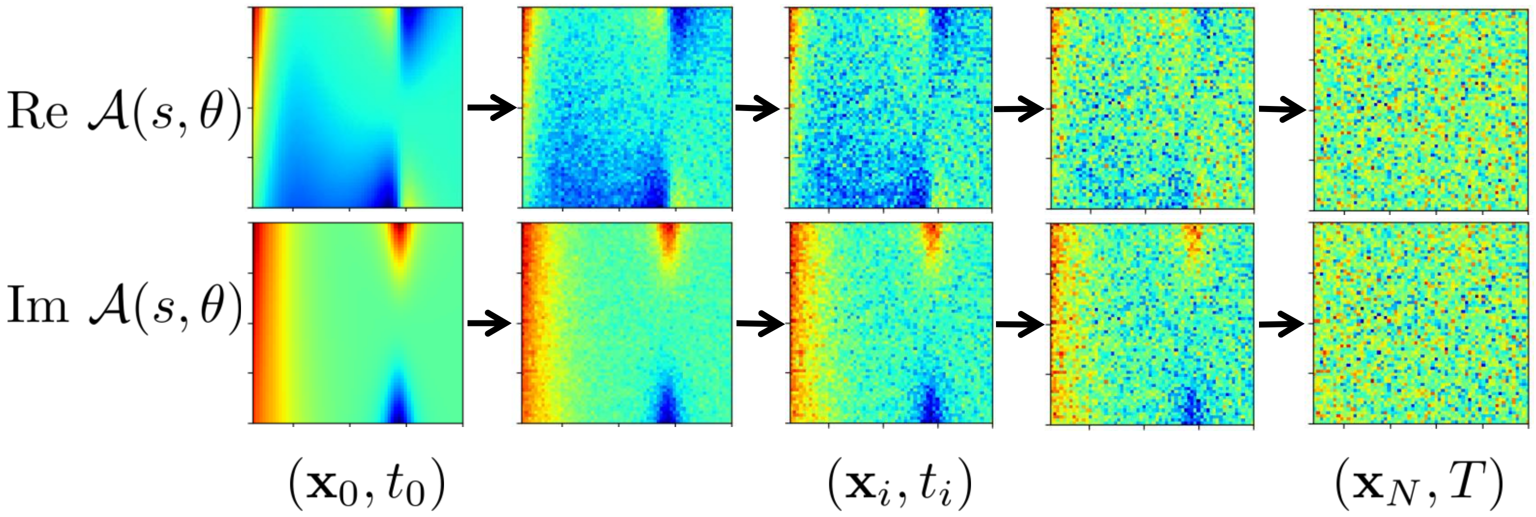
Satisfy unitarity by construction



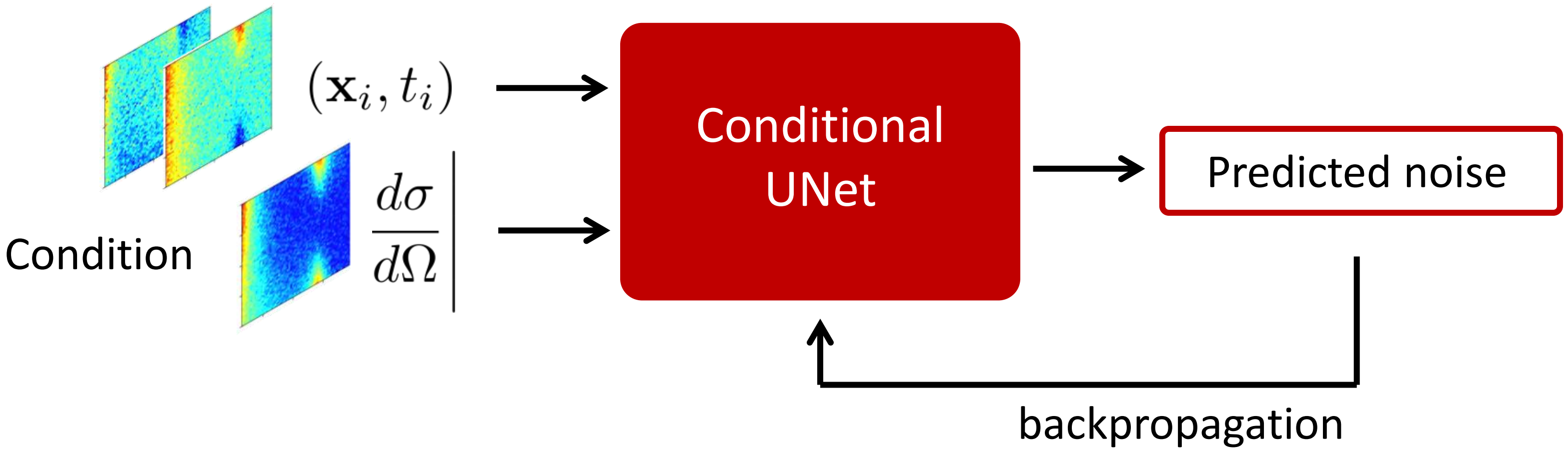
...

Conditional diffusion model

Forward diffusion process

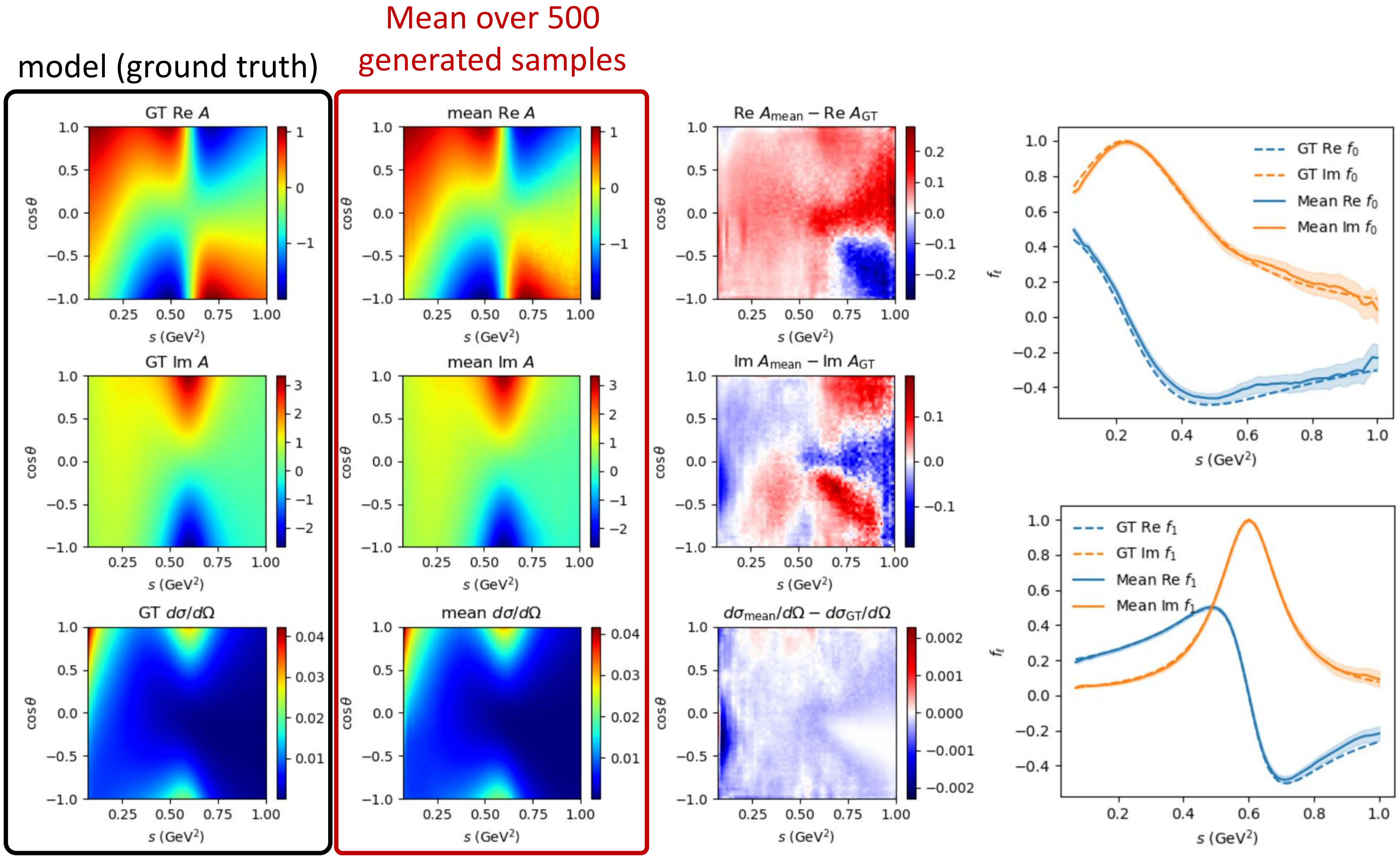


Training



Conditional diffusion model

- ⊳ With std=0.01



Conditional diffusion model

- ⊳ With std=0.05