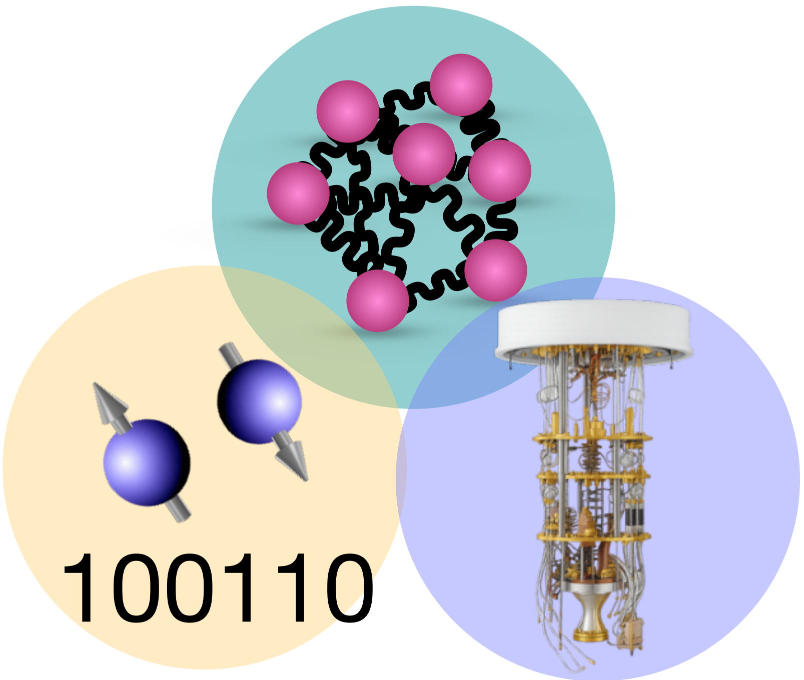
HADNUCMAT workshop

Exploring the QCD phase diagram: from hadrons and nuclei to matter in extreme conditions

Universitat de Barcelona — January 26-28, 2026

Quantum Complexity and Quantum Simulations of Nuclear Many-Body Systems

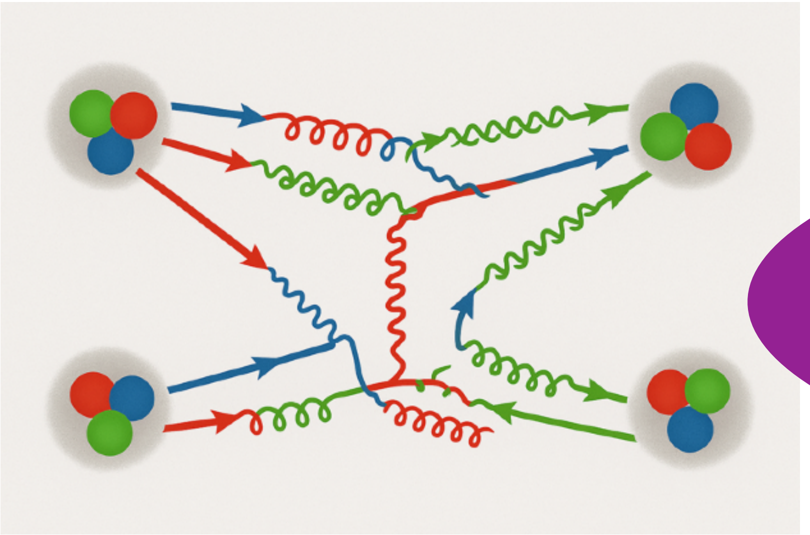


Caroline Robin

UNIVERSITÄT BIELEFELD
Fakultät für Physik

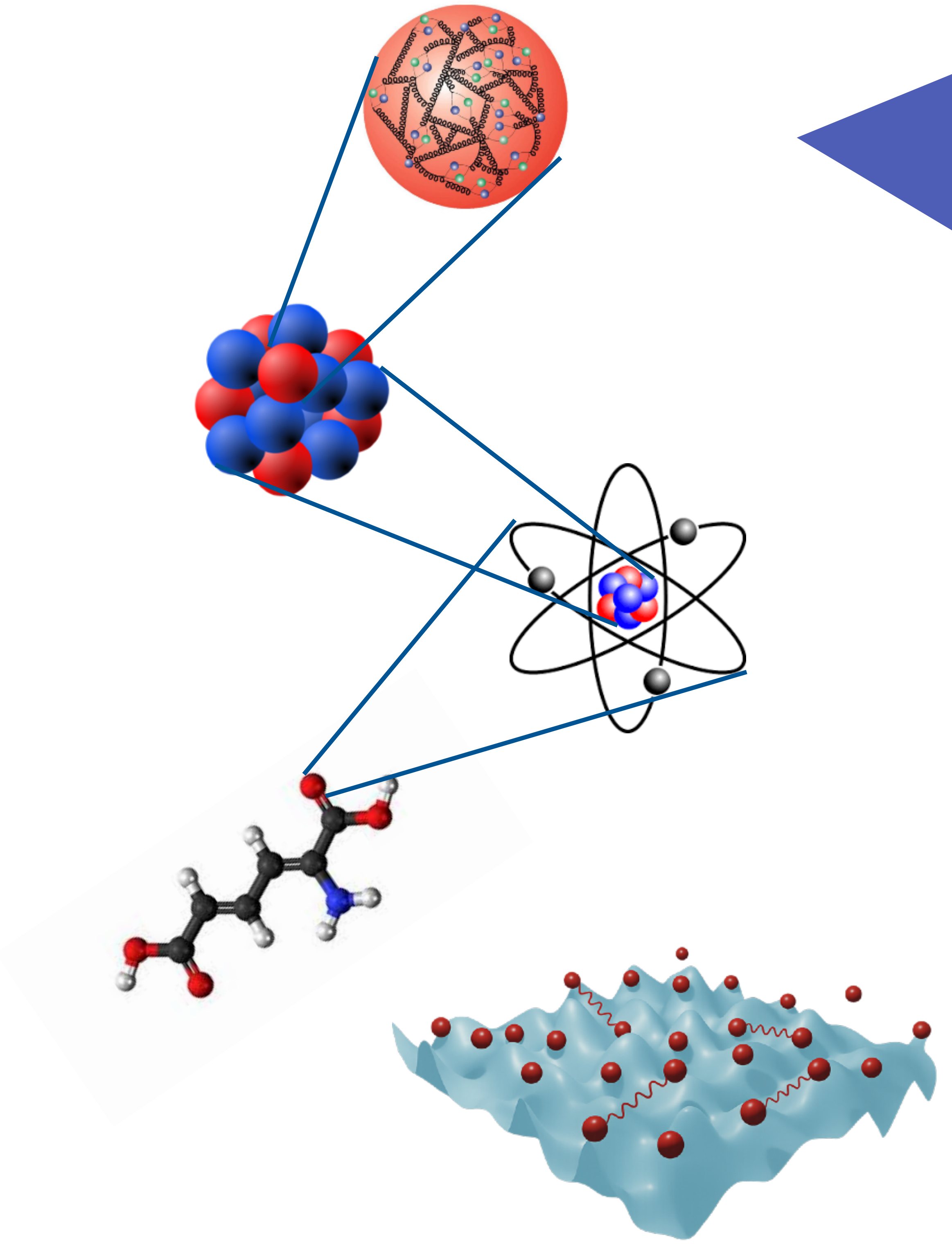
GSI

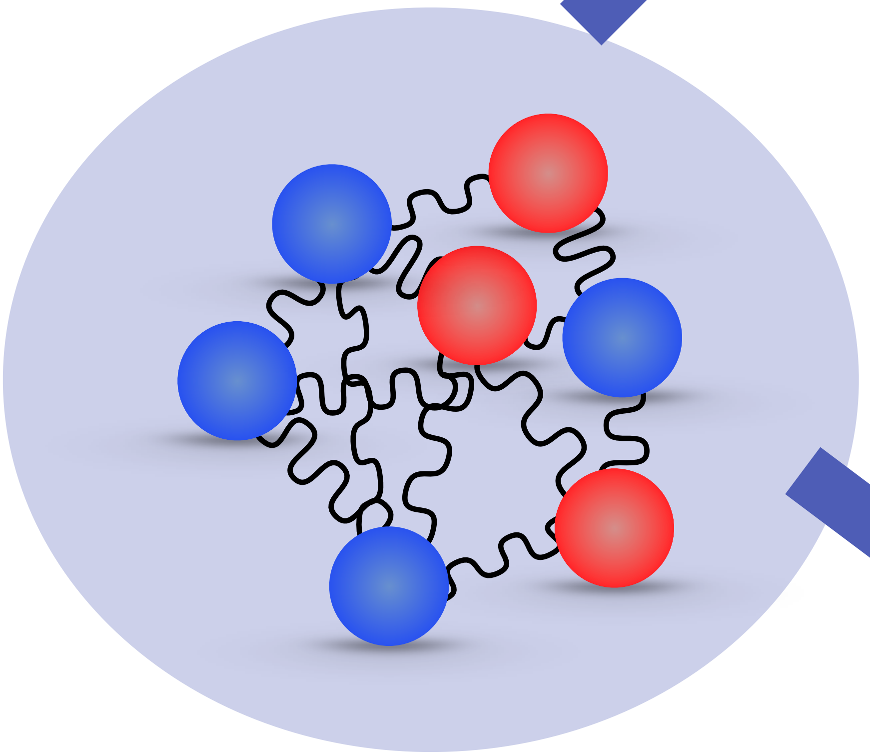
Nuclei to address fundamental science questions

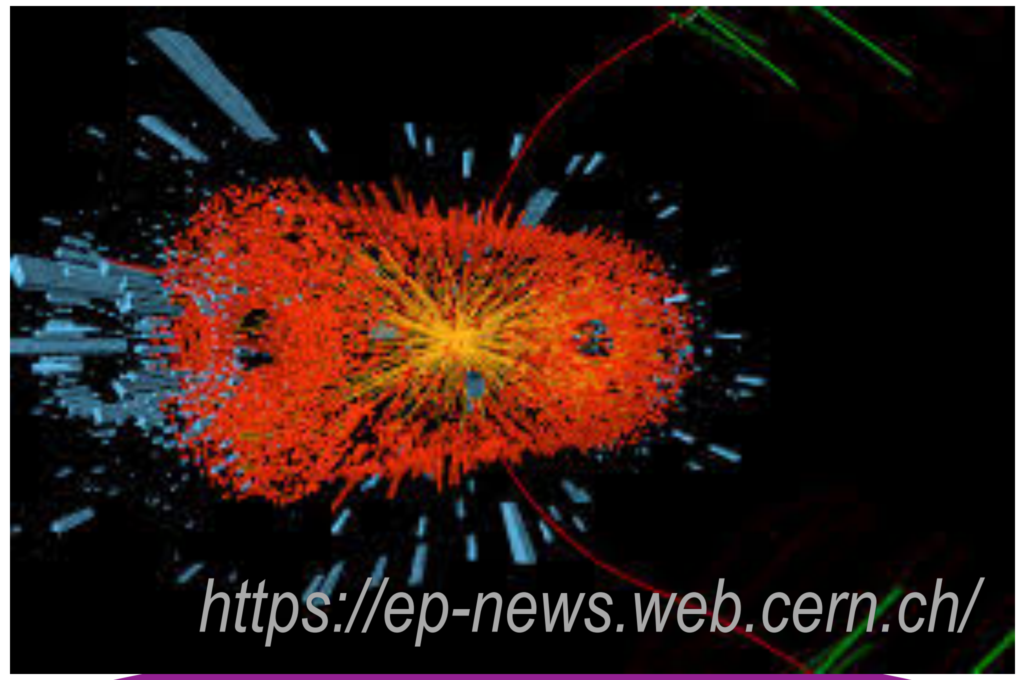


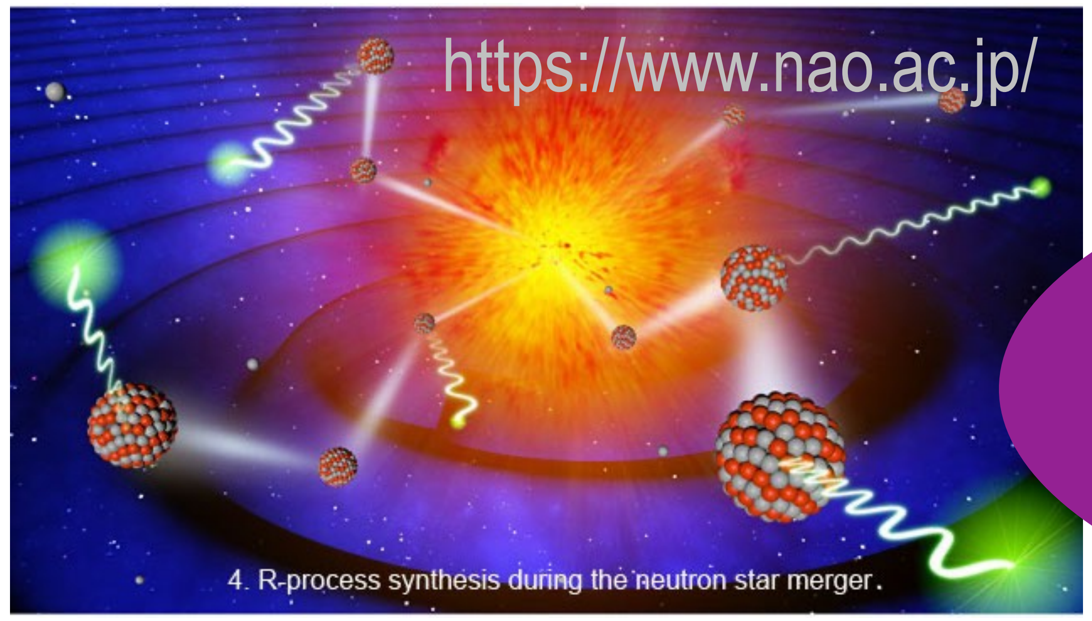
fundamental interactions & symmetries

Organization of matter & emergent phenomena



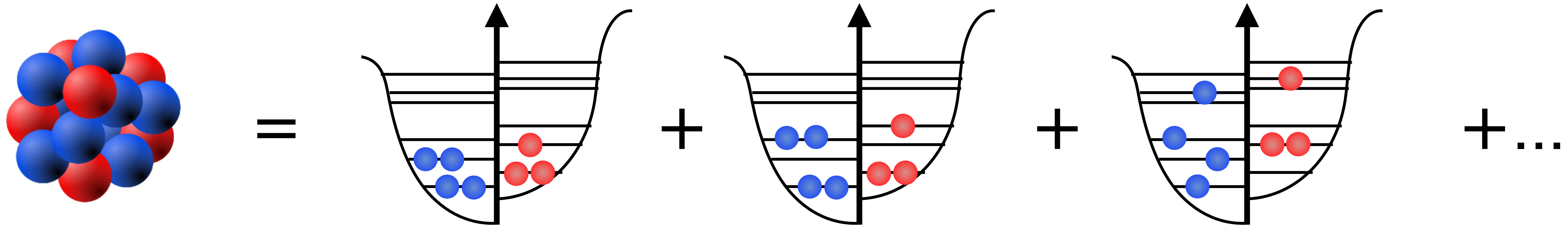






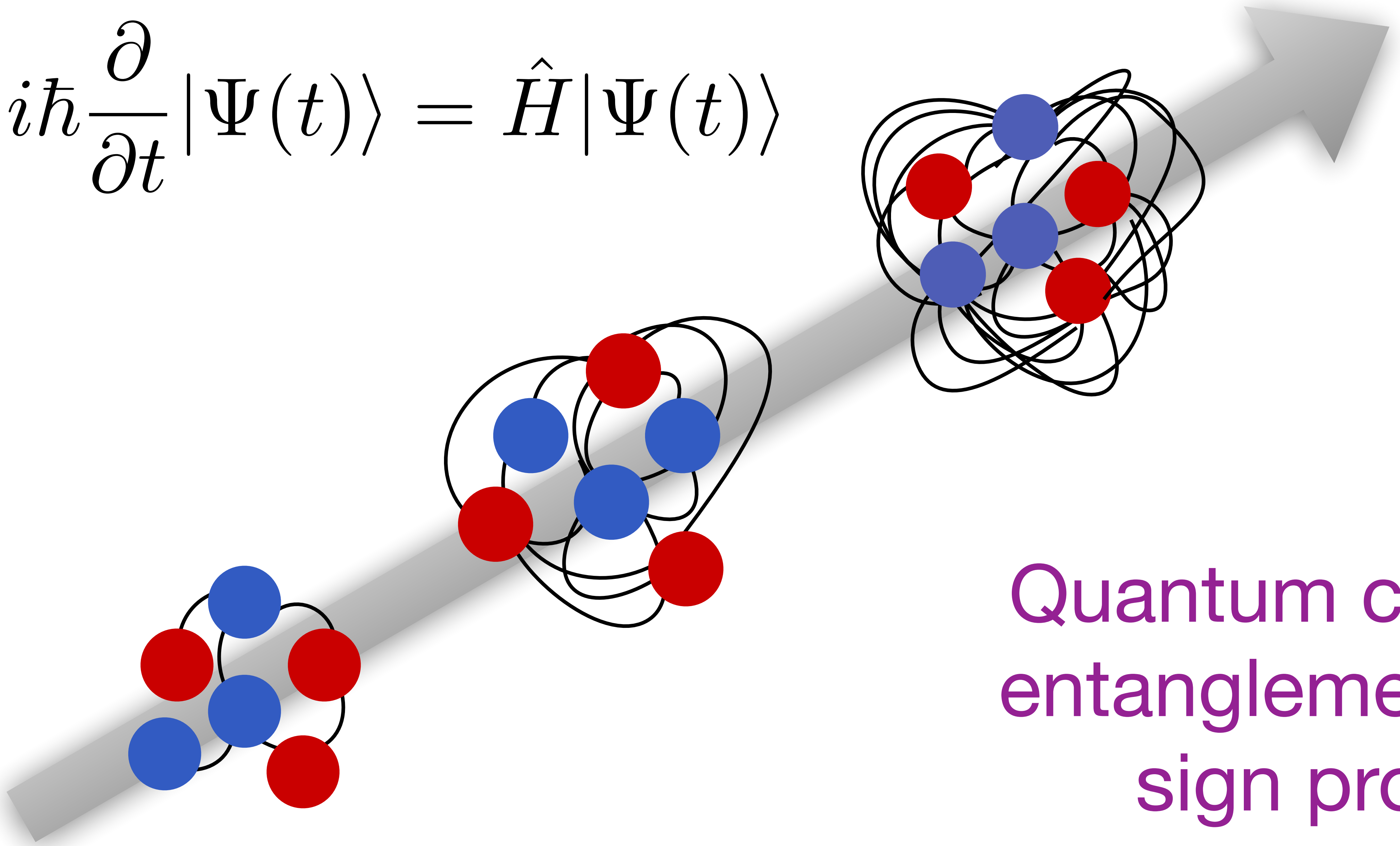
origin of the elements in the cosmos & early-universe

(Nuclear) Quantum Many-Body Problems

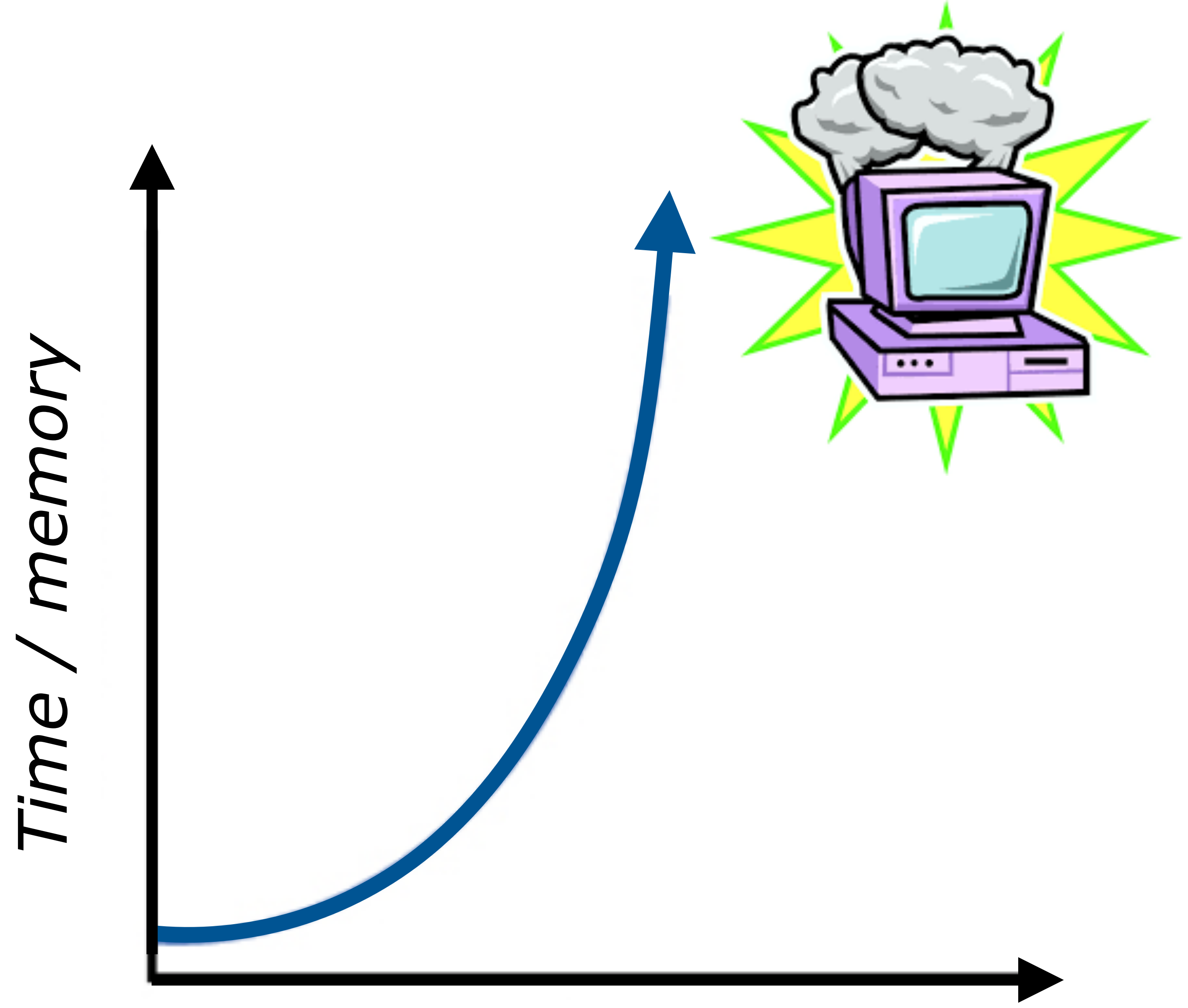


$$|\Psi\rangle = \sum_{i}^{exp(N)} C_i |\Phi_i\rangle$$

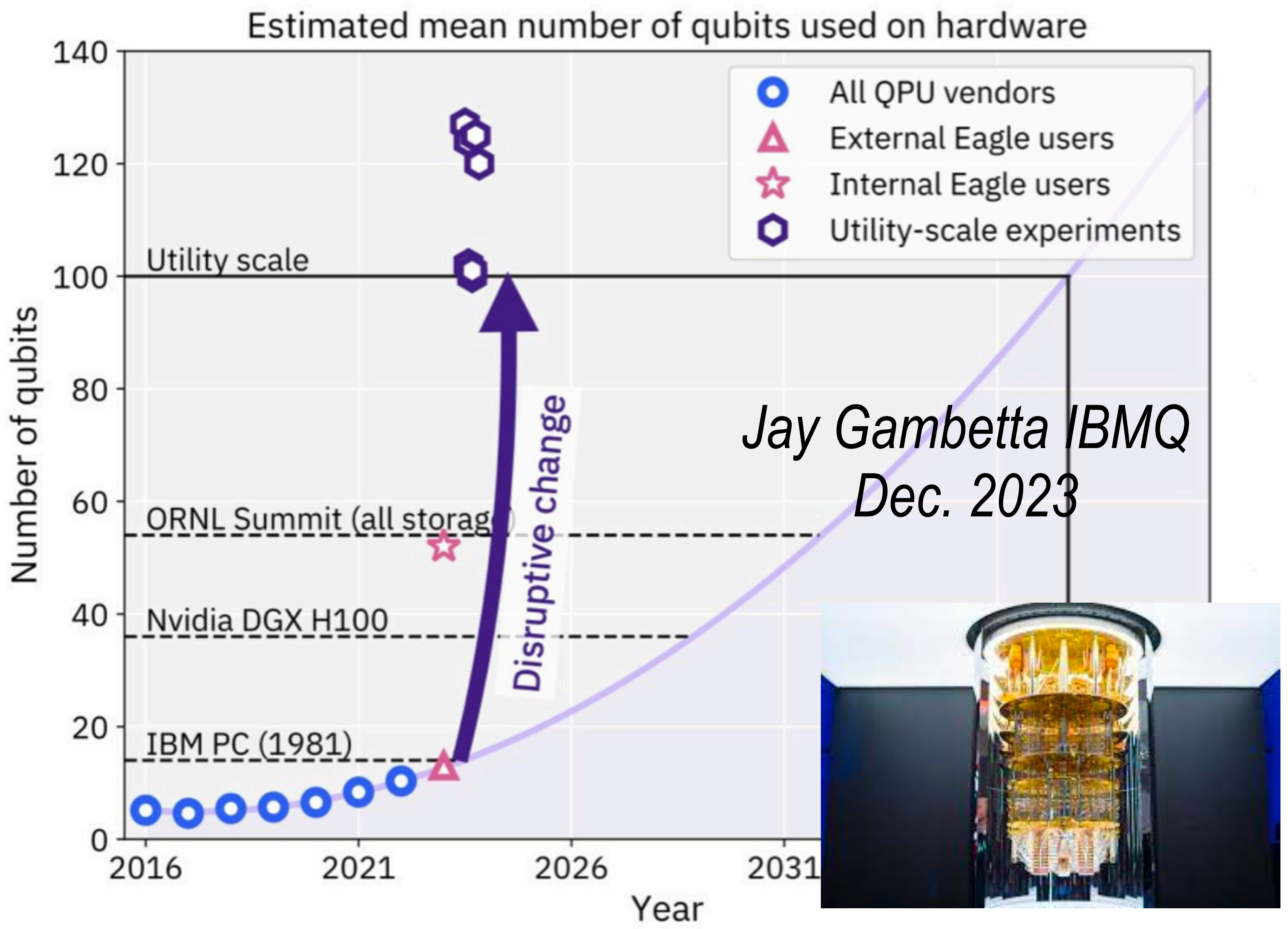
$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle$$



Quantum complexity,
entanglement growth,
sign problems



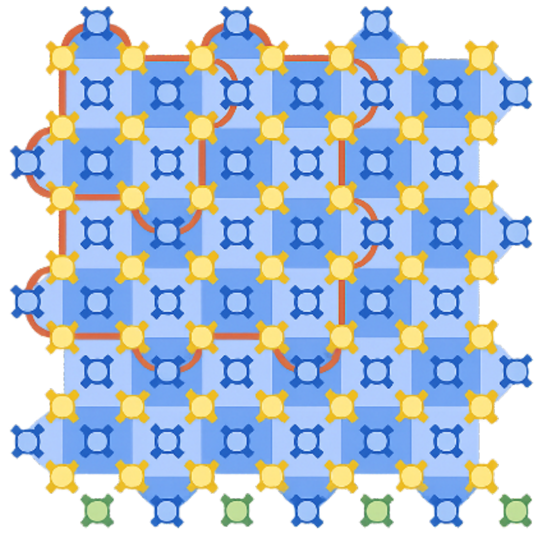
Quantum Computers as a New Opportunity for Many-Body Physics



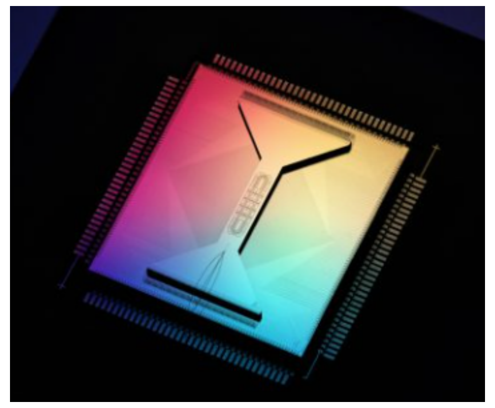
Jay Gambetta IBMQ Dec. 2023

Superconductors

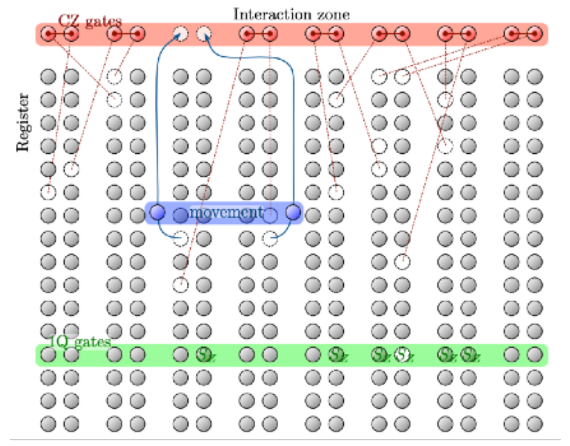
...



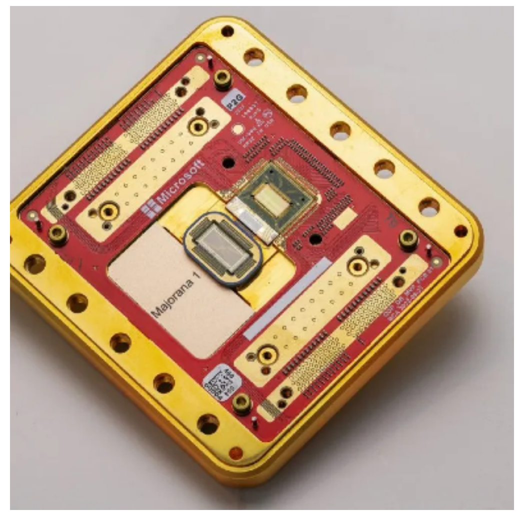
Surface error correction code ≳50 qubits



12 logical qubits (Quantinuum)

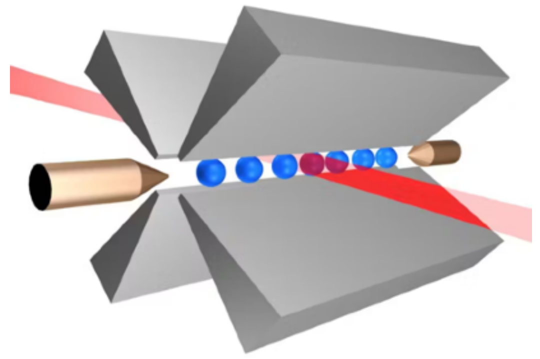


28 logical qubits (Atom Computing/ Microsoft)

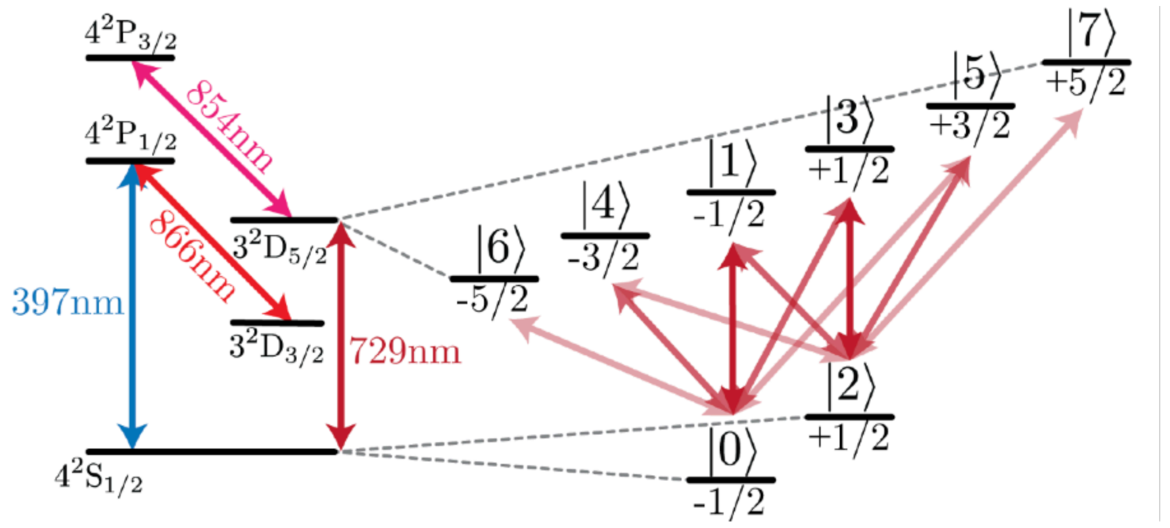


Microsoft Majorana topological qubits

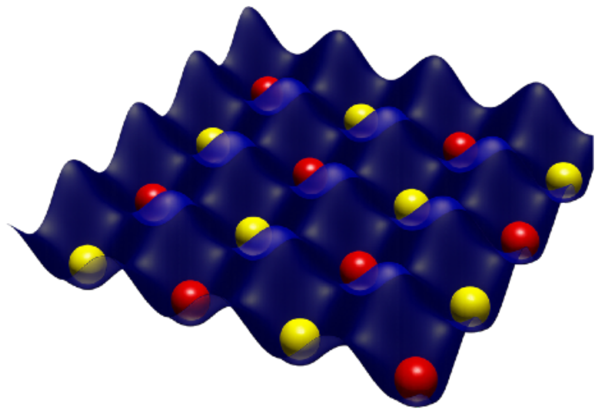
...



Trapped ions



Qudits *(Innsbruck)*



Cold atoms

From NISQ towards fault-tolerant quantum computing

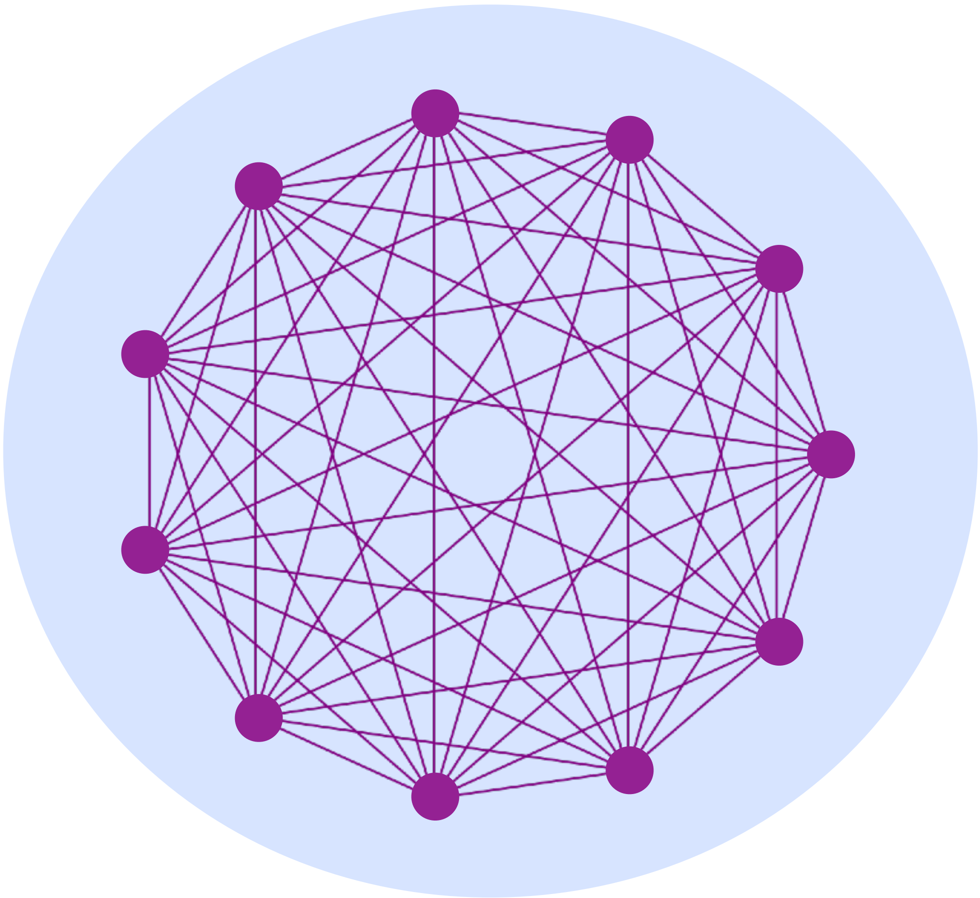
Towards Quantum Computations of (Nuclear) Many-Body Systems

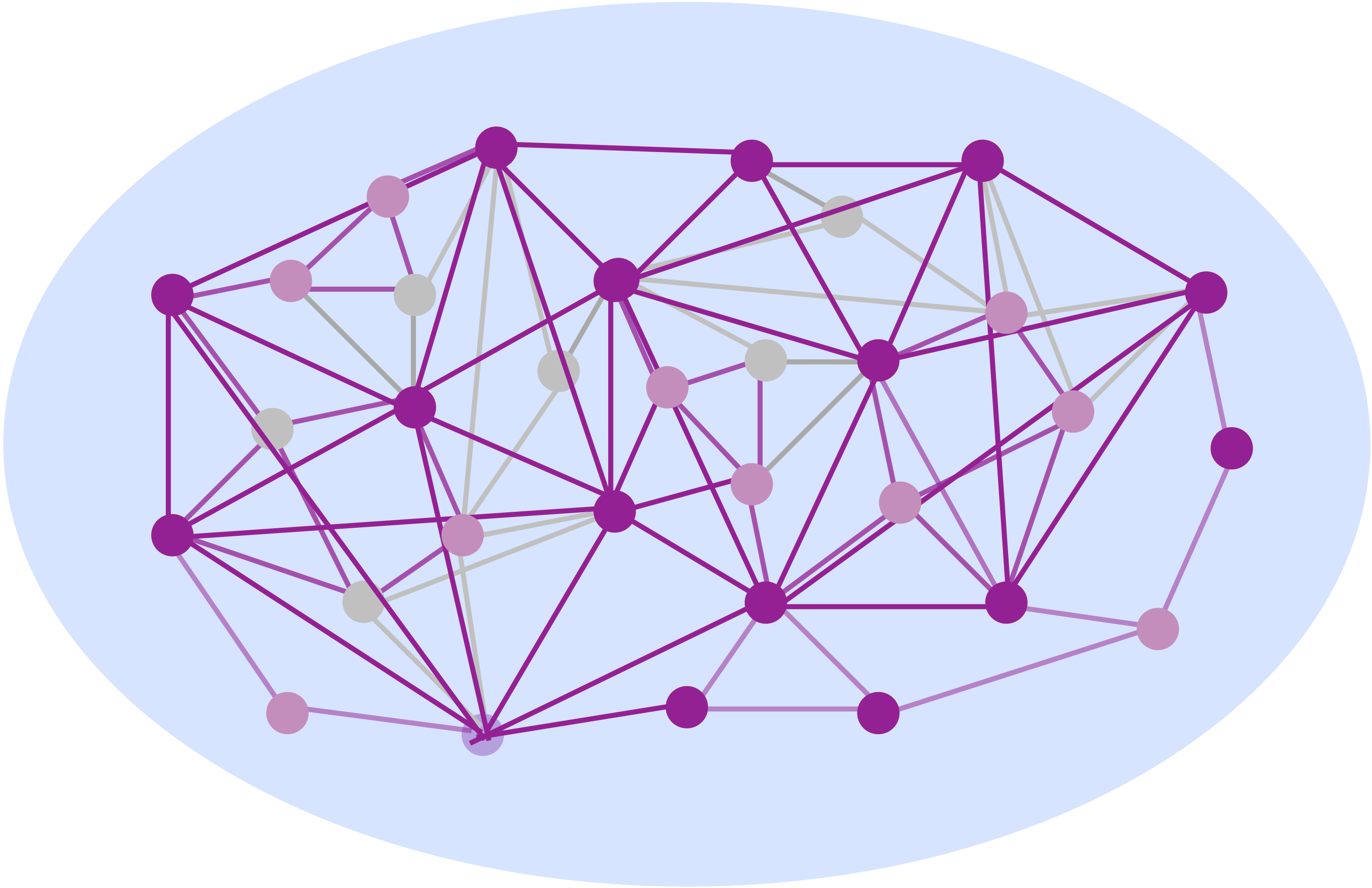
Inspired by "Quantum simulation of fundamental particles and forces", Bauer, Davoudi, Klco, Savage, Nature Rev. Phy. 5, 420 (2023)

Many-Body Quantum Complexity

Complexity of a quantum state ⟷ amount of information needed to represent it

(I) Quantum Entanglement + *(II) Non-Stabilizerness (Quantum "Magic")*





→ highly-entangled "stabilizer states"
~ graph states ~ N^2 bits

"Simple"

Arbitrary magic & entangled state:
~ exp(N) bits

"Complex"

Quantum Computational Complexity

Universal Quantum Gate Set (Clifford+T gate resource theory):

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad \mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

+

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

Entanglement

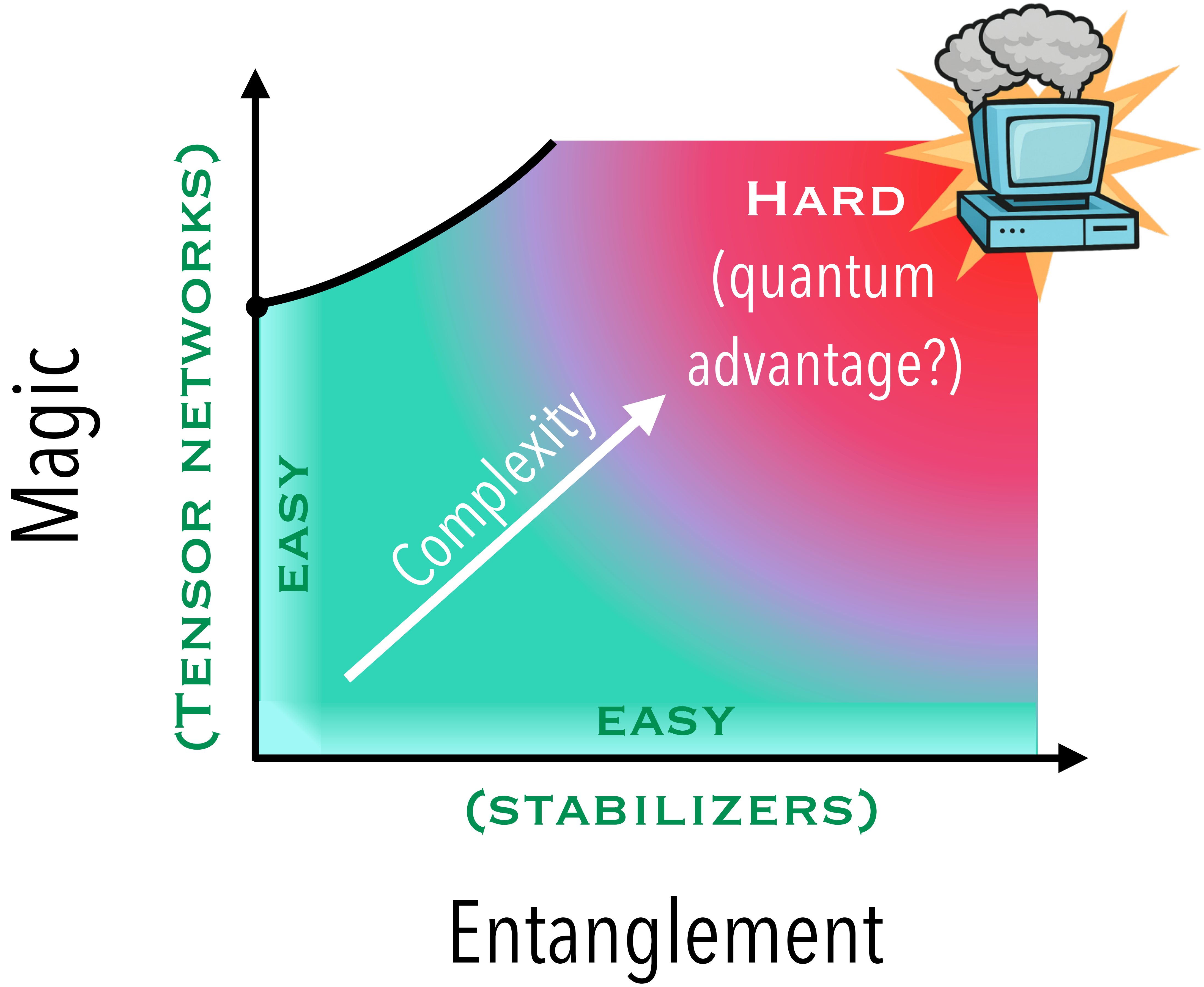
- Efficient classically (stabilizer formalism)
  — *Gottesman Knill (1998)*
- Cheap fault-tolerant implementations
- Non-universal (stabilizer states only)

Non-stabilizerness (Magic)

- Classical resources scale ~ exp(# T gates)
  — *Aaronson & Gottesman (2004)*
- Expensive fault-tolerant implementations
- Needed for universality & quantum advantage

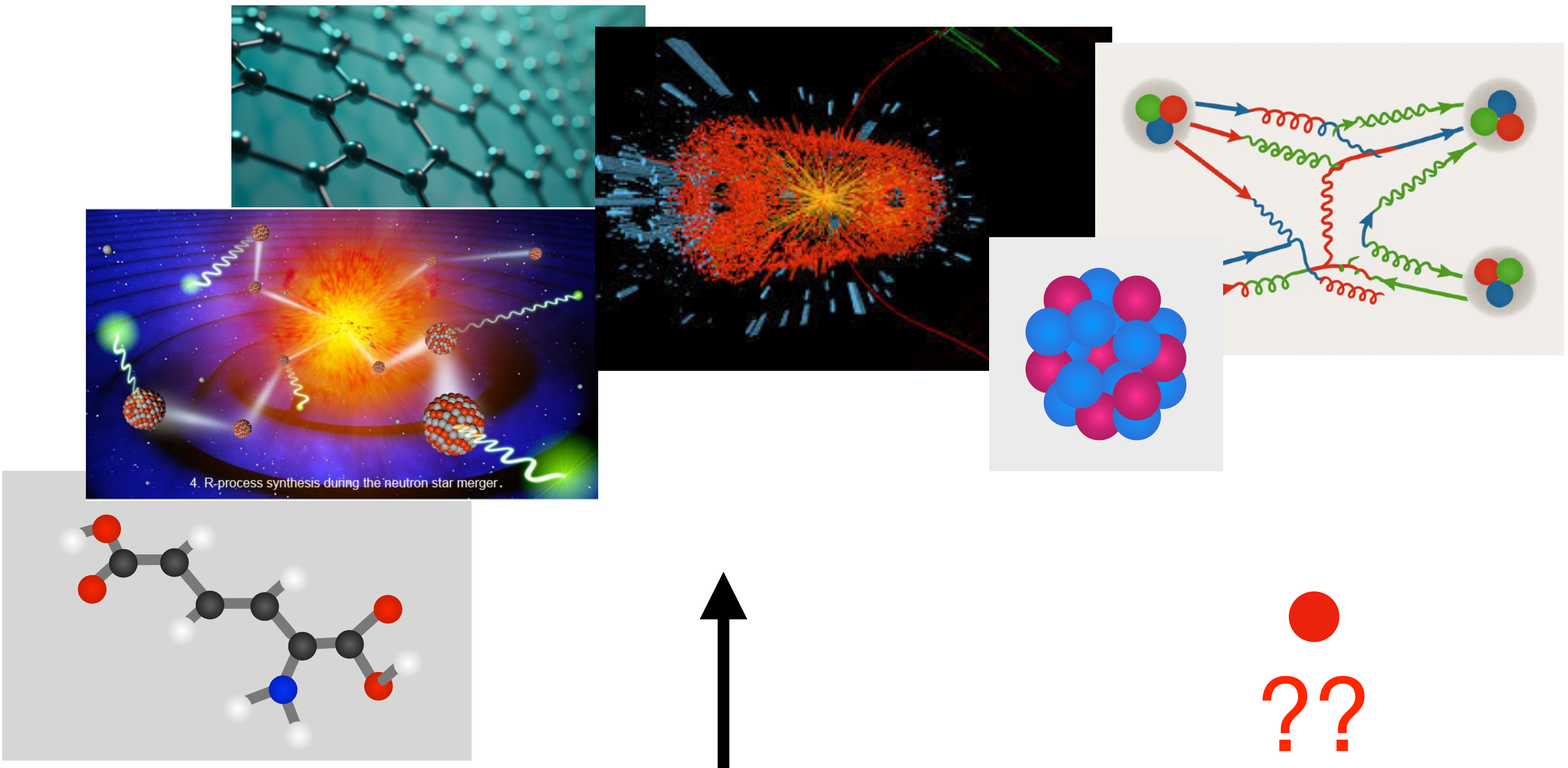
Complexity of Physical Systems — when is a Quantum Computer required?

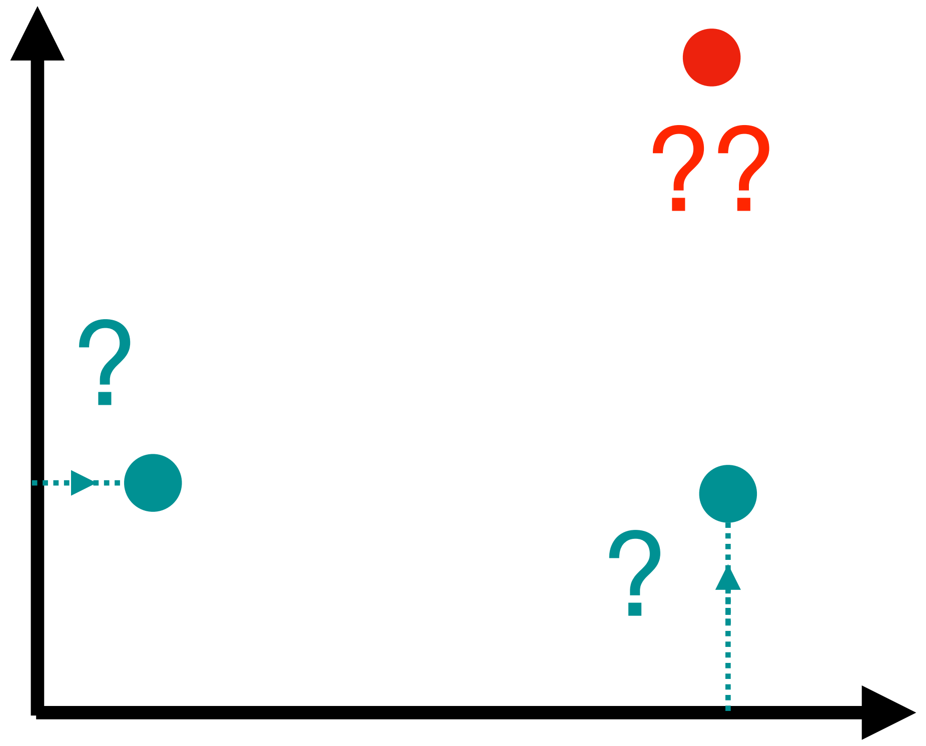
Complexity phase diagram, hardness of classical simulations



Inspired by Alioscia Hamma, Qmeets seminar,
https://www.youtube.com/watch?v=UrOCCGpL_go

Motivational Questions & Goals





- Where do physical states of interest stand in the quantum complexity phase diagram?

- How is quantum complexity generated in dynamical processes?

→ *Optimal paths, architectures and algorithms for targeted quantum simulations*

→ *Search for quantum advantage*

→ *How are many-body phenomena are rooted in quantum information*

Outline

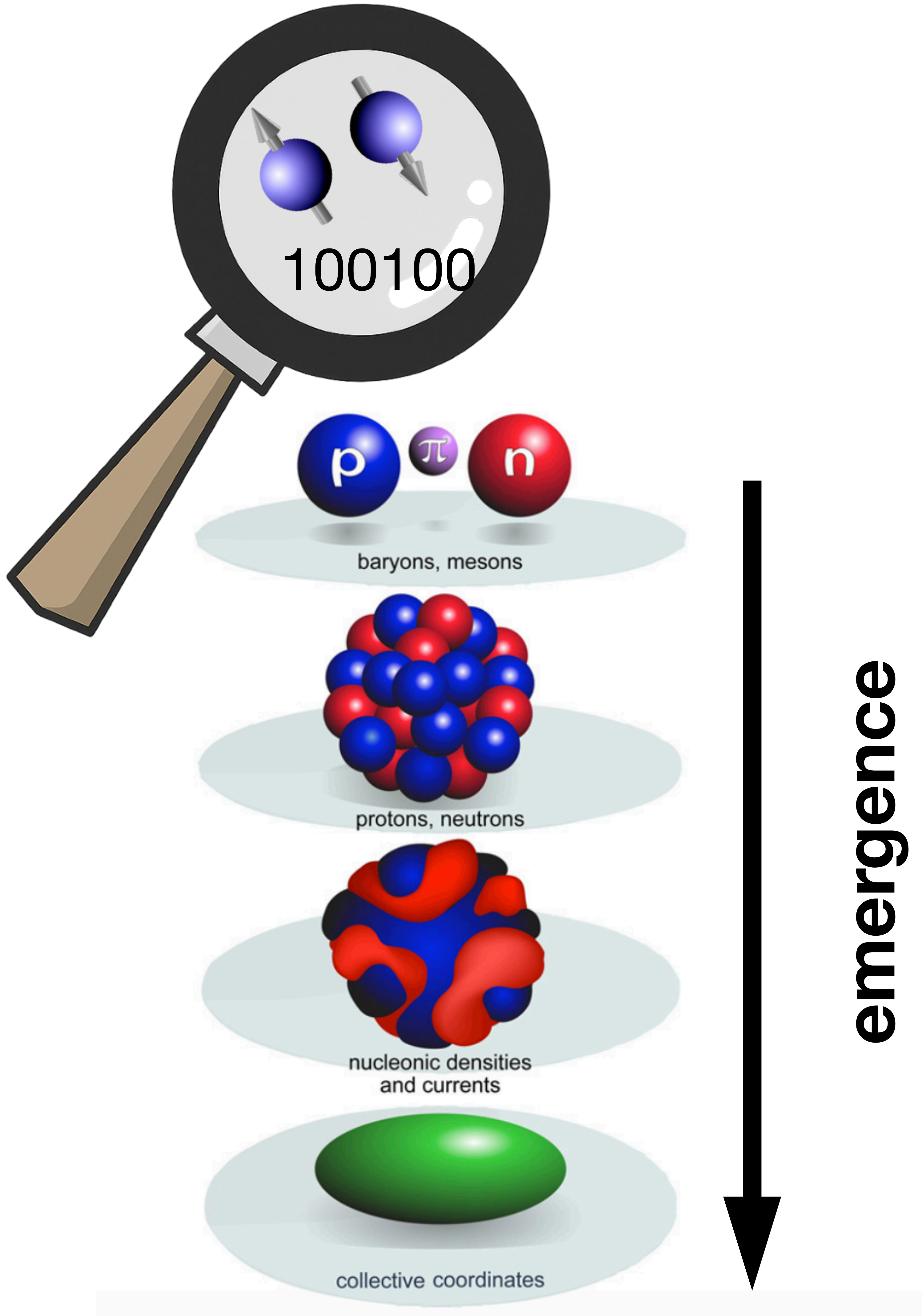
★ Quantum complexity and the emergence of collectivity in nuclear many-body systems

→ *Entanglement, non-stabilizerness and deformation*

★ Complexity-guided ground-state finding algorithms

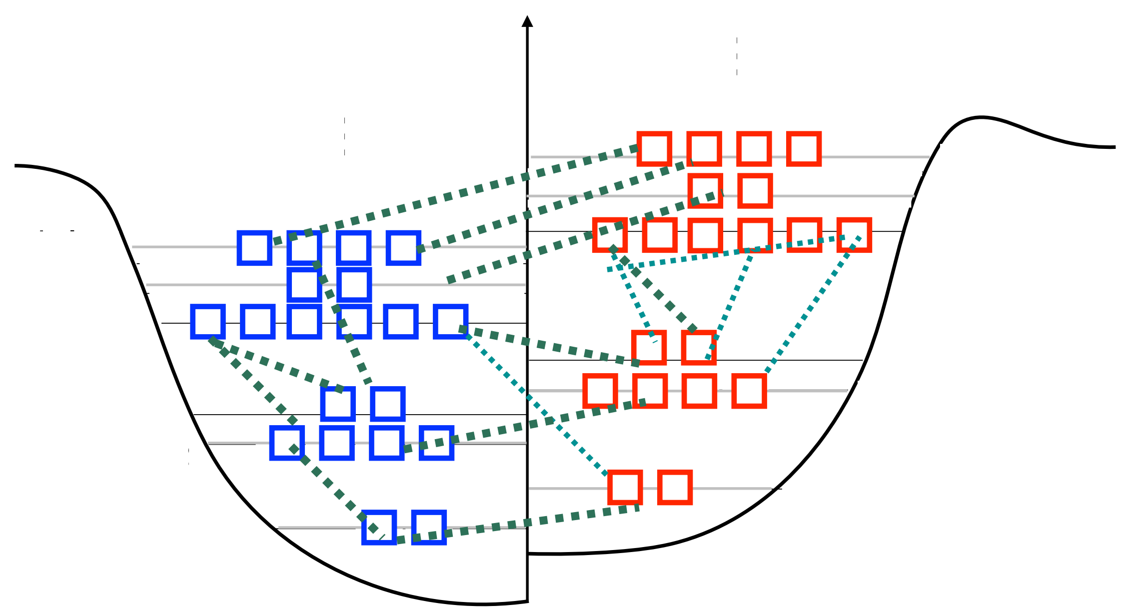
→ *Stabilizer-accelerated many-body ground state estimations*

→ *Information and complexity rearrangement with Hamiltonian-learning-VQE*



Adopted from
Bertsch, Dean, Nazarewicz SciDAC review (2007)

Multi-Body Entanglement and Magic in Shell-Model Nuclei



Orbital-to-qubit JW mapping

$$a_i^\dagger \rightarrow (\prod_{j<i} \hat{\sigma}_z^{(j)})(\hat{\sigma}_x^{(i)} - i\hat{\sigma}_y^{(i)})/2$$

Multi-Partite entanglement via n-tangles

Wong, Christensen, PRA 63, 044301 (2001)

$$\tau^{(n)}_{(i_1 \ldots i_n)} = \left| \langle \Psi | \hat{\sigma_y}^{(i_1)} \otimes \ldots \otimes \hat{\sigma_y}^{(i_n)} | \Psi^* \rangle \right|^2$$

⇒ n-tangles are related to n/2-body entanglement

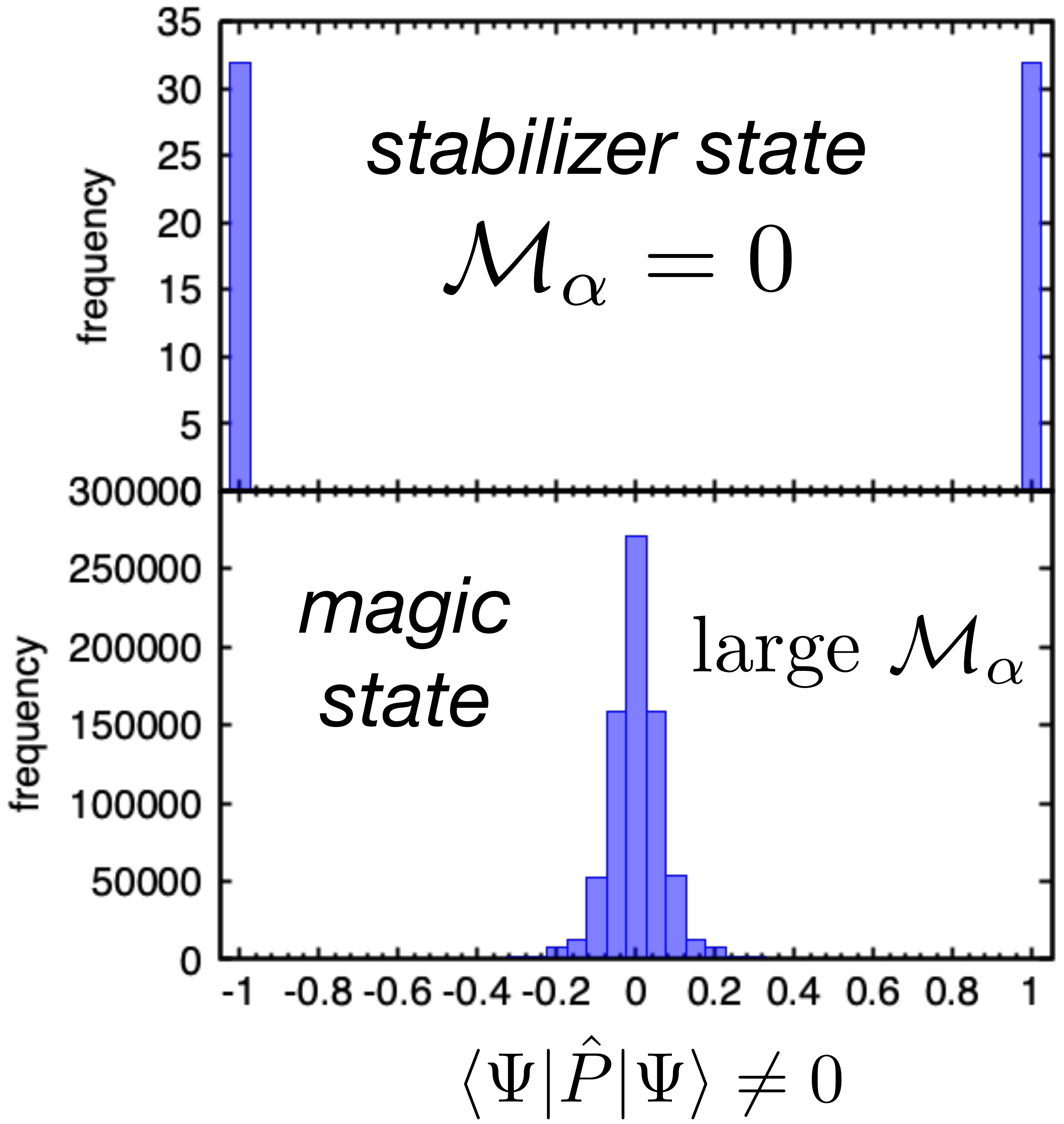
Magic via Stabilizer Rényi Entropy:

Leone, Oliviero, Hamma, PRL 128, 050402 (2022)

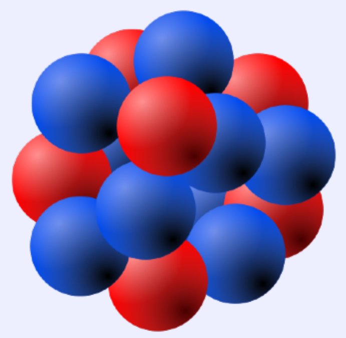
$$\mathcal{M}_\alpha(|\Psi\rangle) = -\log(d) + \frac{1}{1-\alpha} \log\left(\sum_P \frac{\langle \Psi | \hat{P} | \Psi \rangle^{2\alpha}}{d^\alpha} \right)$$

⇒ ~ distance to the nearest stabilizer state

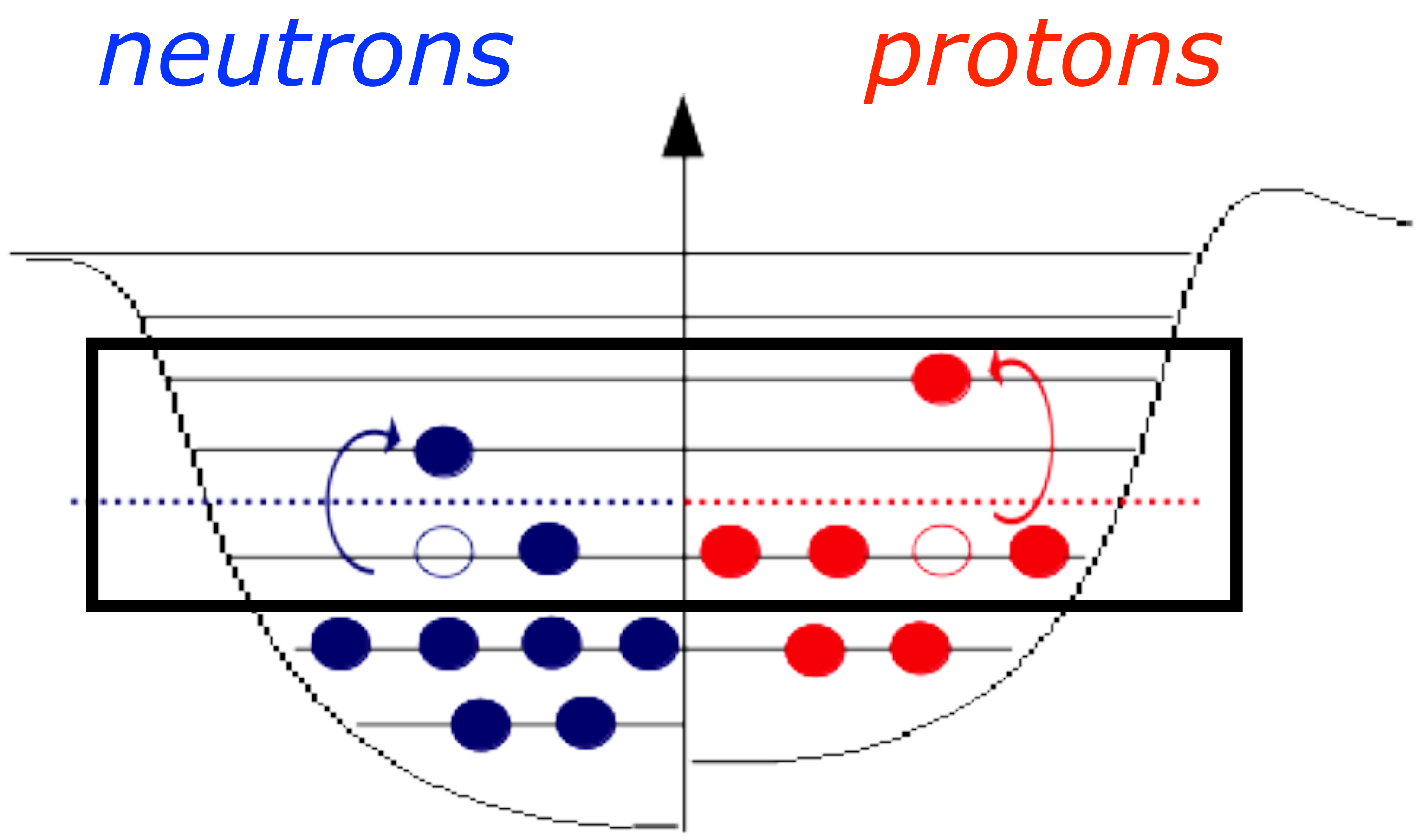
$d = 2^{n_{qubits}}$ $\quad \hat{P} =$ Tensor product of Pauli operators



Multi-Body Entanglement and Magic in Shell-Model Nuclei

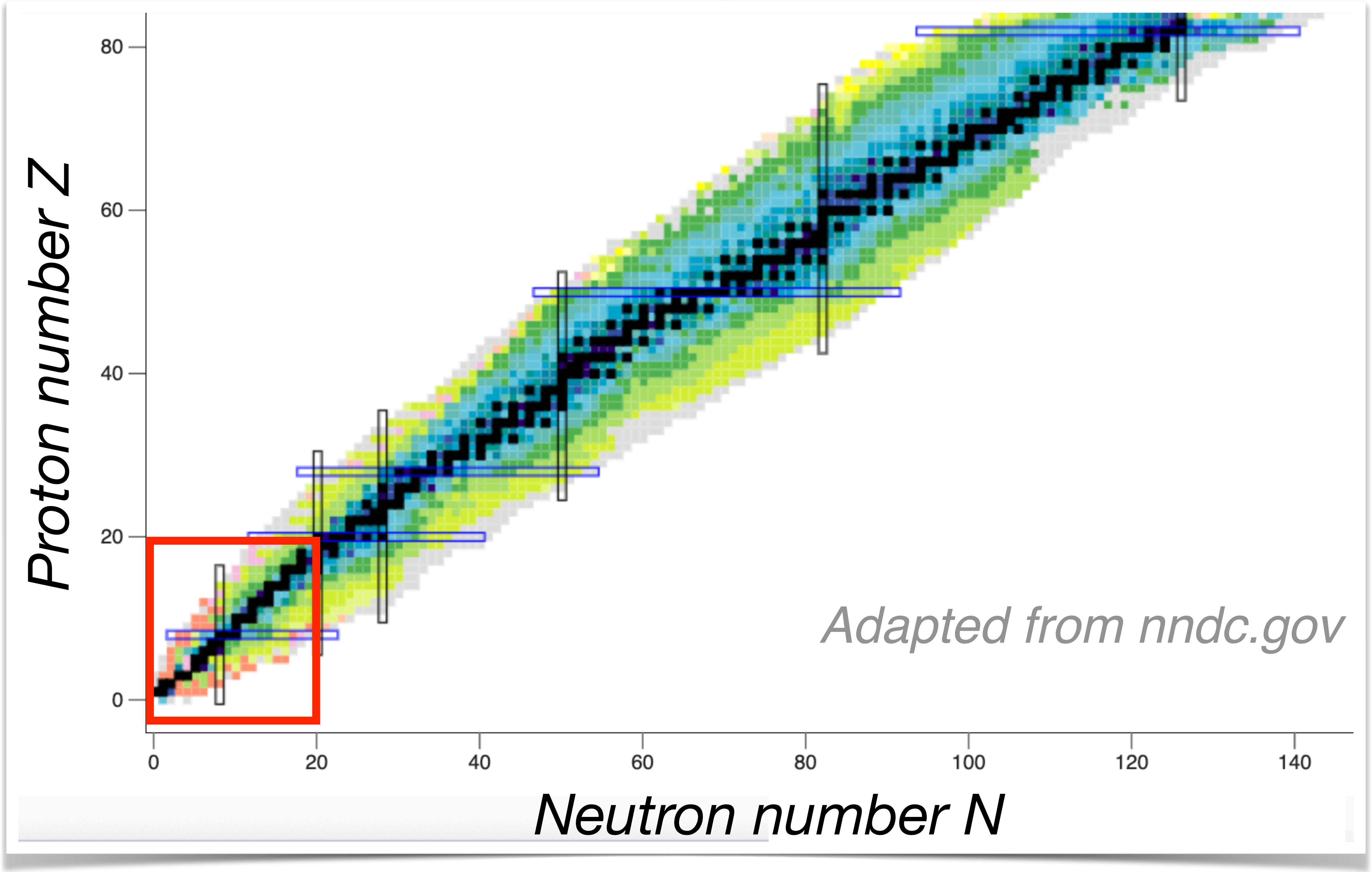


$$|\Psi\rangle = \sum_{\pi\nu} C_{\pi\nu} |\phi_\pi\rangle \otimes |\phi_\nu\rangle$$



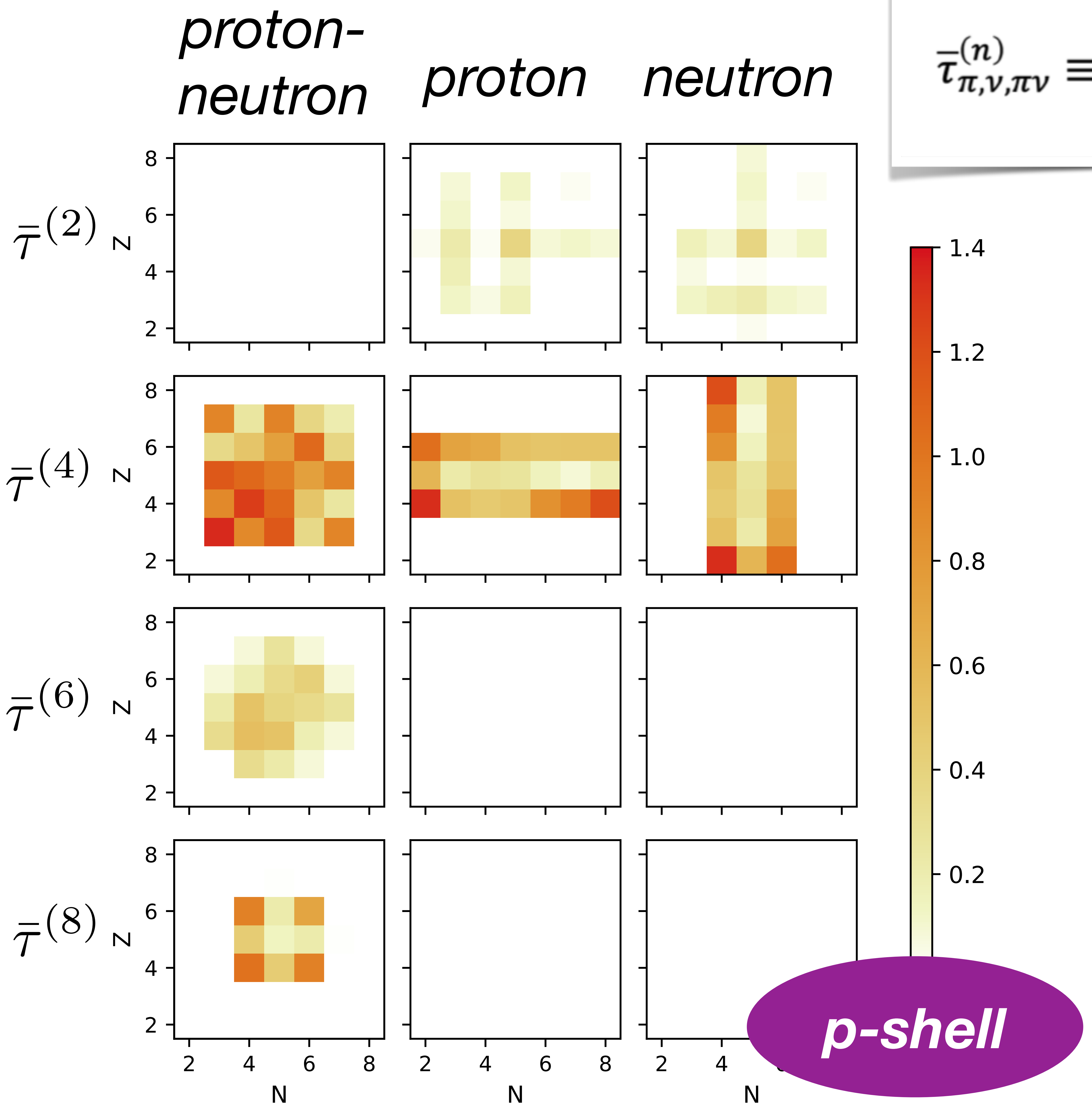
shell-model calculations

$$\hat{H} = \sum_{ij} \varepsilon_i a_i^\dagger a_j + \frac{1}{4} \sum_{ijkl} \bar{v}_{ijkl} a_i^\dagger a_j^\dagger a_l a_k$$

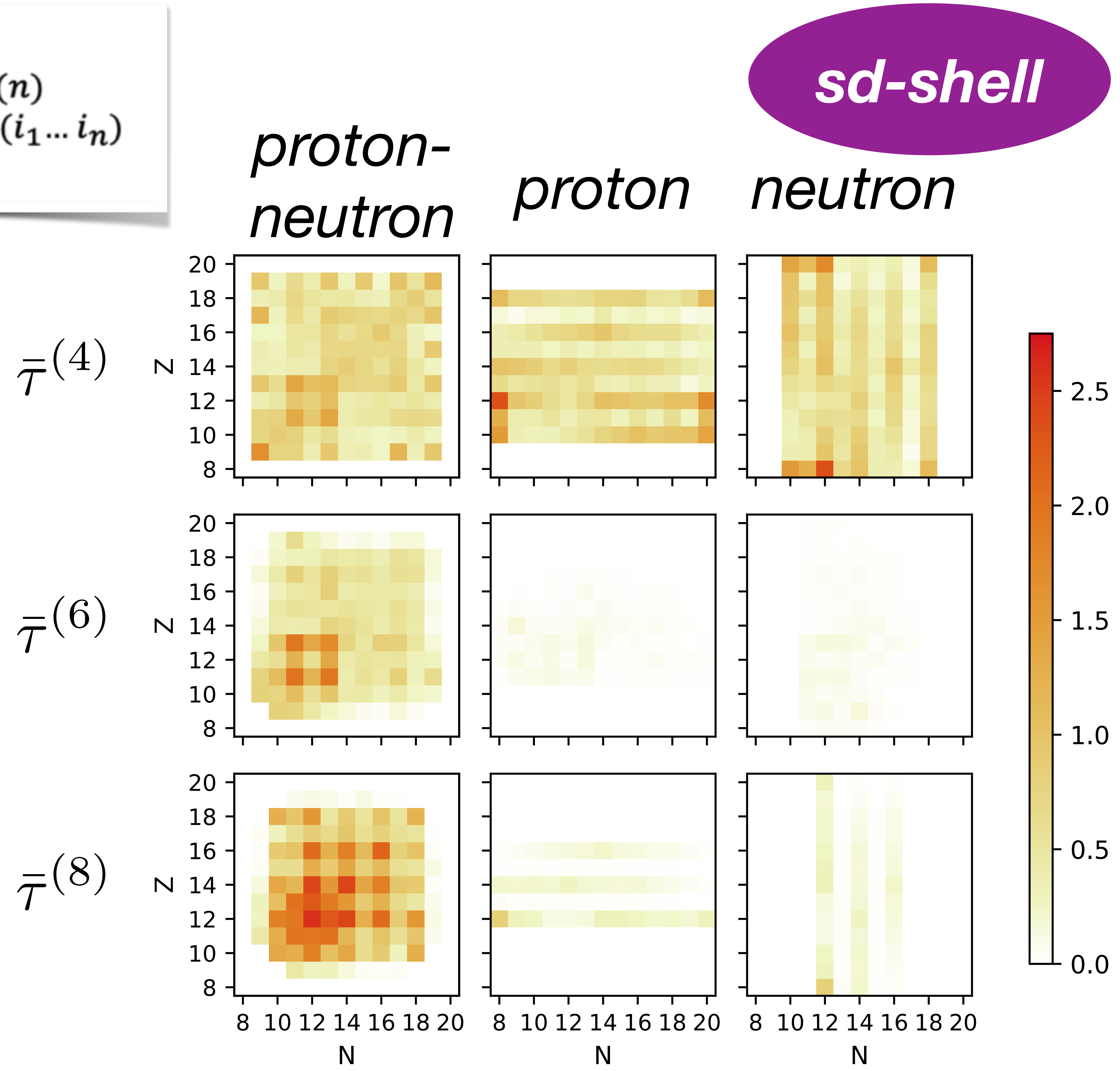


p-and sd-shell nuclei 2 < Z,N < 20
→ mapped onto 12 and 24 qubits

Multi-Partite Entanglement in Shell-Model Nuclei



$$\bar{\tau}^{(n)}_{\pi,\nu,\pi\nu} \equiv \sum_{i_1,i_2,..i_n \in \pi,\nu,\pi\nu} \tau^{(n)}_{(i_1 \dots i_n)}$$

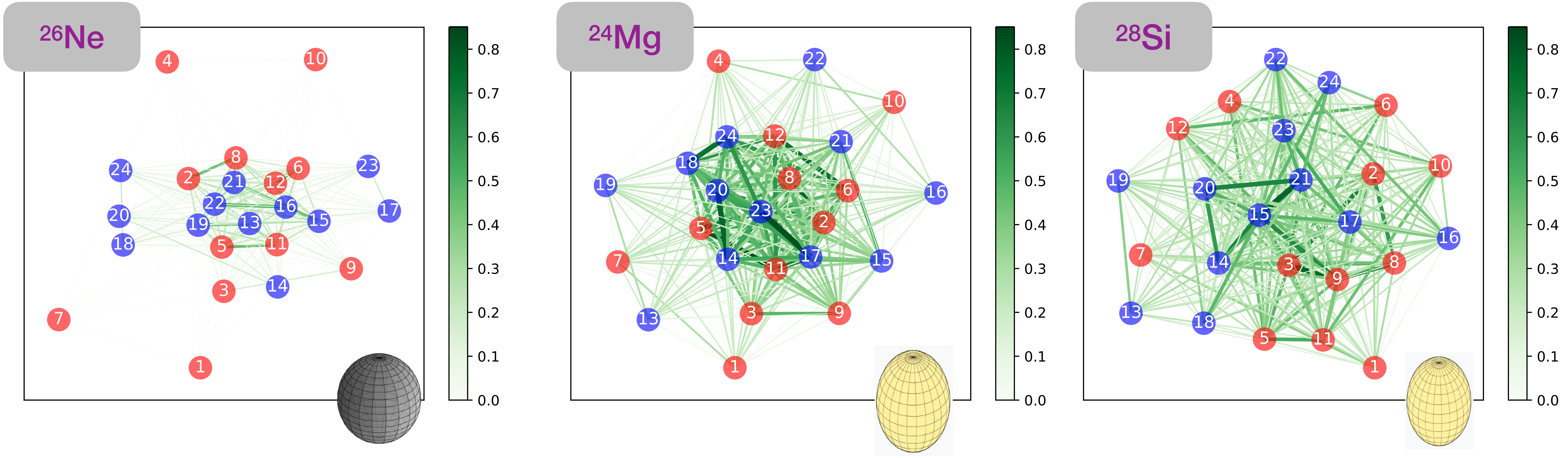


Large many-body entanglement when allowed by model space and symmetries

Multi-Partite Entanglement in sd-shell Nuclei

Brökemeier, CR+, PRC 111, 034317 (2025)

Collective entanglement networks (8-tangles)

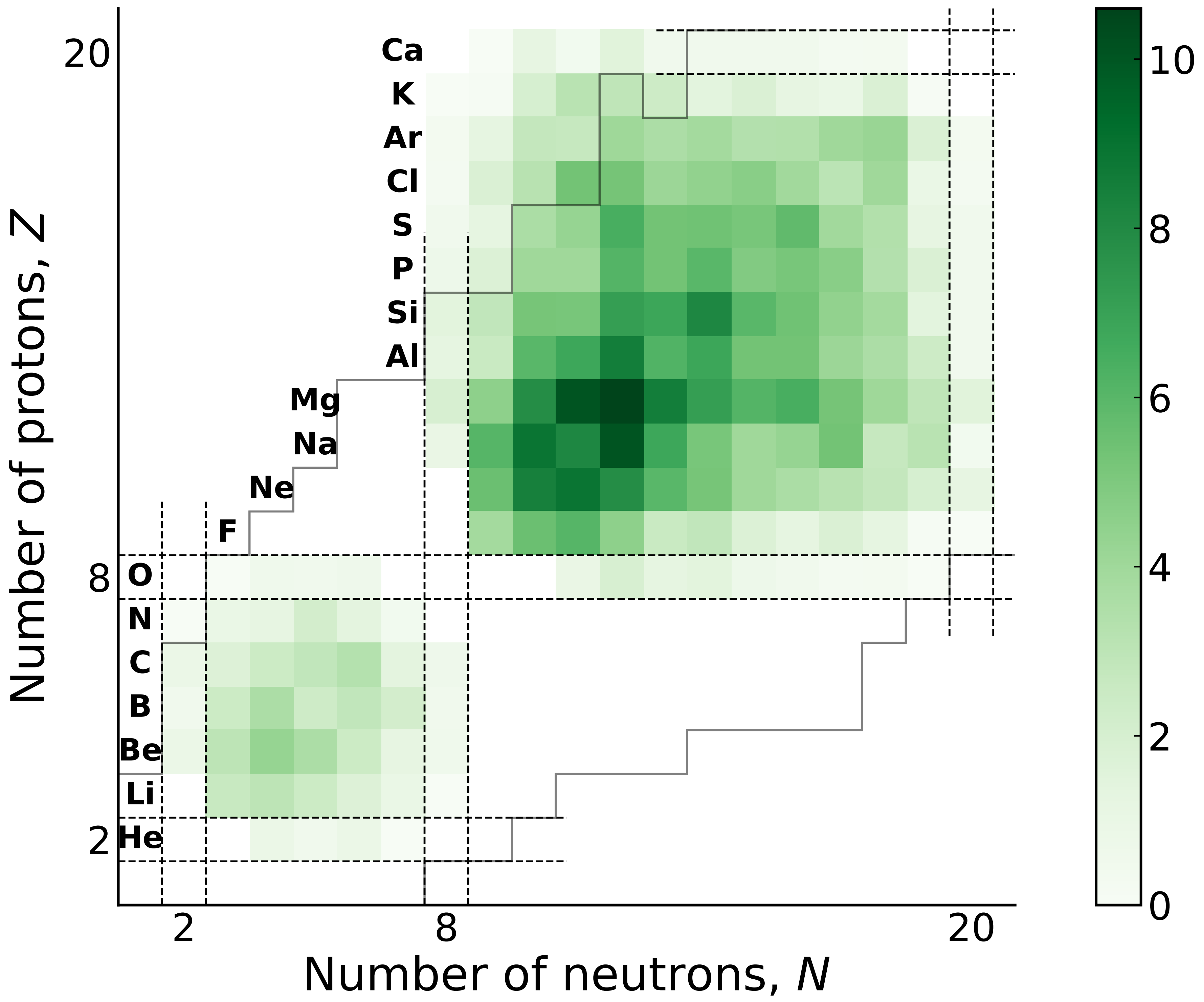


Edge values $$e^{(8)}_{i_1 i_2} = \sum_{i_3<i_4<i_5<i_6<i_7<i_8} \tau^{(8)}_{(i_1,i_2,i_3,i_4,i_5,i_6,i_7,i_8)}$$

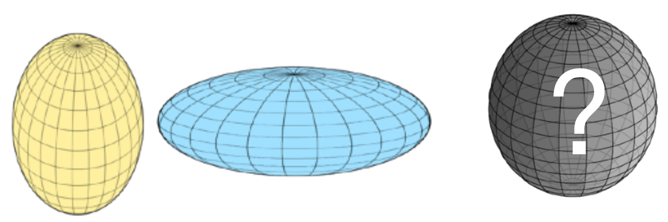
Signature of Shape Collectivity in Magic and Multi-pn Entanglement

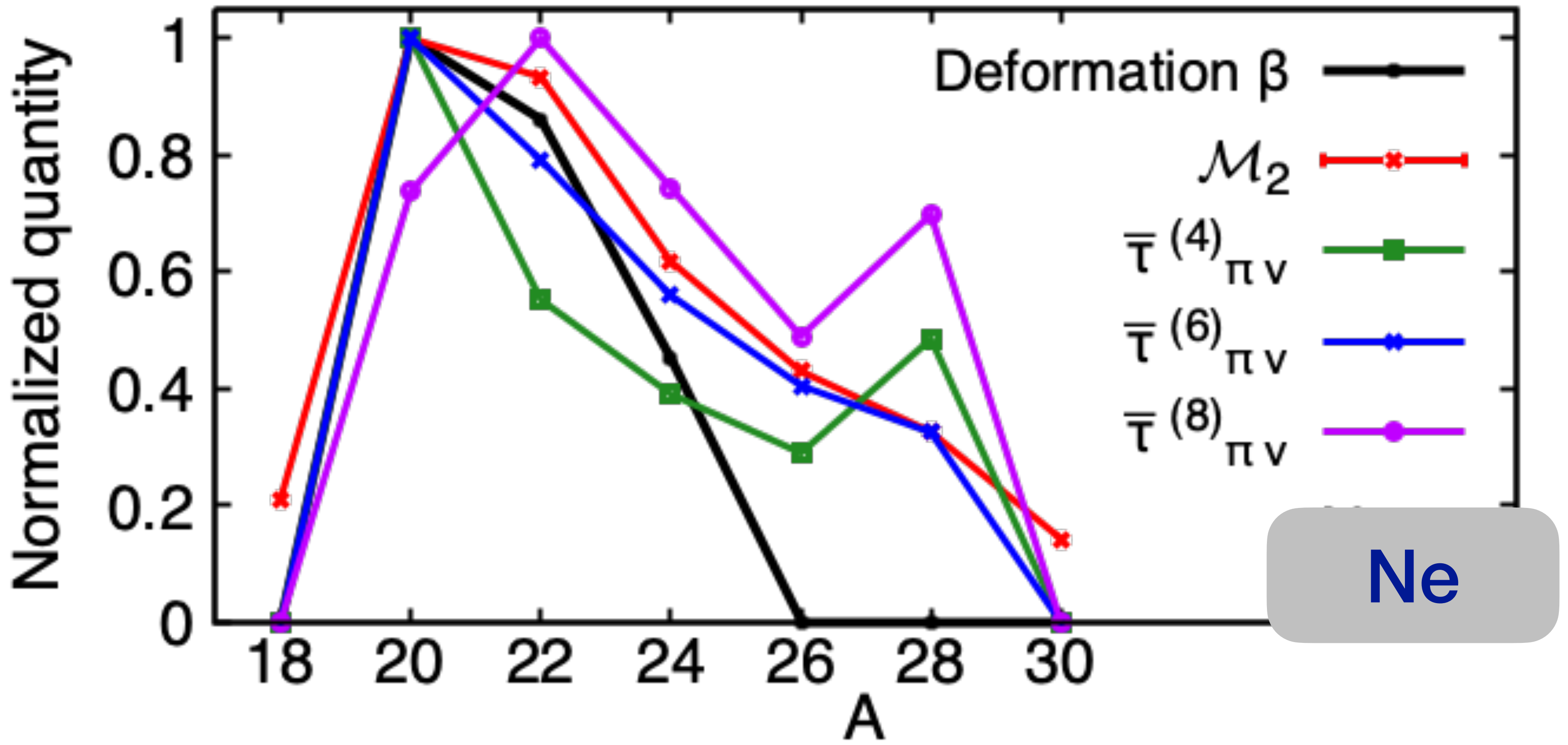
Stabilizer 2-Rényi Entropy $\mathcal{M}_2$

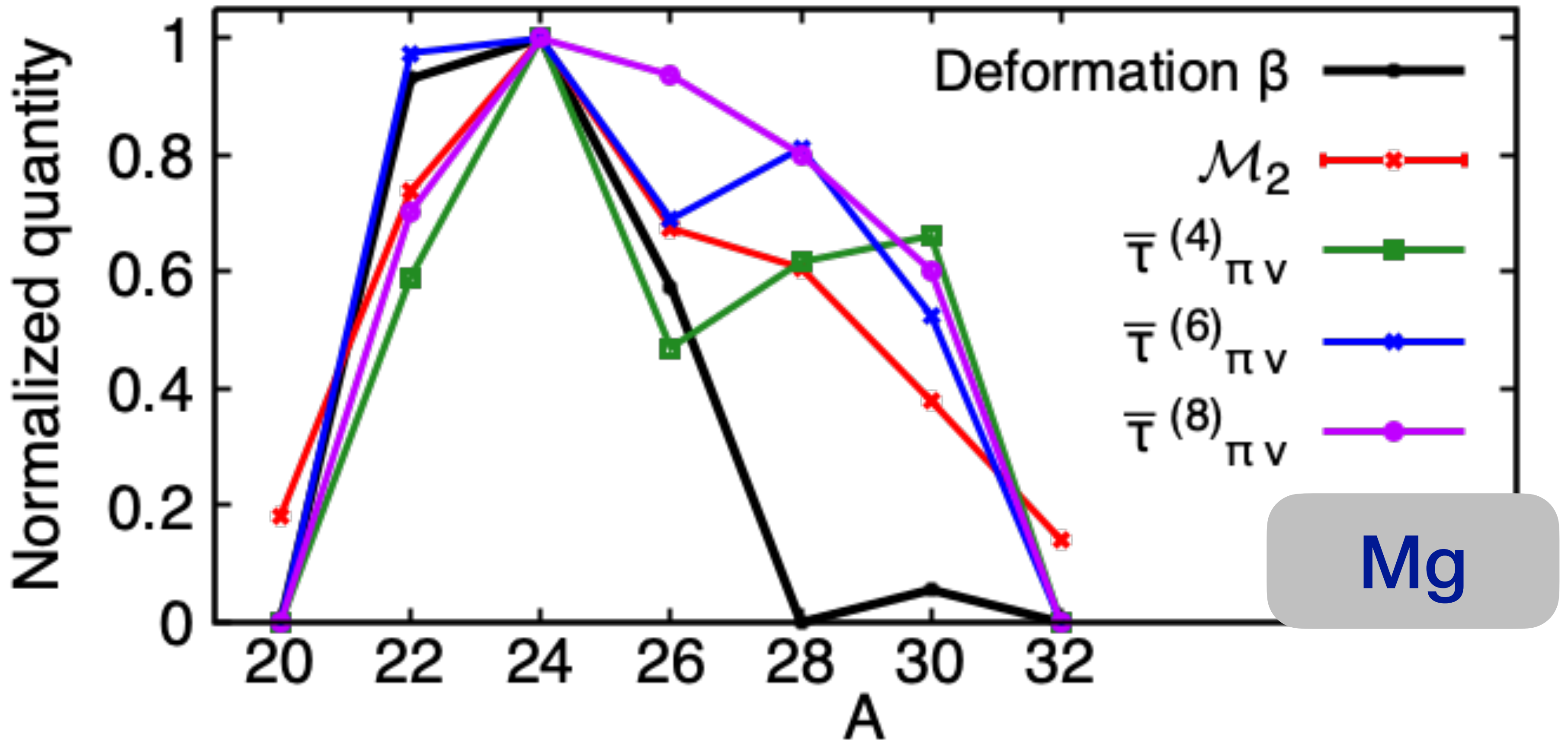
Exact and MCMC computations



PRC 111, 034317 (2025)



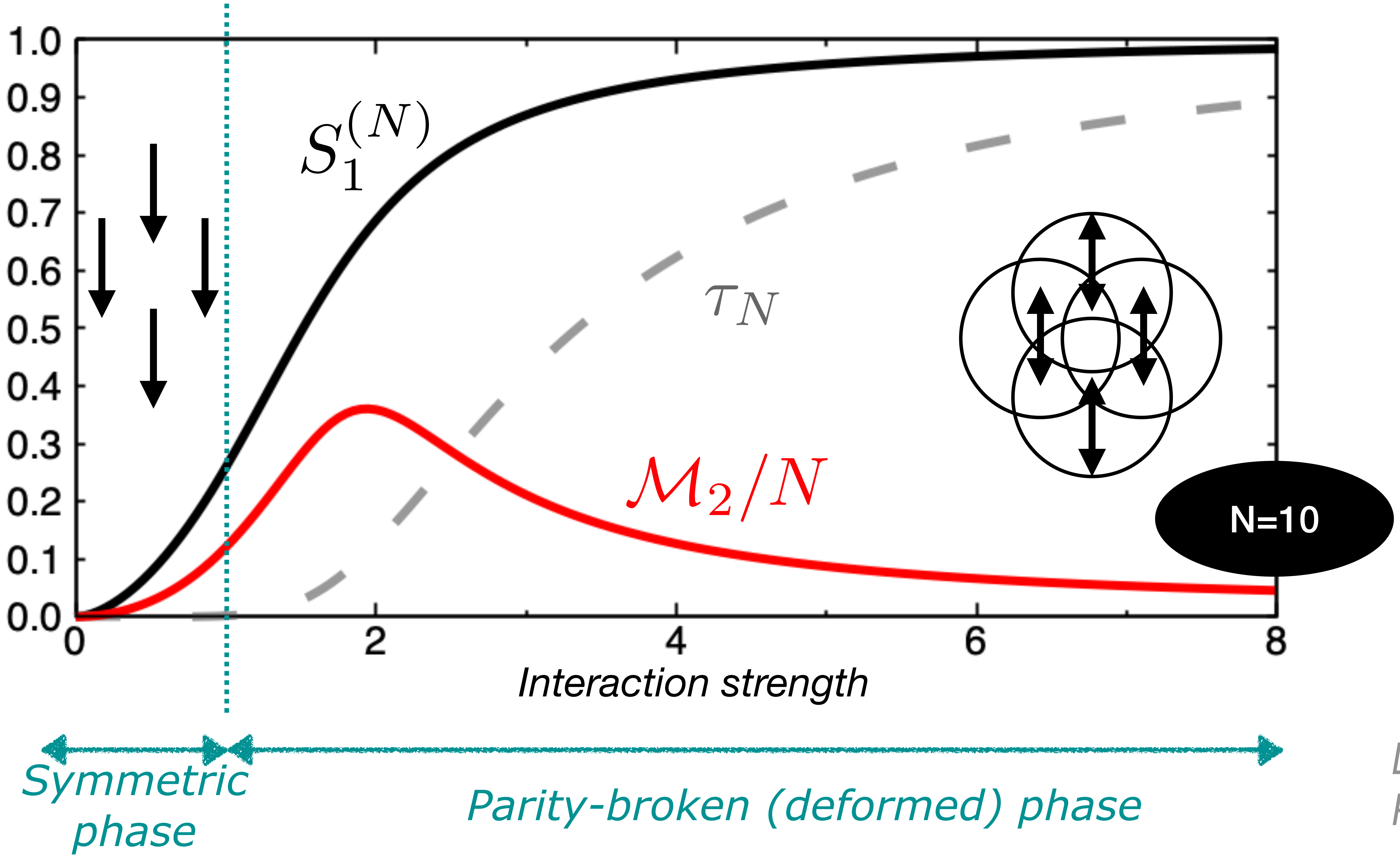




- Maximal magic and proton-neutron tangles coincide with maximal deformation in nuclei
- Magic and tangles also persist in the region where axial deformation vanishes (shape co-existence, other correlations?)

Singling out Deformation: the Lipkin-Meshkov-Glick model

Relevance for nuclear physics, condensed matter, trapped-ion quantum computing, quantum sensing/metrology



$S_1^{(N)}$ = One-spin entanglement entropy

τ_N = N-tangle

$\mathcal{M}_2$ = Stabilizer 2-Rényi entropy

Latorre+ (2005); Hengstenberg, CR+ (2023), Passarelli+ (2024), CR arXiv:2505.02923…

Singling out Deformation: the Lipkin-Meshkov-Glick model

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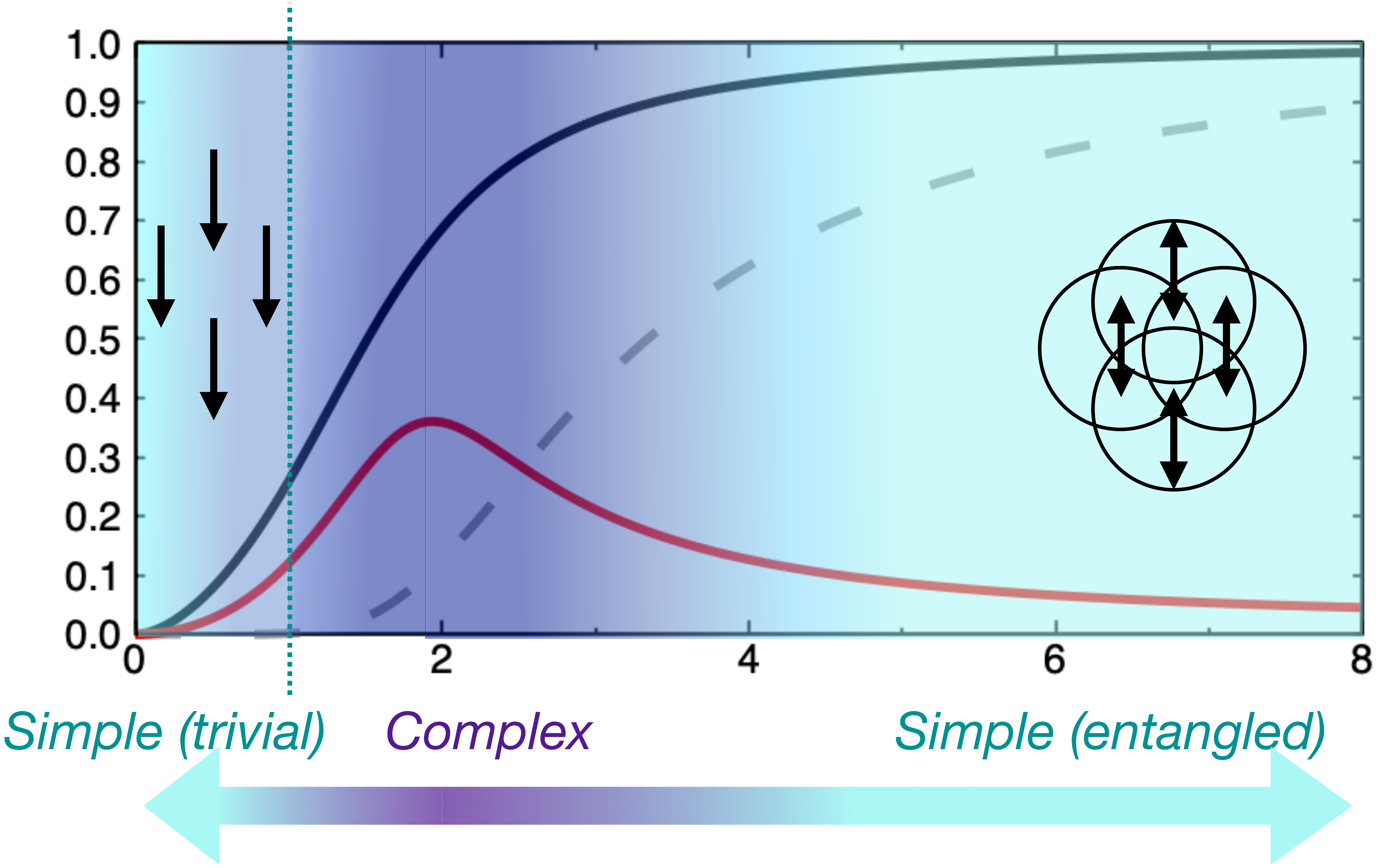

Can stabilizer states capture deformation in an efficient & symmetry-preserving way?

CR arXiv:2505.02923

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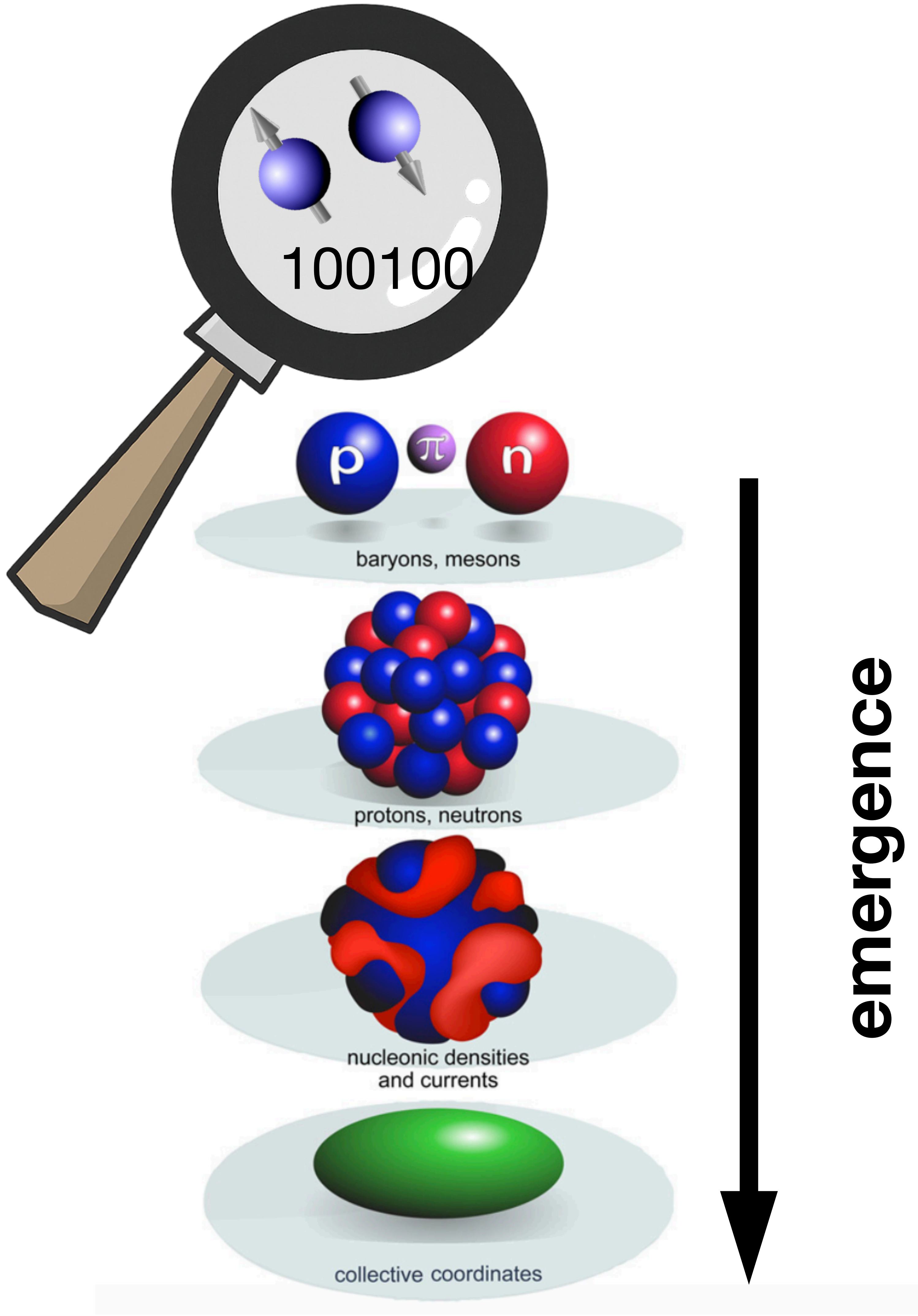
★ Quantum complexity and the emergence of collectivity in nuclear many-body systems

→ *Entanglement, non-stabilizerness and deformation*

★ Complexity-guided ground-state finding algorithms

→ *Stabilizer-accelerated many-body ground state estimations*

→ *Information and complexity rearrangement with Hamiltonian-learning-VQE*

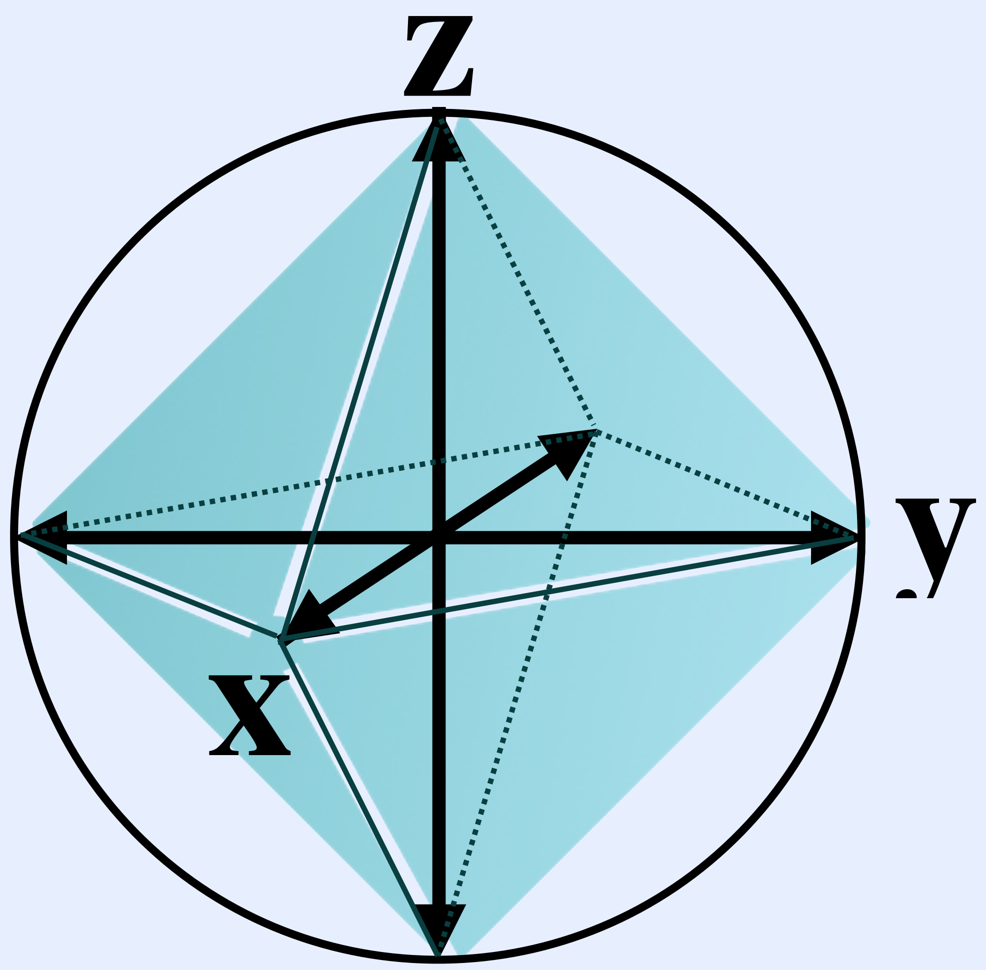


Adopted from Bertsch, Dean, Nazarewicz SciDAC review (2007)

Stabilizer Ground States

- one qubit:

  6 stabilizer states



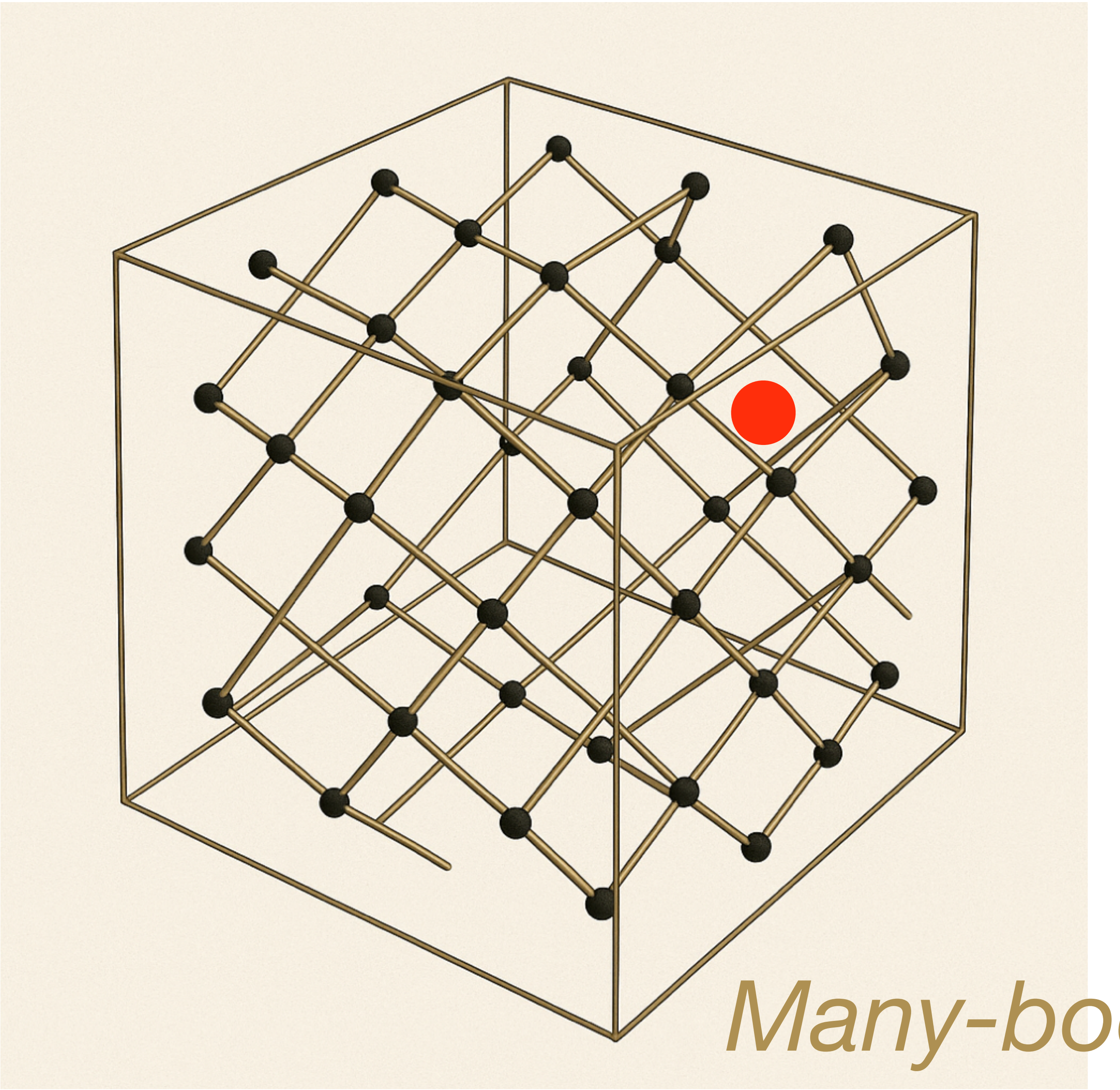
- two qubits: 60 stabilizer states
  incl. 24 entangled states

  e.g. $\frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$, $\frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$

- Three qubits: 1080, Four qubits 36720…

- n qubits: $N_{stab} = 2^{(1/2+o(1))n^2}$

Aaronson Gottesman 2004



Many-body Hilbert space

→ How to determine the closest stabilizer state to the physical ground state?

Stabilizer Ground-State Method

☞ *Strategy:*

CR PRA 112, 052408 (2025)

$$\hat{H} = \underbrace{\sum_{P\in\mathcal{S}} a_P \hat{P}}_{\hat{H}_{stab}} + \underbrace{\sum_{P\in\mathcal{G}_N(H)\notin\mathcal{S}} a_P \hat{P}}_{\hat{W}} ,$$

Stabilizer part | Magic-inducing part

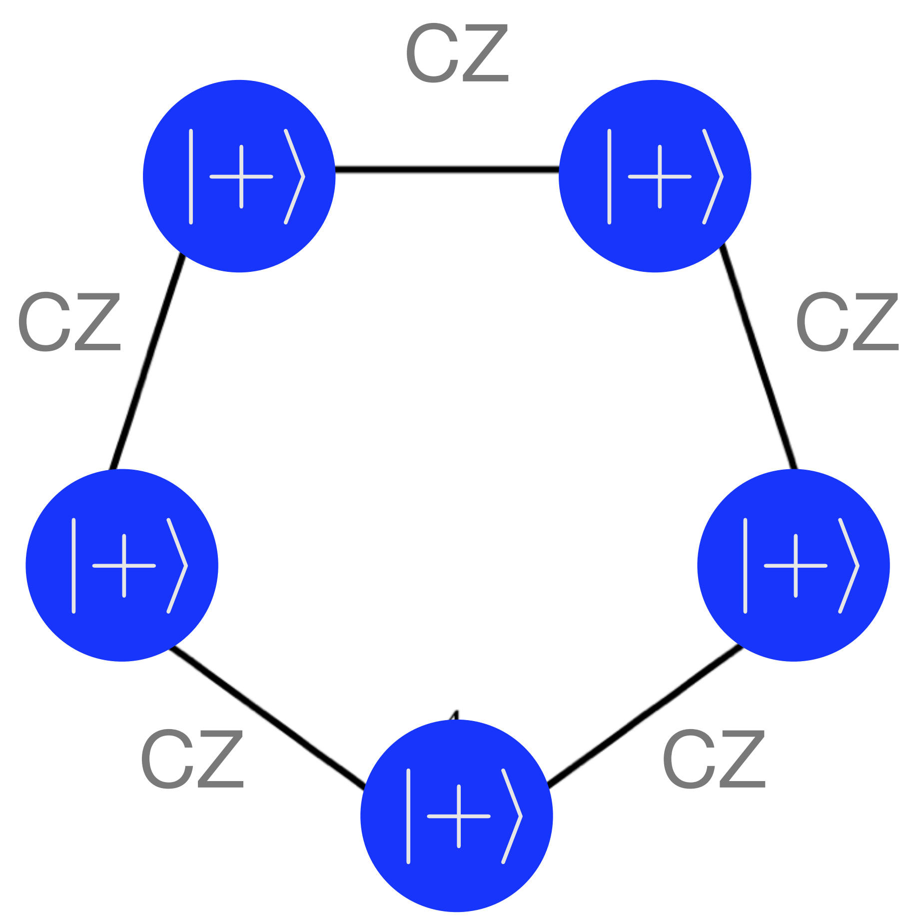
See also Sun+ (2024), Gu+ (2024), Weaving+ (2022)…

$$\mathcal{S} = \langle g_1, g_2, ...g_N \rangle$$

Stabilizer group

- Explore all partitionings, and pick the energy-minimizing solution
- The stabilizer ground state can be prepared using graph states

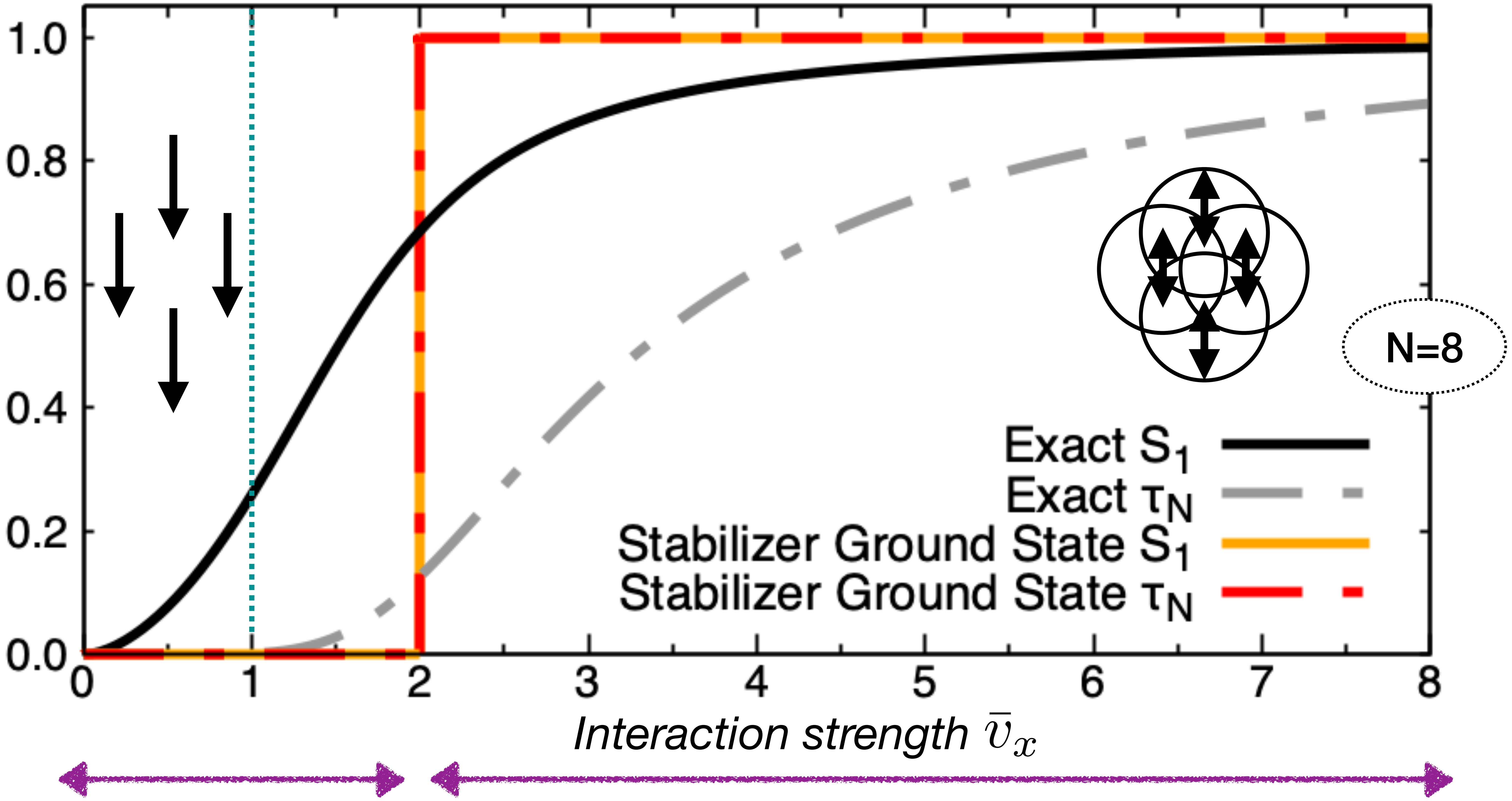
$$|\Psi_s\rangle = \prod_{i=1}^{N} C_i \, |G\rangle \quad \text{where} \quad |G\rangle = \left(\prod_{e\in E} \mathrm{CZ}_e\right) |+\rangle^{\otimes N}$$

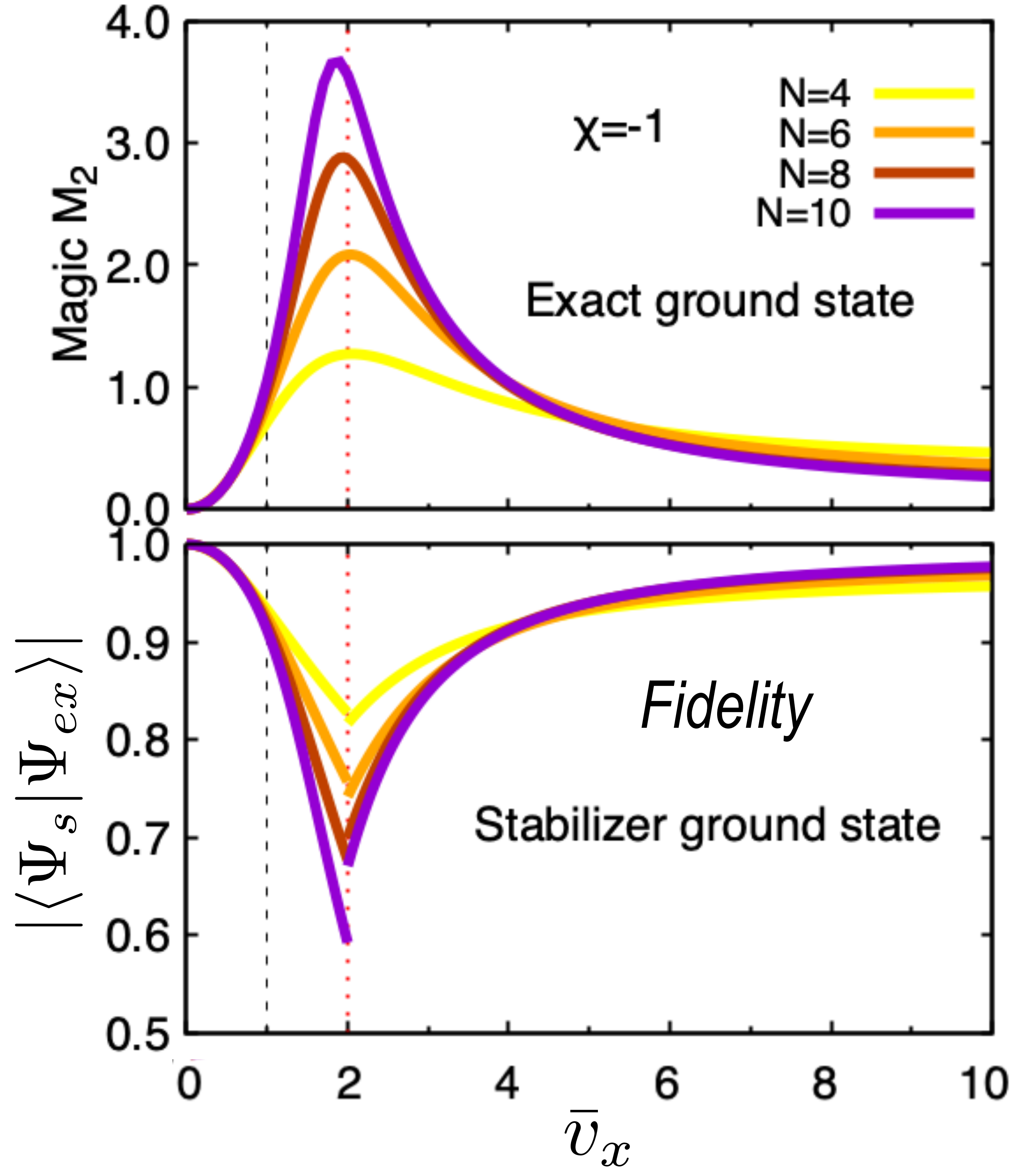


The 1-qubit Clifford C_i's can be determined efficiently, using stabilizer tableaus

see e.g. *Van den Nest+ PRA 69, 022316 (2004).*

Stabilizer Ground State in the LMG model



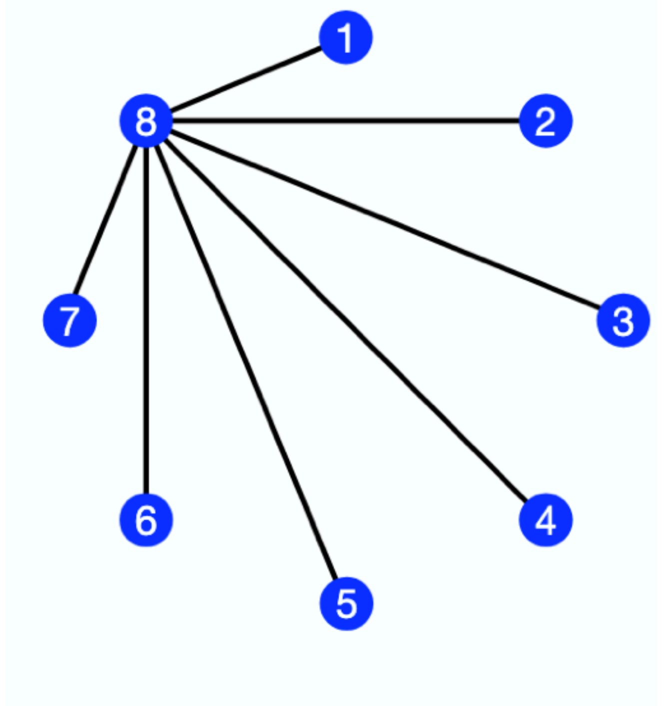


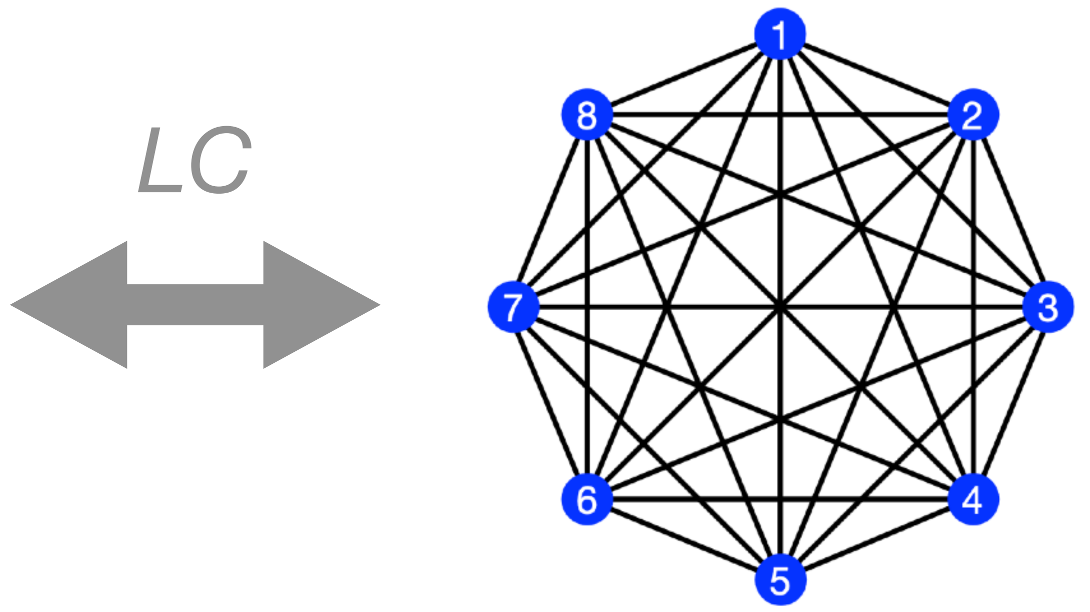
$$|\Psi_s\rangle = |\downarrow\downarrow\dots\downarrow\rangle \qquad |\Psi_s\rangle = (X_1)^{N \bmod 2} H_N |G\rangle$$

Beyond stabilizer ground state?
(Q)ITE, ADAPT-VQE…

CR PRA 112, 052408 (2025)

$$|G\rangle =$$



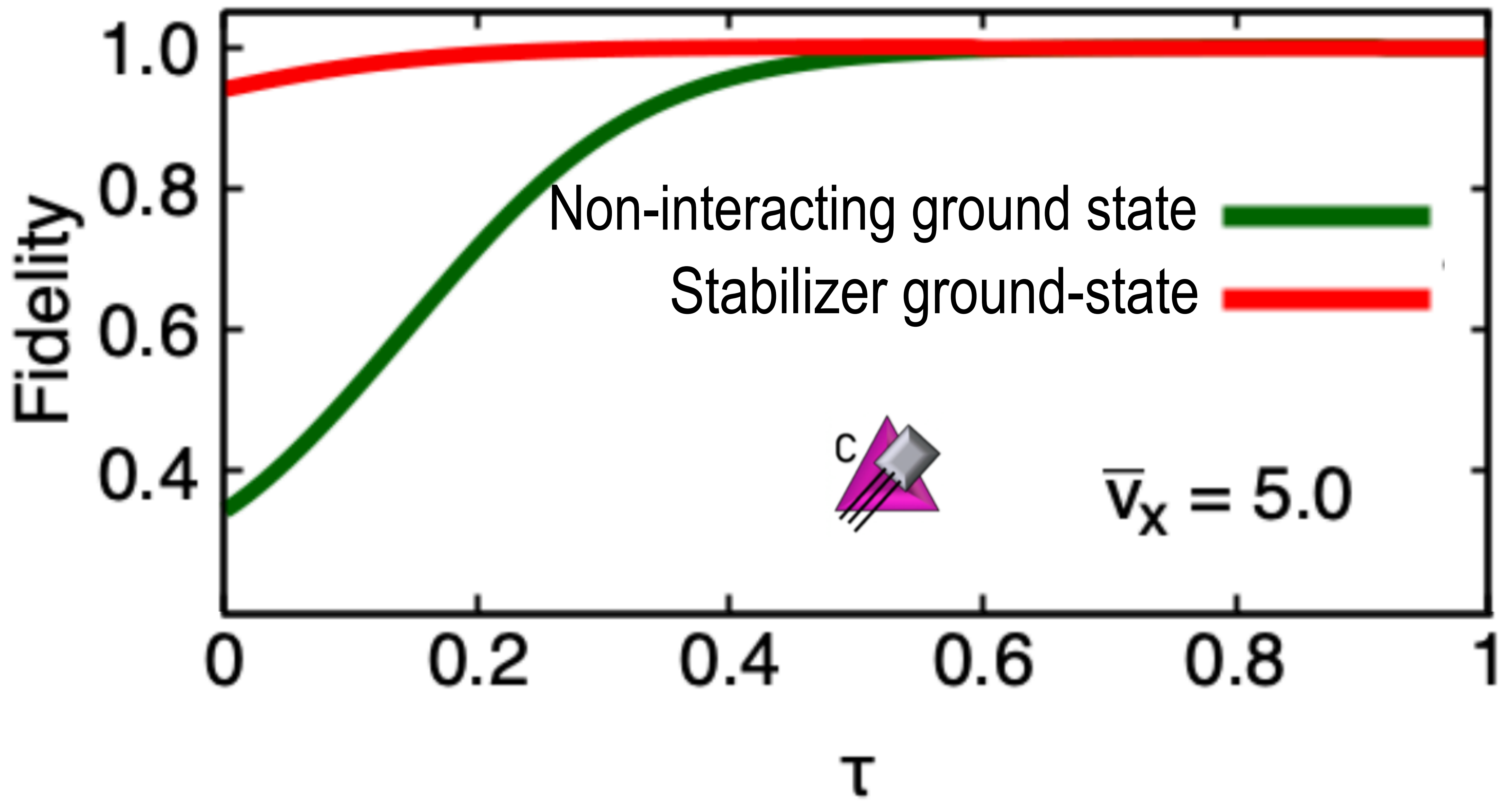


Beyond Stabilizer Ground State with Magic Injection

★ *Example: (Quantum) Imaginary Time Evolutions*

$$e^{-(\hat{H}-\bar{E}_0)\tau}\,|\eta\rangle = \sum_n e^{-(E_n-\bar{E}_0)\tau}\,\langle\Phi_n|\eta\rangle\,|\Phi_n\rangle \xrightarrow[\tau\to\infty]{} \propto\,|\Phi_0\rangle$$

Convergence with different initial states:



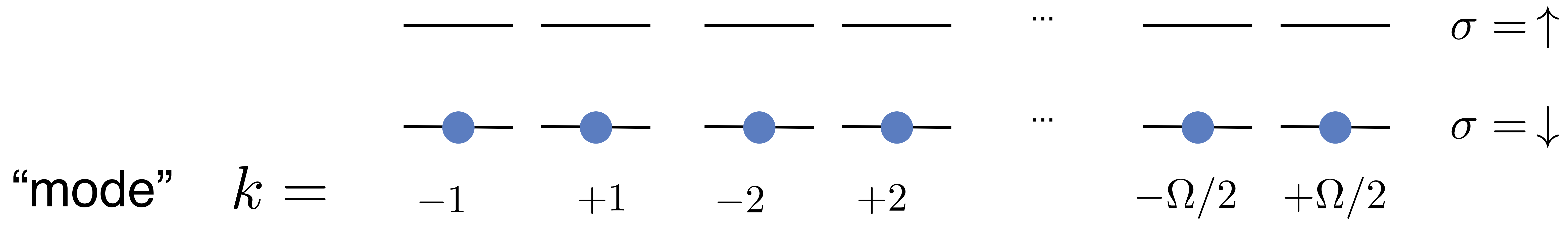
CR PRA 112, 052408 (2025)

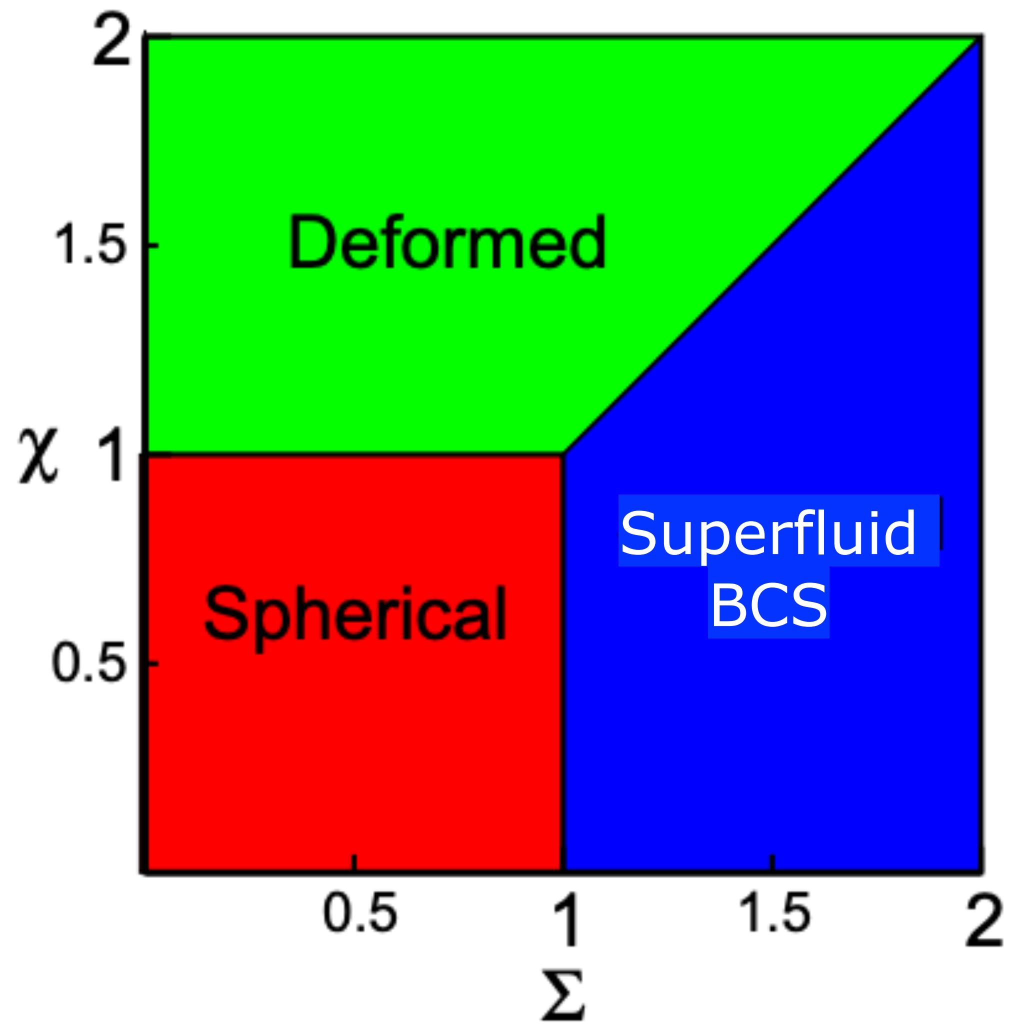
Many-Body Complexity, Deformation and Superfluid pairing

*D. Agassi, Nucl. Phys. A 116, 49 (1968)

★The Agassi model

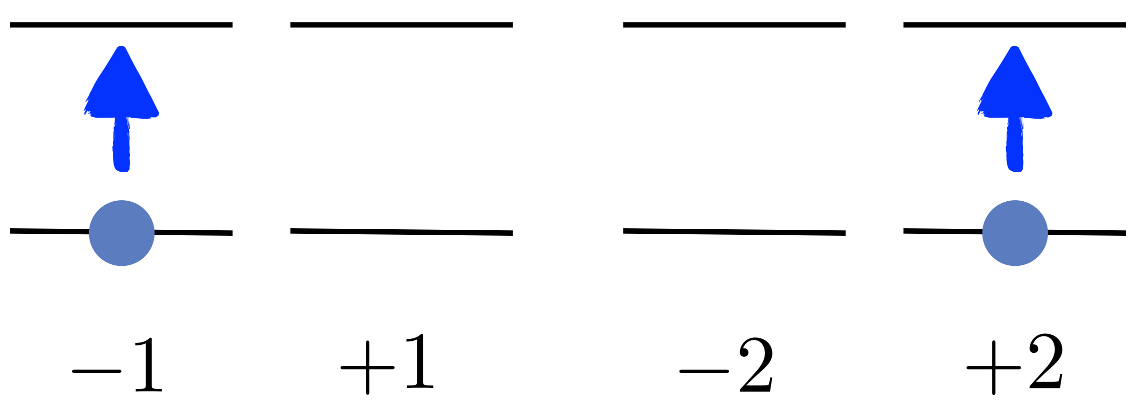
= extension of the LMG model with superfluid pairing



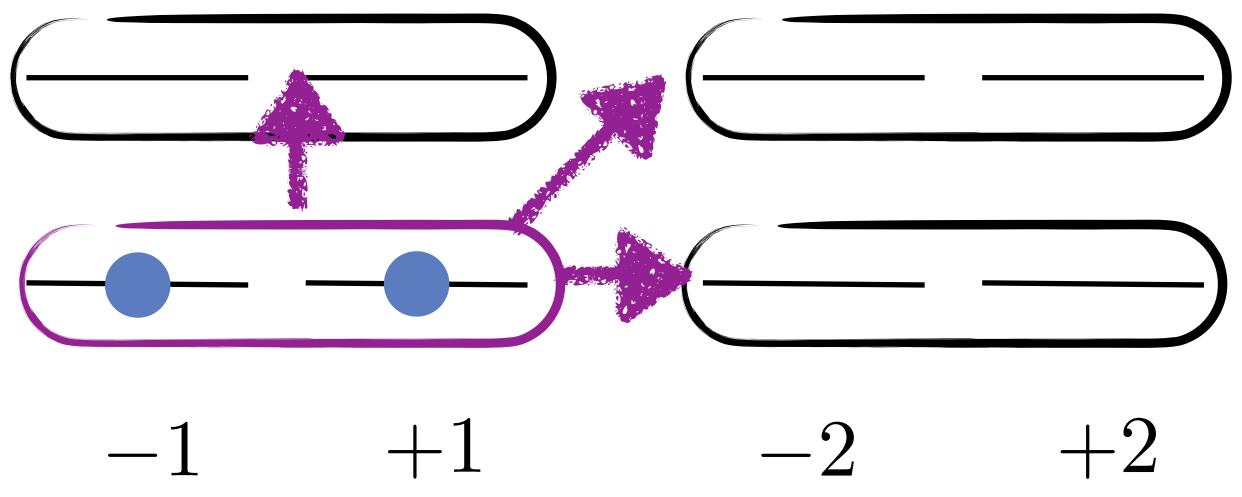


[Pérez-Fernández+ PLB 829 137133 (2022)]

$$\hat{H} = \varepsilon\, \hat{J}_z - \frac{V}{2}\left(\hat{J}_+^2 + \hat{J}_-^2\right) - g \sum_{\sigma\sigma'} \hat{B}^\dagger_\sigma \hat{B}_{\sigma'}$$



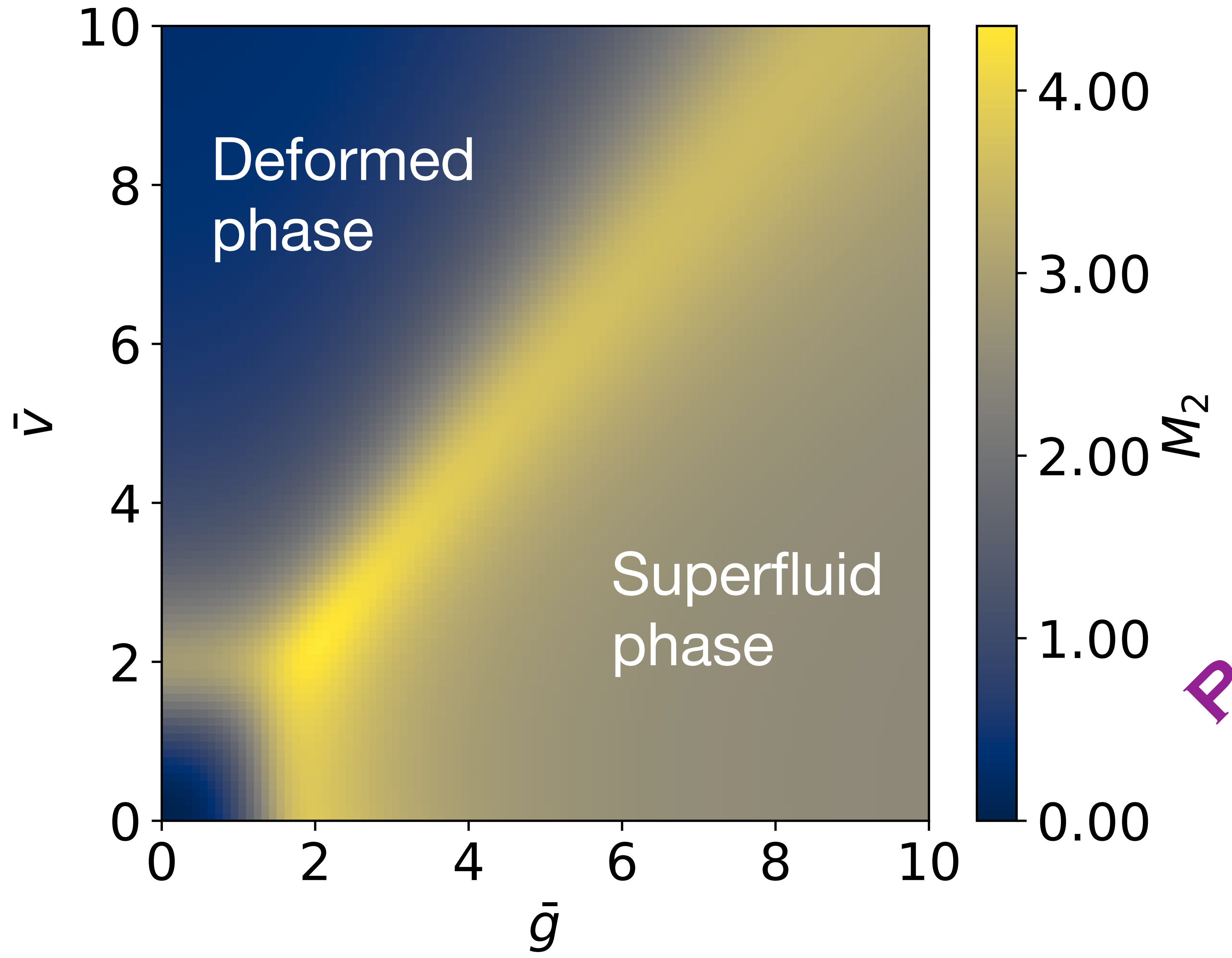
particle-hole interaction V



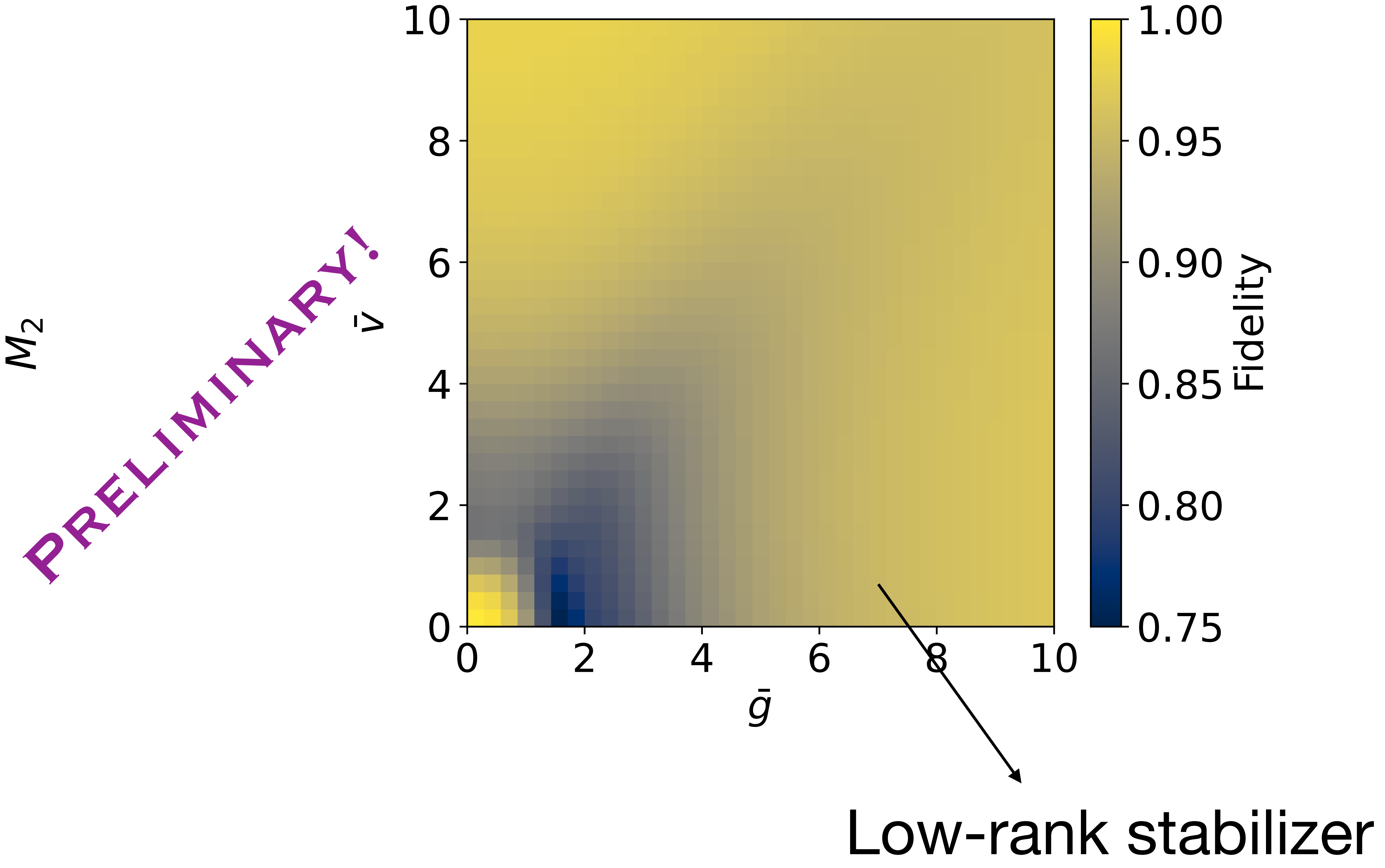
pairing interaction g

Many-Body Complexity, Deformation and Superfluid pairing

Magic (Stabilizer Rényi Entropy $\mathcal{M}_2$)



Stabilizer ground state fidelity



Hengstenberg, Robin, in prep.

Figures courtesy of Momme Hengstenberg

Outline

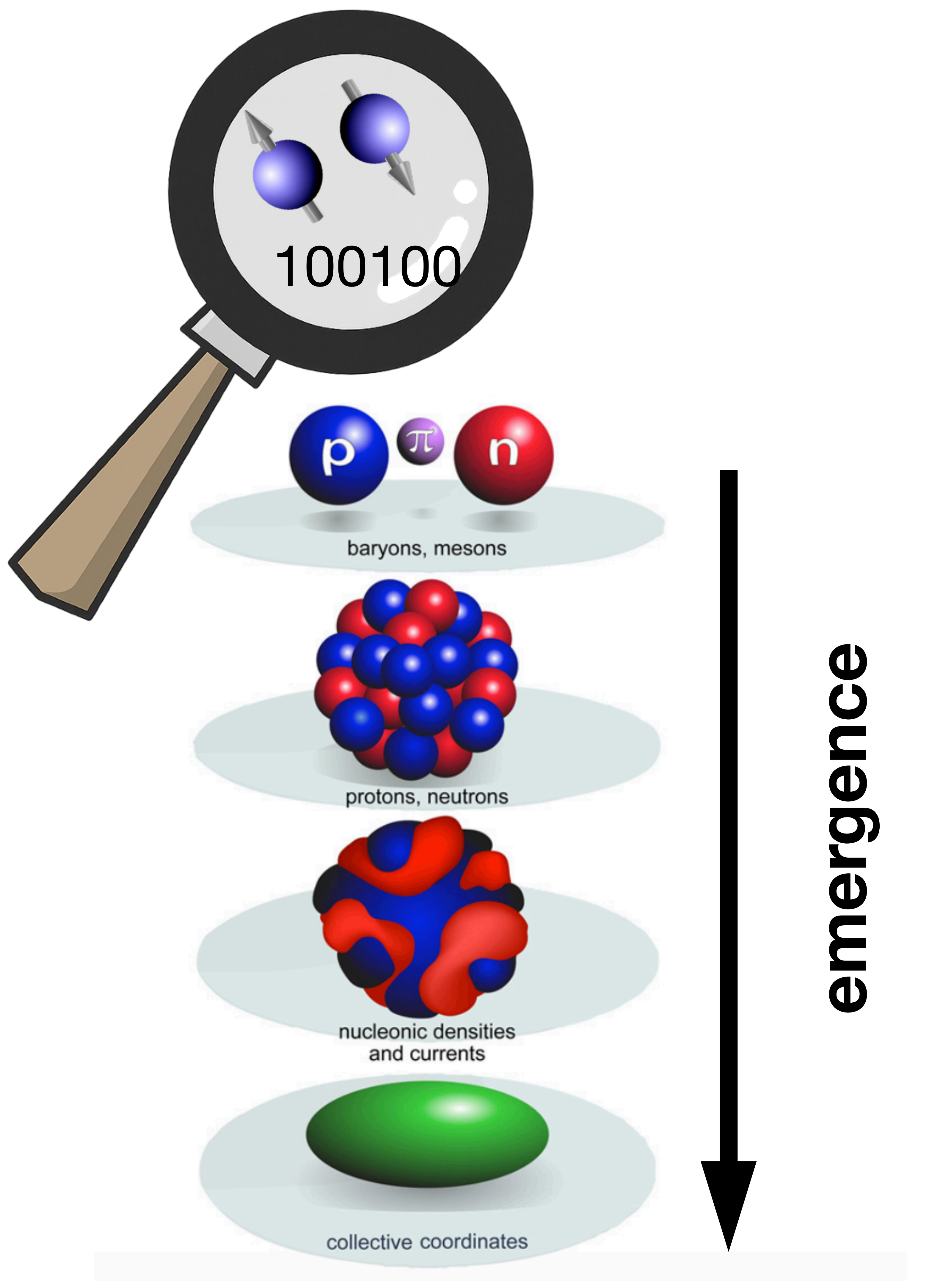
★ Quantum complexity and the emergence of collectivity in nuclear many-body systems

→ *Entanglement, non-stabilizerness and deformation*

★ Complexity-guided ground-state finding algorithms

→ *Stabilizer-accelerated many-body ground state estimations*

→ *Information and complexity rearrangement with Hamiltonian-learning-VQE*

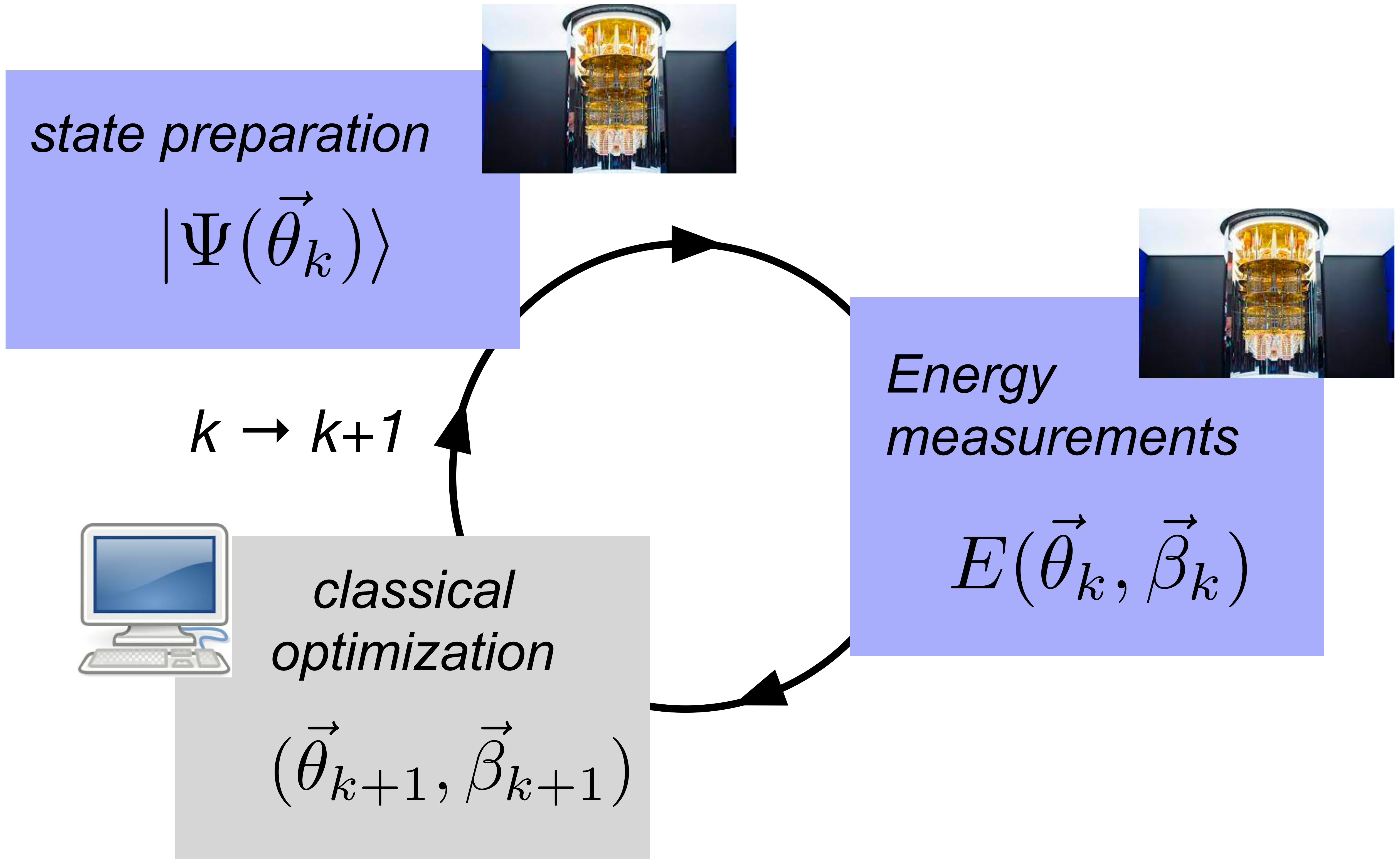


Adopted from
Bertsch, Dean, Nazarewicz SciDAC review (2007)

Information Rearrangement in Classical and Quantum Computations

★ *Hamiltonian-Learning-VQE Algorithm for ground-state finding:*

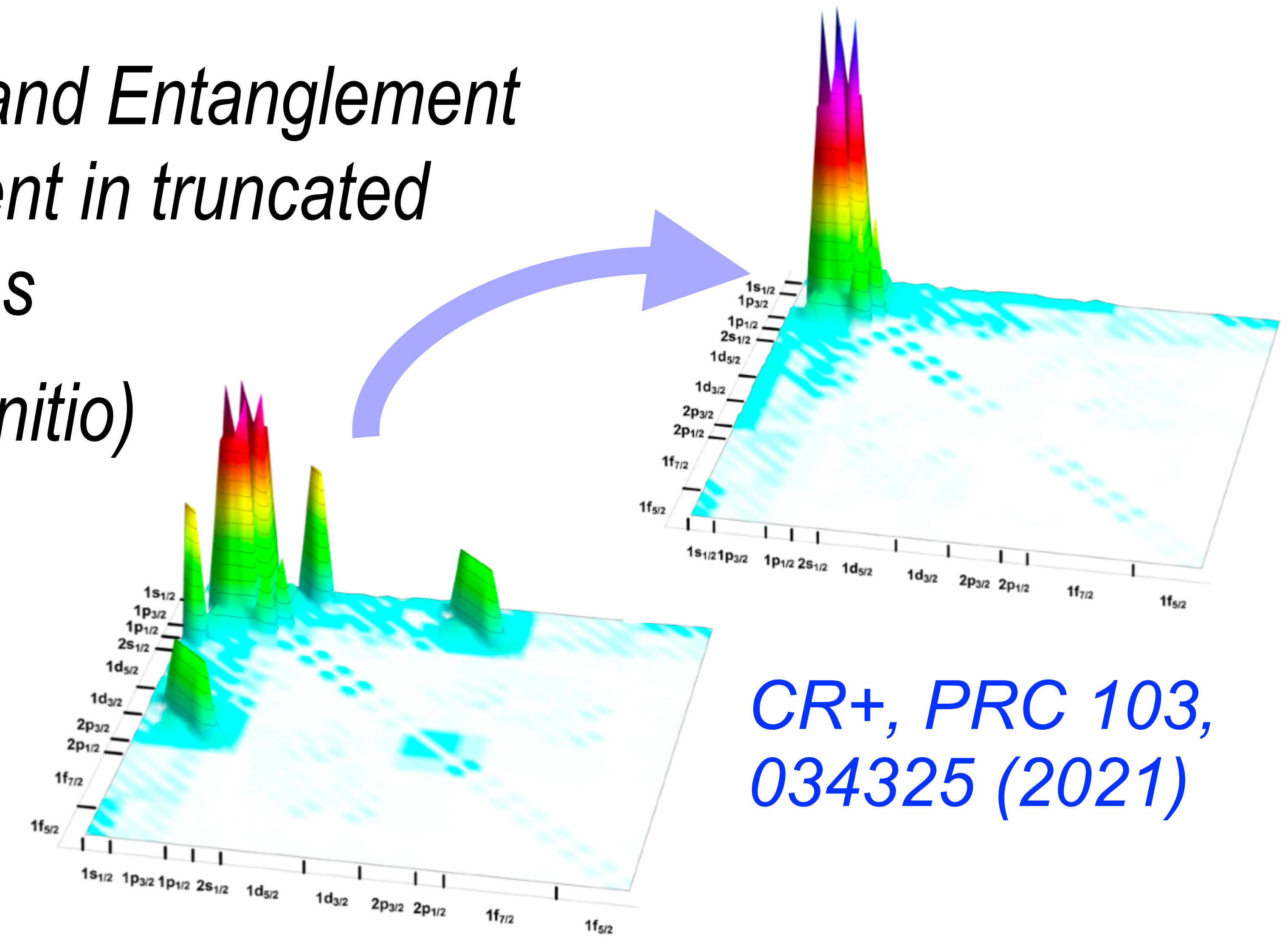
CR, Savage PRC 108, 024313 (2023)



⇒ learns the effective Hamiltonian and identifies the associated ground state simultaneously

Information and Entanglement rearrangement in truncated model spaces

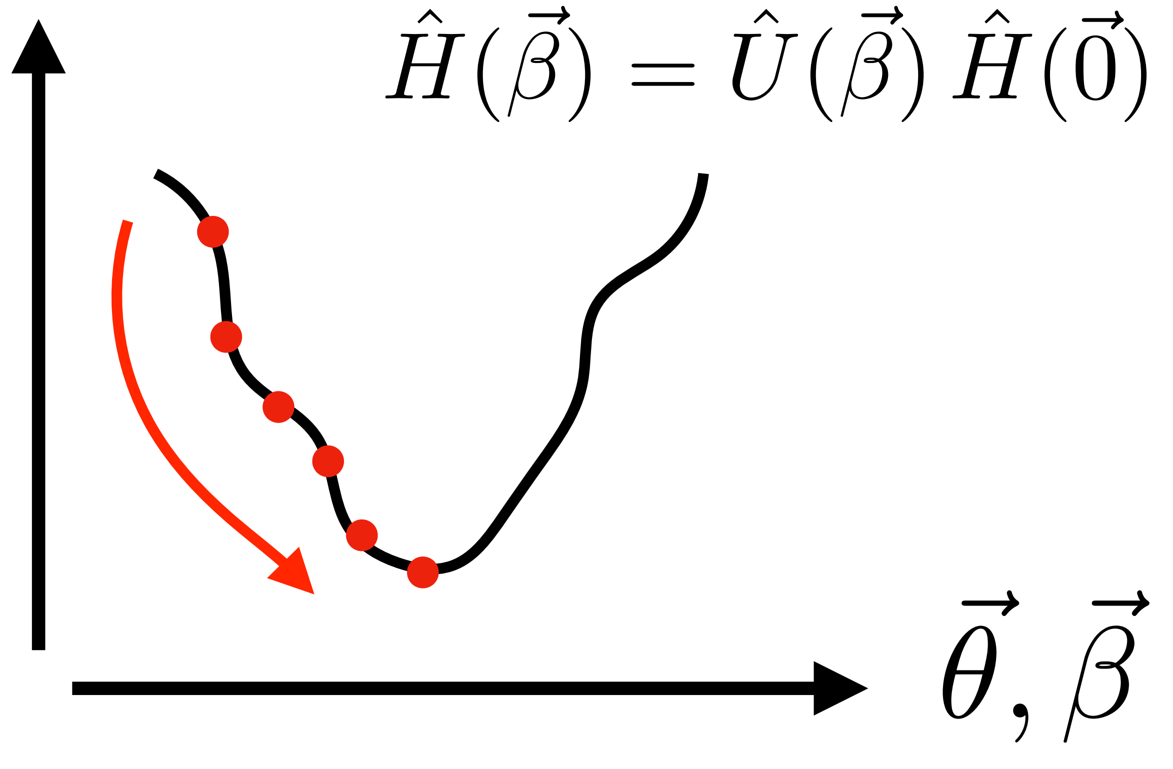
(^{6}He, ab-initio)



CR+, PRC 103, 034325 (2021)

$$E(\vec{\theta}, \vec{\beta}) = \langle \Psi(\vec{\theta}) | \hat{H}(\vec{\beta}) | \Psi(\vec{\theta}) \rangle$$

$$\hat{H}(\vec{\beta}) = \hat{U}(\vec{\beta})\, \hat{H}(\vec{0})\, \hat{U}^\dagger(\vec{\beta})$$



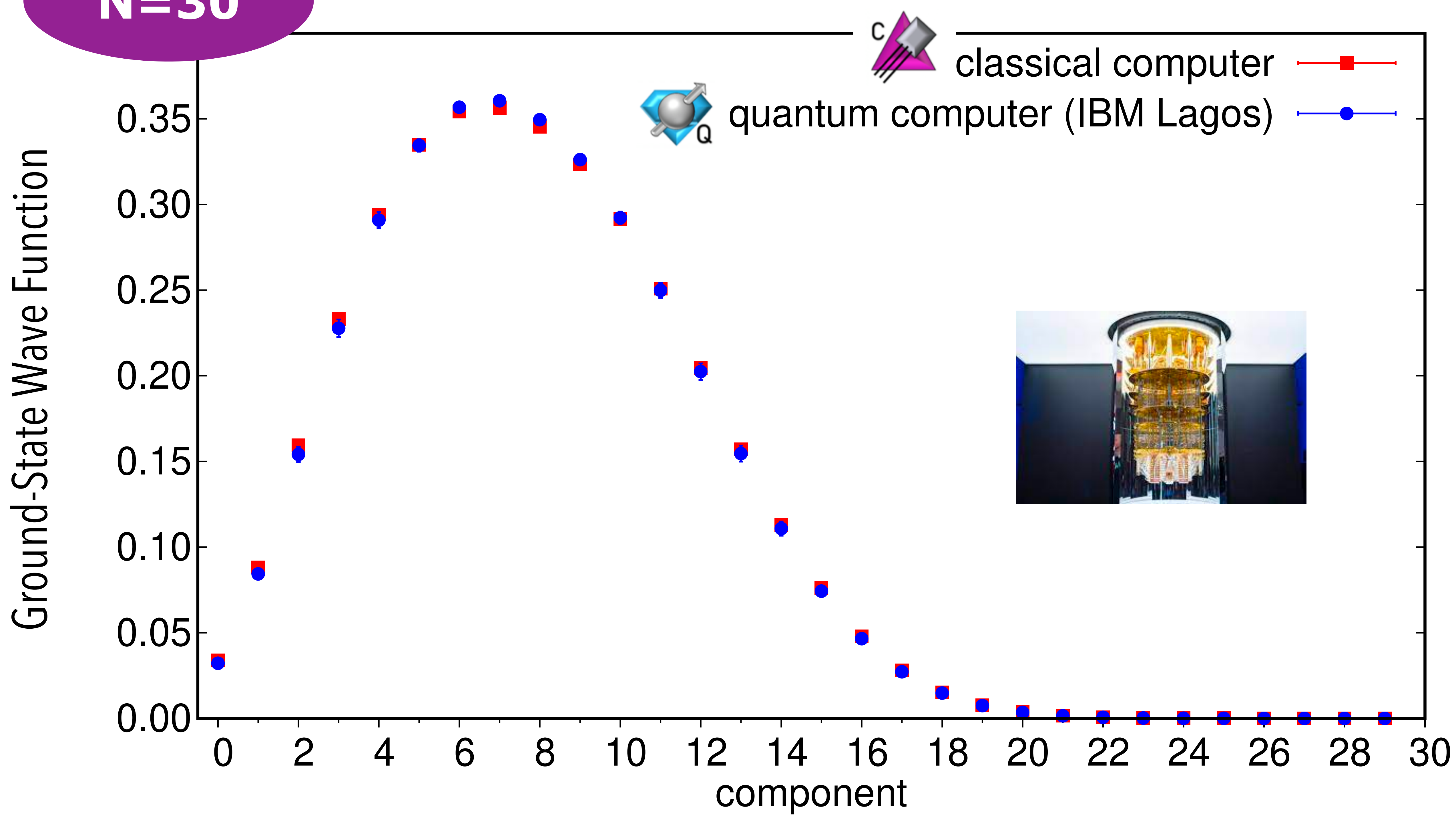
Information Rearrangement in Classical and Quantum Computations

→ *Application to the LMG model:*

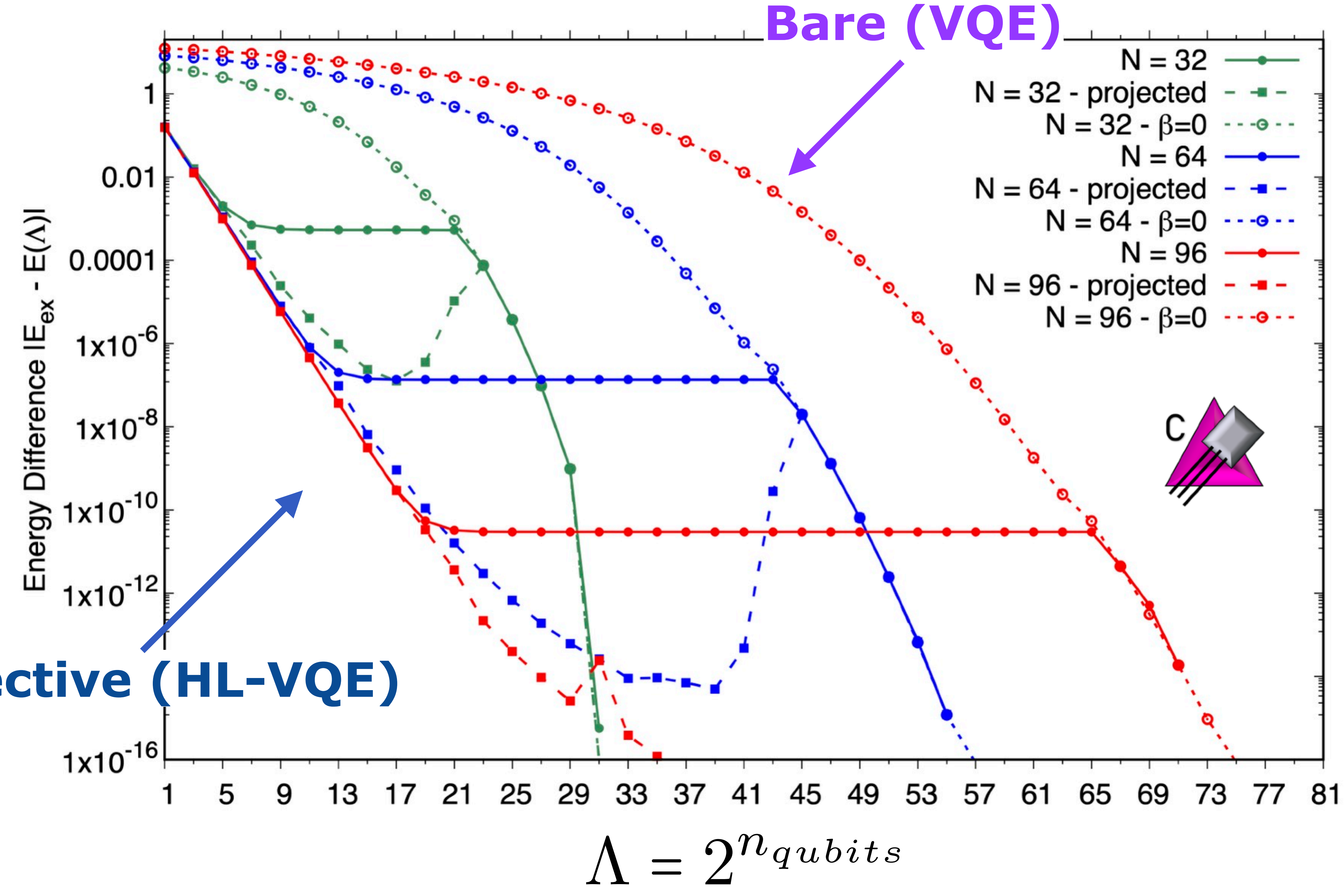
CR, Savage PRC 108, 024313 (2023)

Wave function extracted from IBM quantum computer

N=30



Exponential Acceleration in the expected convergence:



Conclusions

★ Concepts of QIS bring new insights into many-body phenomena and underlying forces

→ Better understand the underlying mechanisms governing the emergence of collective behaviours and other phenomena

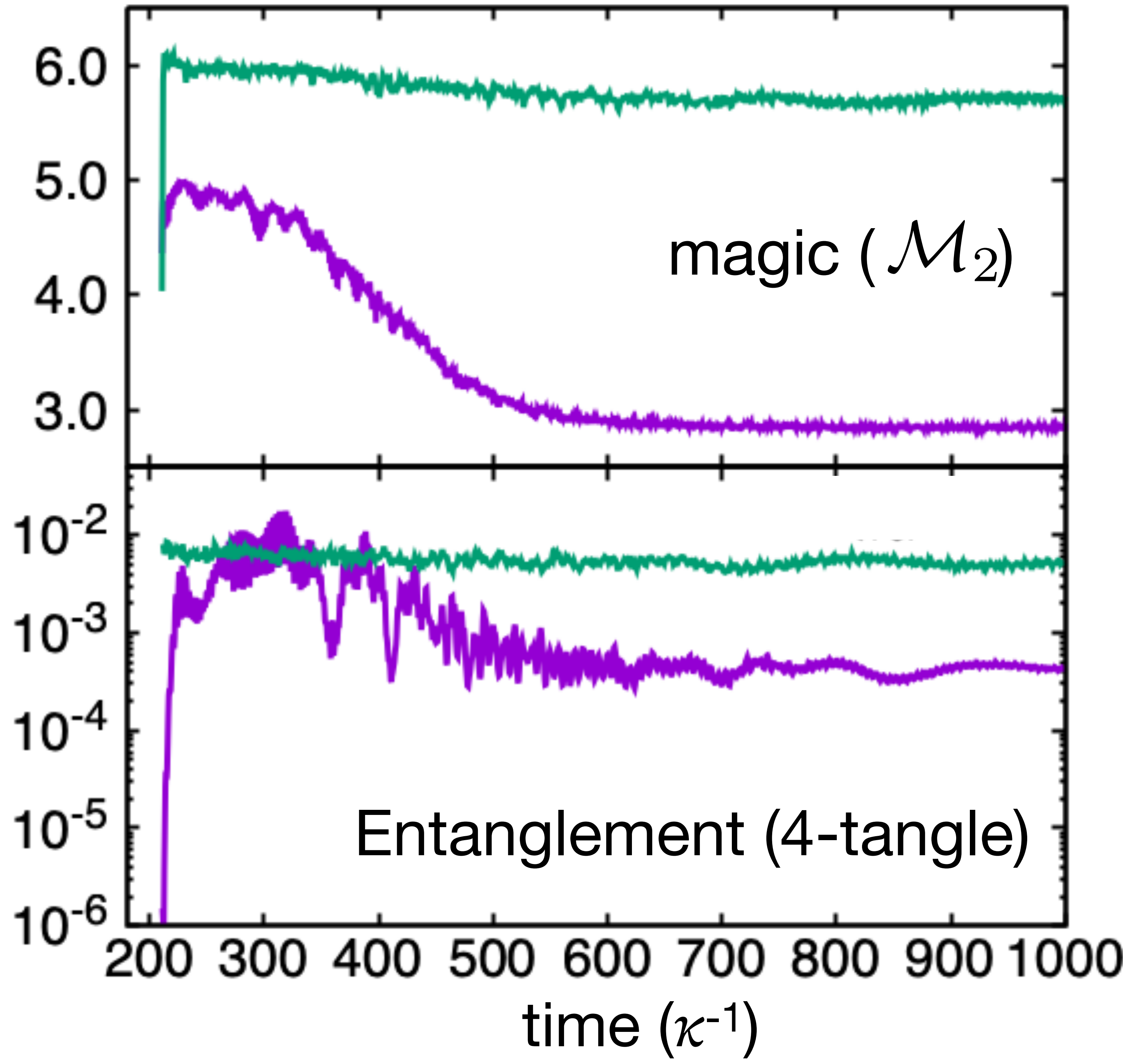
3-flavour neutrino dynamics

1-flavour $|\nu_e\nu_e\nu_e\nu_e\nu_e\rangle$

vs

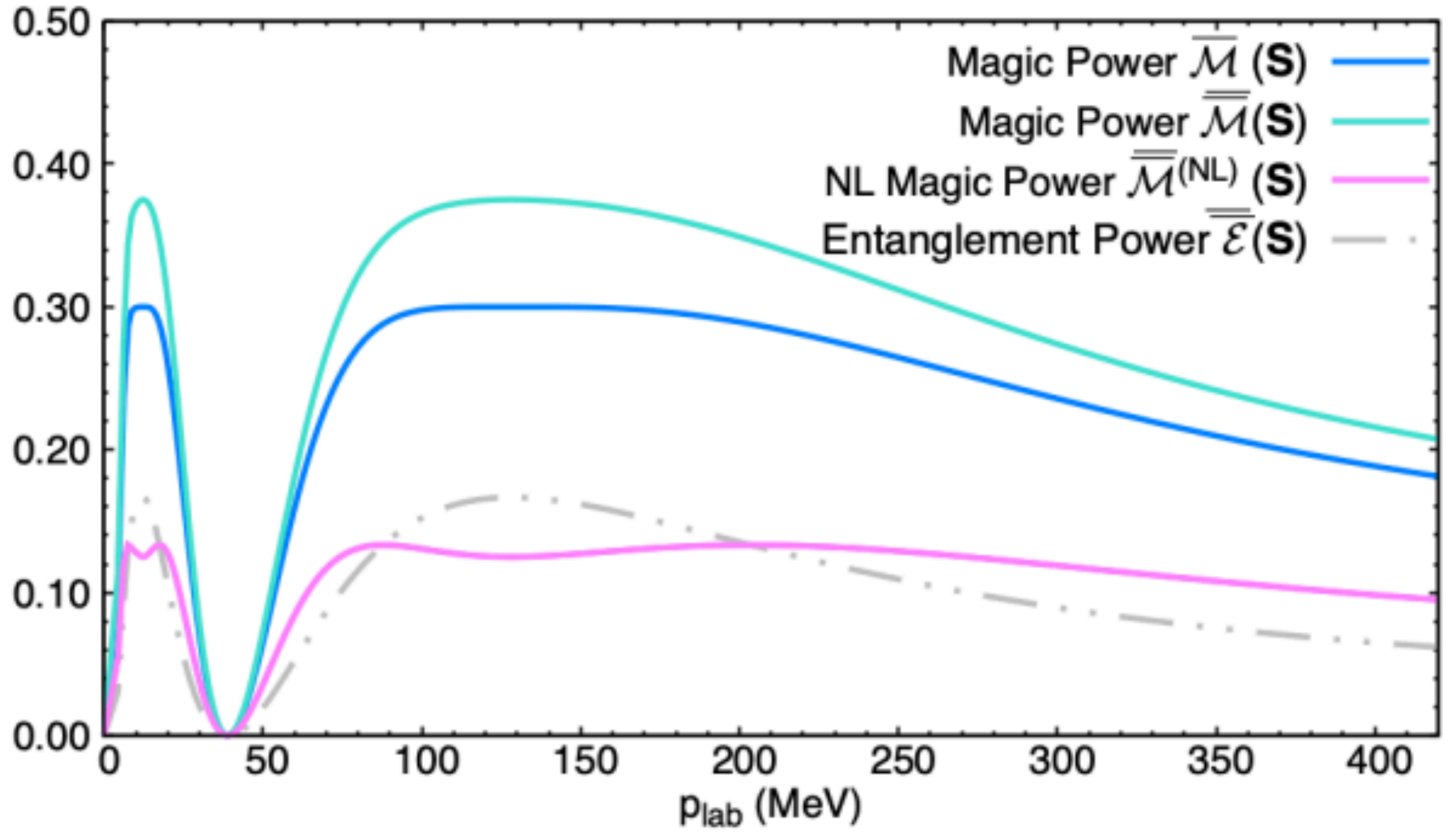
3-flavour $|\nu_e\nu_e\nu_\mu\nu_\mu\nu_\tau\rangle$

initial state



Chernyshev, CR, Savage PRR 7 023228 (2025)

Non-local magic in NN scattering:



Robin & Savage, arXiv:2510.23426

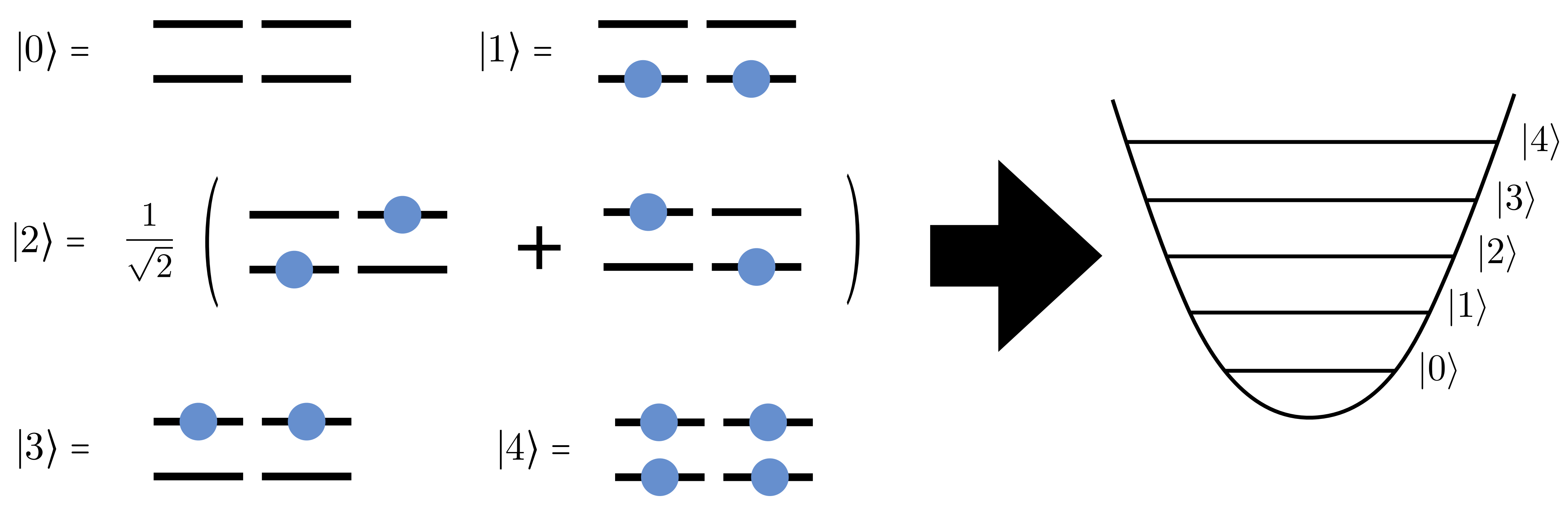
…. much more to explore!

Conclusions

★ Entanglement, Magic and Symmetries are key ingredients for designing resource efficient quantum computations of many-body structure and real-time dynamics

Symmetries & Qudits to reduce computational (gate) complexity

*e.g. Meth+ Nature Physics 21, 570–576 (2025), Calajó+ PRX Quantum **5**, 040309 (2024), Illa, CR, Savage, PRC 108, 064306 (2023), & PRD 110, 014507 (2024), Kürkçüoglu+ arXiv:2410.16414, Fromm+ EPJ Quant. Technol. **12**, 92 (2025), Turro+ PRD 111, 043038 (2025)…*

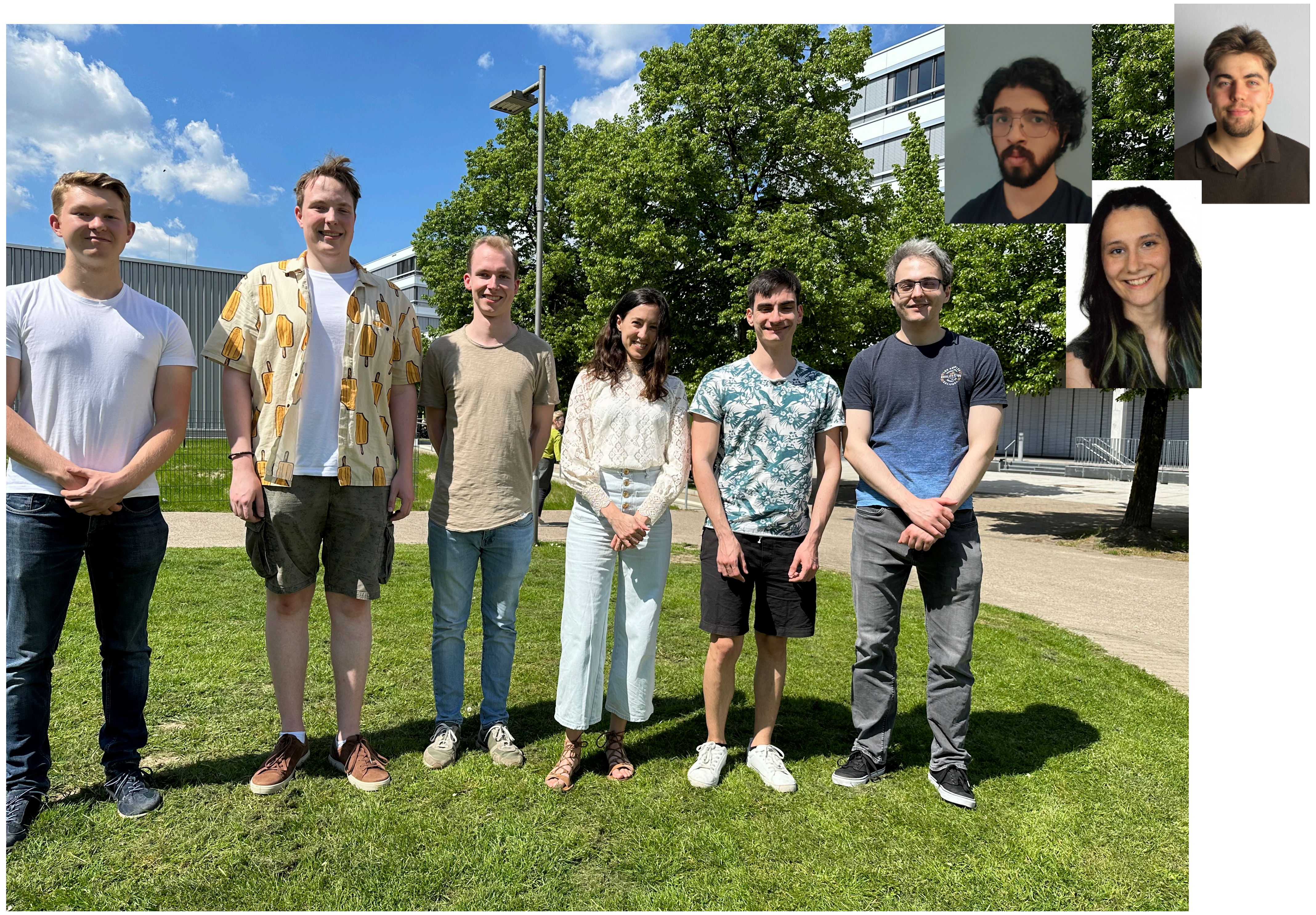


Agassi model with qu5its
Illa, CR, Savage, PRC 108, 064306 (2023)

→ Combine these aspects to develop efficient algorithms for NISQ and fault-tolerant quantum computers

Thanks to Collaborators!

Uni Bielefeld group



From left to right:
Erik Müller, Florian Brökemeier, Momme Hengstenberg, CR, Federico Rocco, James Keeble, Niranjan Jayakrishnan, Elisabeth Hahm, Felix Schunder

IQuS @ UW Seattle

Martin Savage

Marc Illa
(Now at PNNL)

Ivan Chernyshev
(Now at LANL)

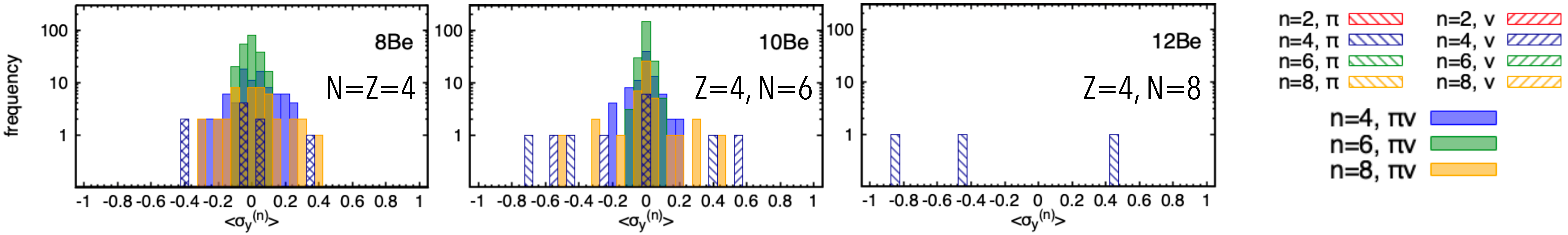
Thank You!

Backup

Multi-Body Entanglement in p-shell (12-qubit) Nuclei

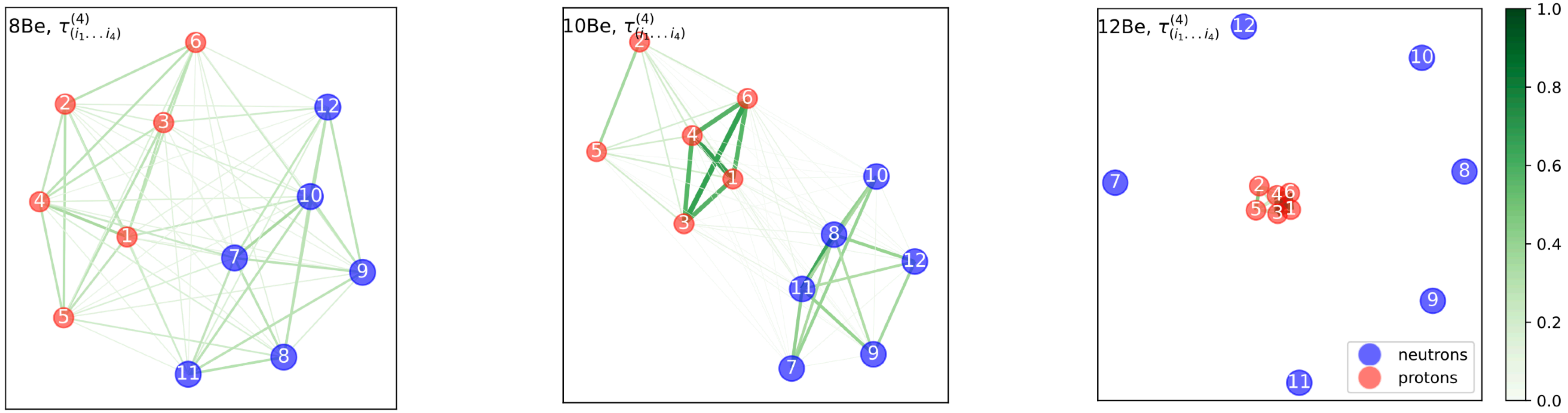
Brökemeier, CR+, PRC 111, 034317 (2025)

✶*Distribution of the $\sigma_y^{\otimes n}$ Pauli strings expectation values*



- proton-neutron entanglement is more collective than pure proton or neutron entanglement
- large proton-neutron 8-tangles → hint of alpha correlations?

✶*Network plots*

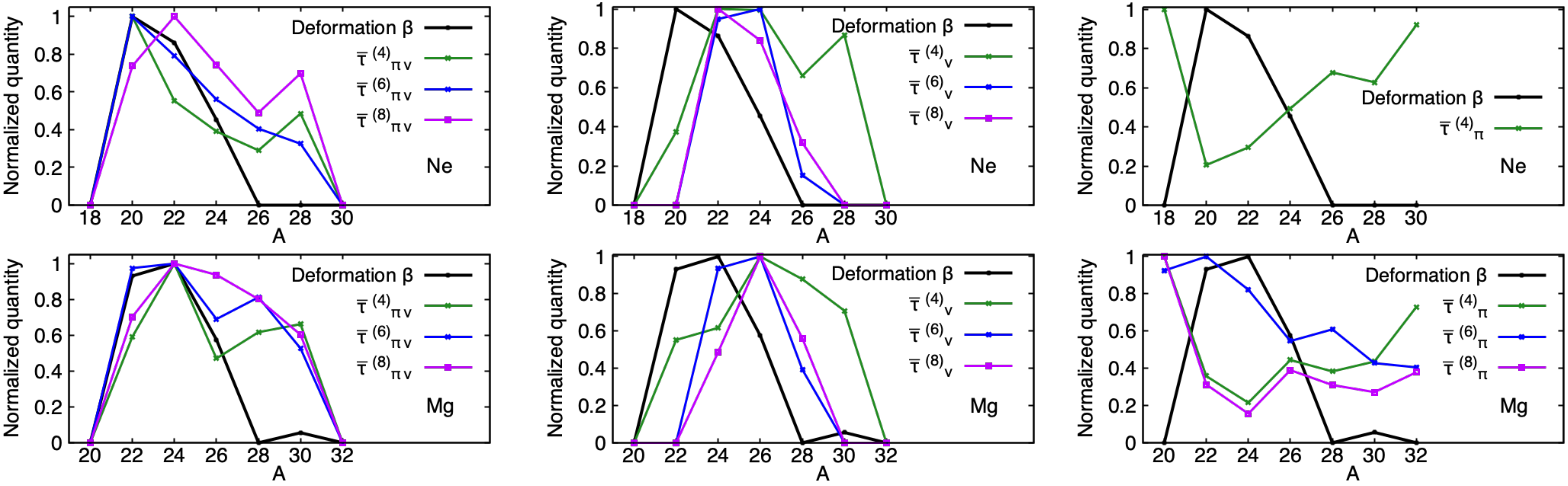


edge values

$$e^{(4)}_{i_1 i_2} = \sum_{i_3 < i_4} \tau^{(4)}_{(i_1,i_2,i_3,i_4)}$$

Protons become more entangled as neutrons are added → in accordance with Pérez-Obiol+ Eur. Phys. J. A 59, 240 (2023)

Multi-Partite Entanglement in Shell-Model Nuclei



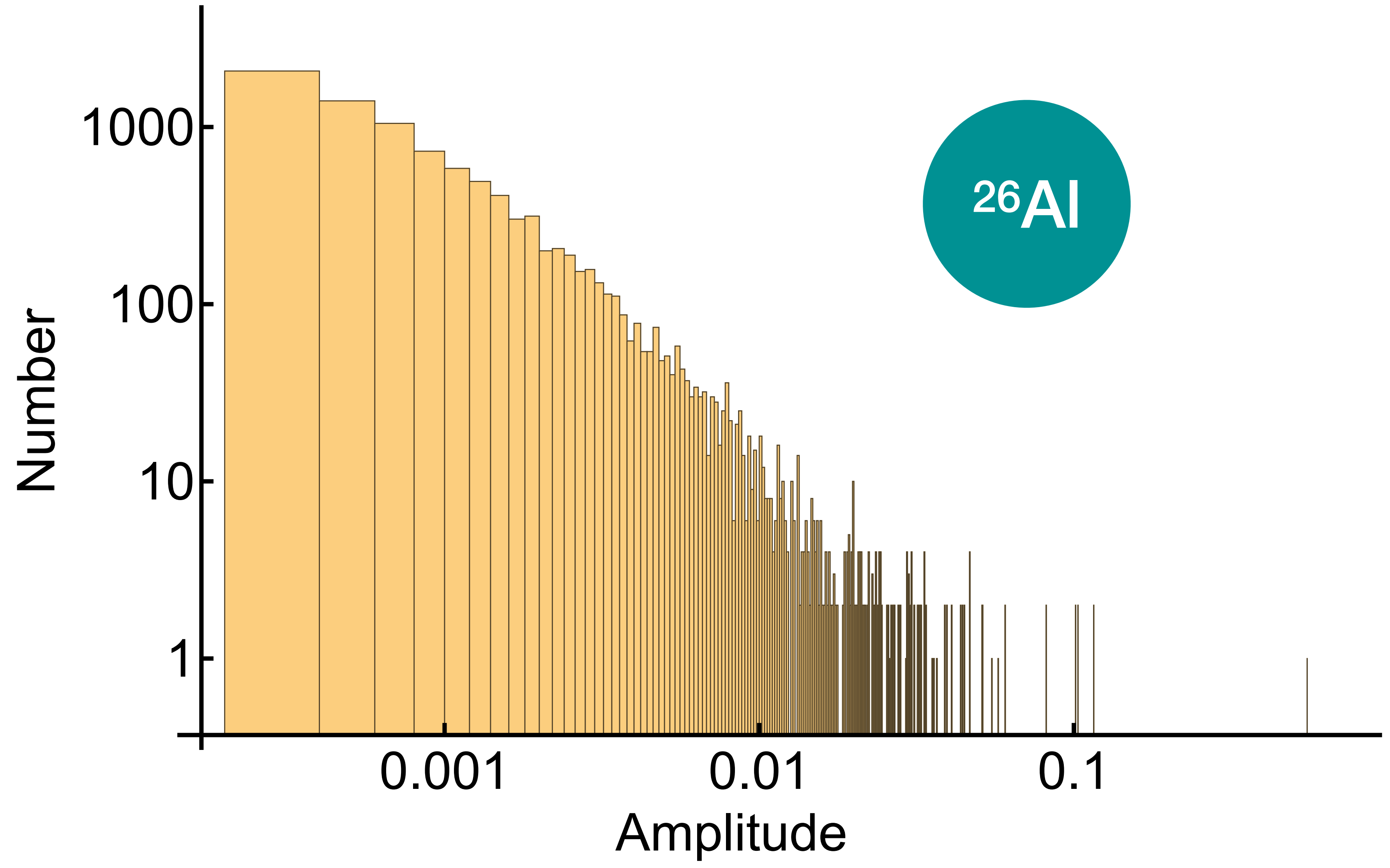
PRC 111, 034317 (2025)

Magic in sd-shell nuclei

- SREs still require $d^2 = 4^{n_{qubits}}$ expectation values

- MCMC techniques can be used to compute SREs in large systems *Tarabunga et al PRX Quantum 4, 040317 (2023)*

- However the distribution of amplitudes in the wave function of collective nuclei slows down the convergence of MCMC

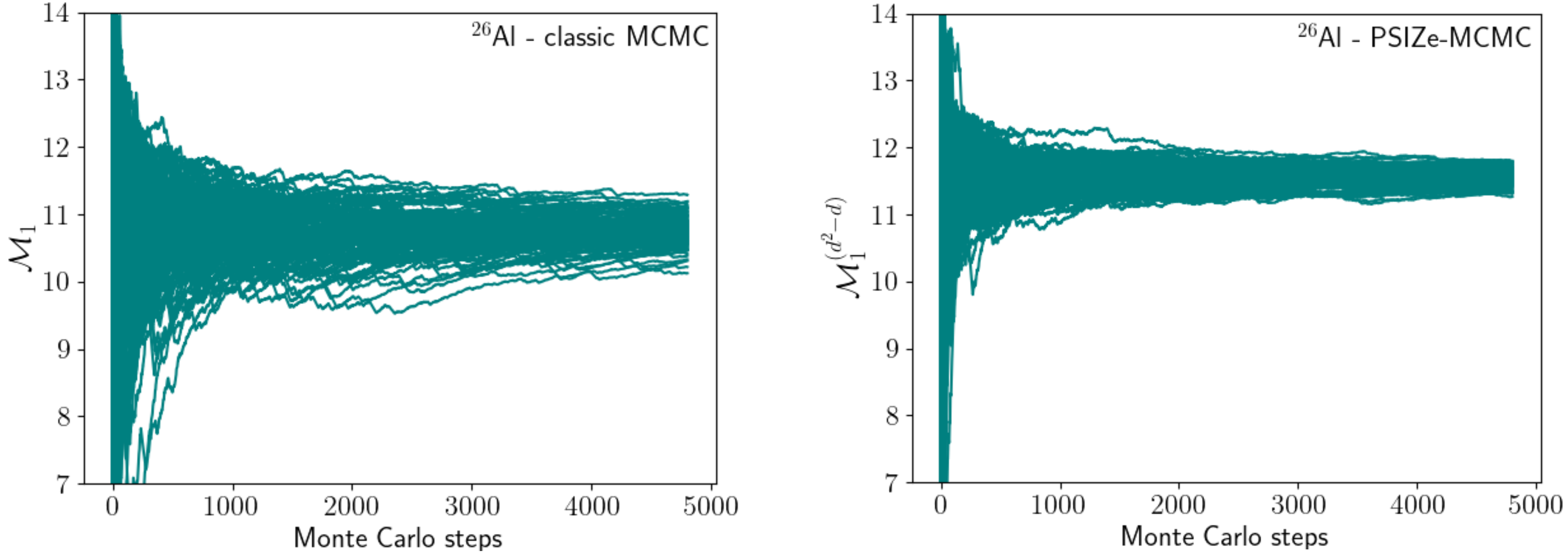


➡ "Pauli-String IZ exact MCMC" (PSIZe-MCMC) algorithm:

Expectation values of IZ strings computed exactly, MCMC samples to remaining space

Brökemeier, Hengstenberg, Keeble, CR, Rocco & Savage, arXiv:2409.12064

Magic in Nuclei: PSIZe-MCMC algorithm

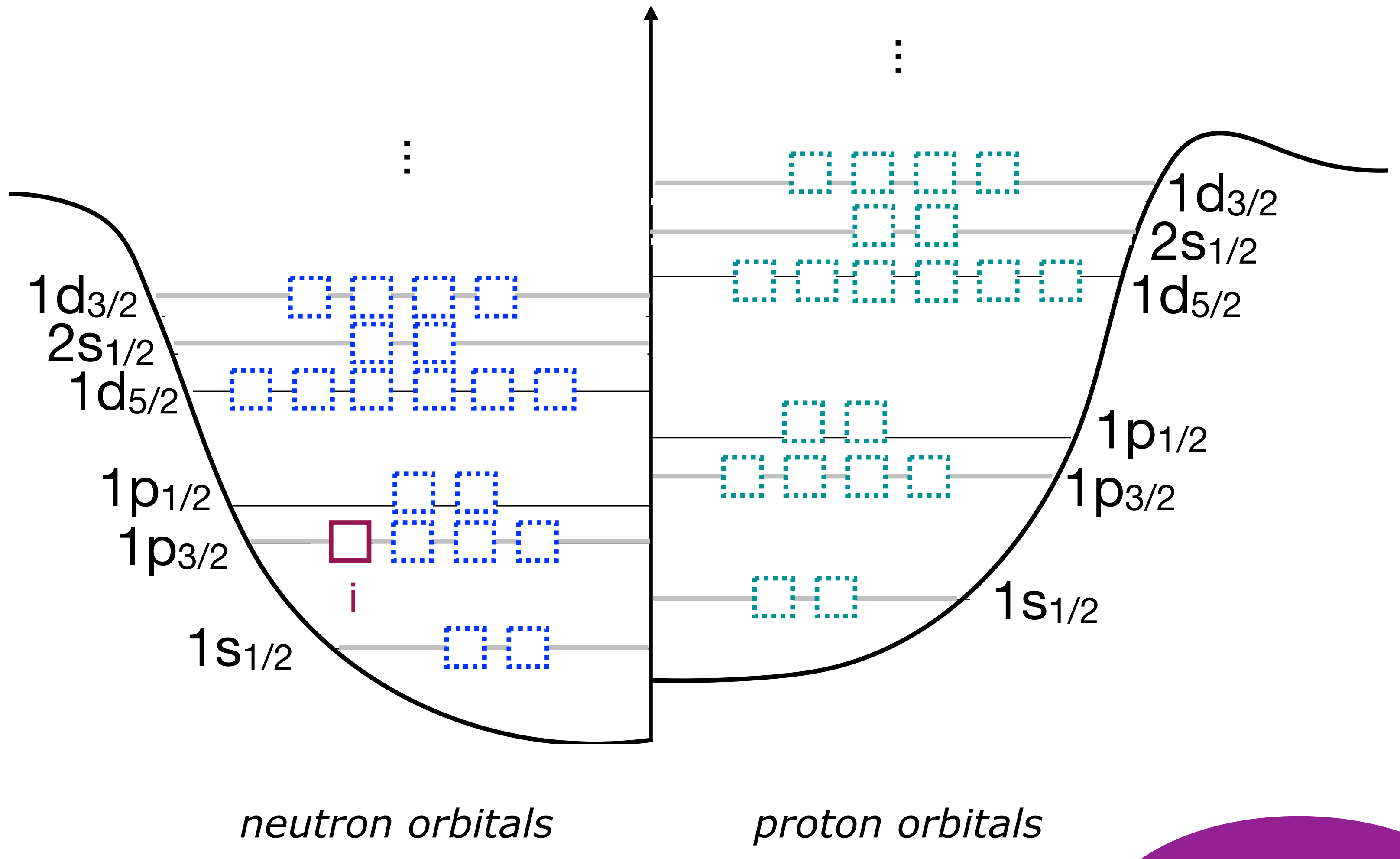


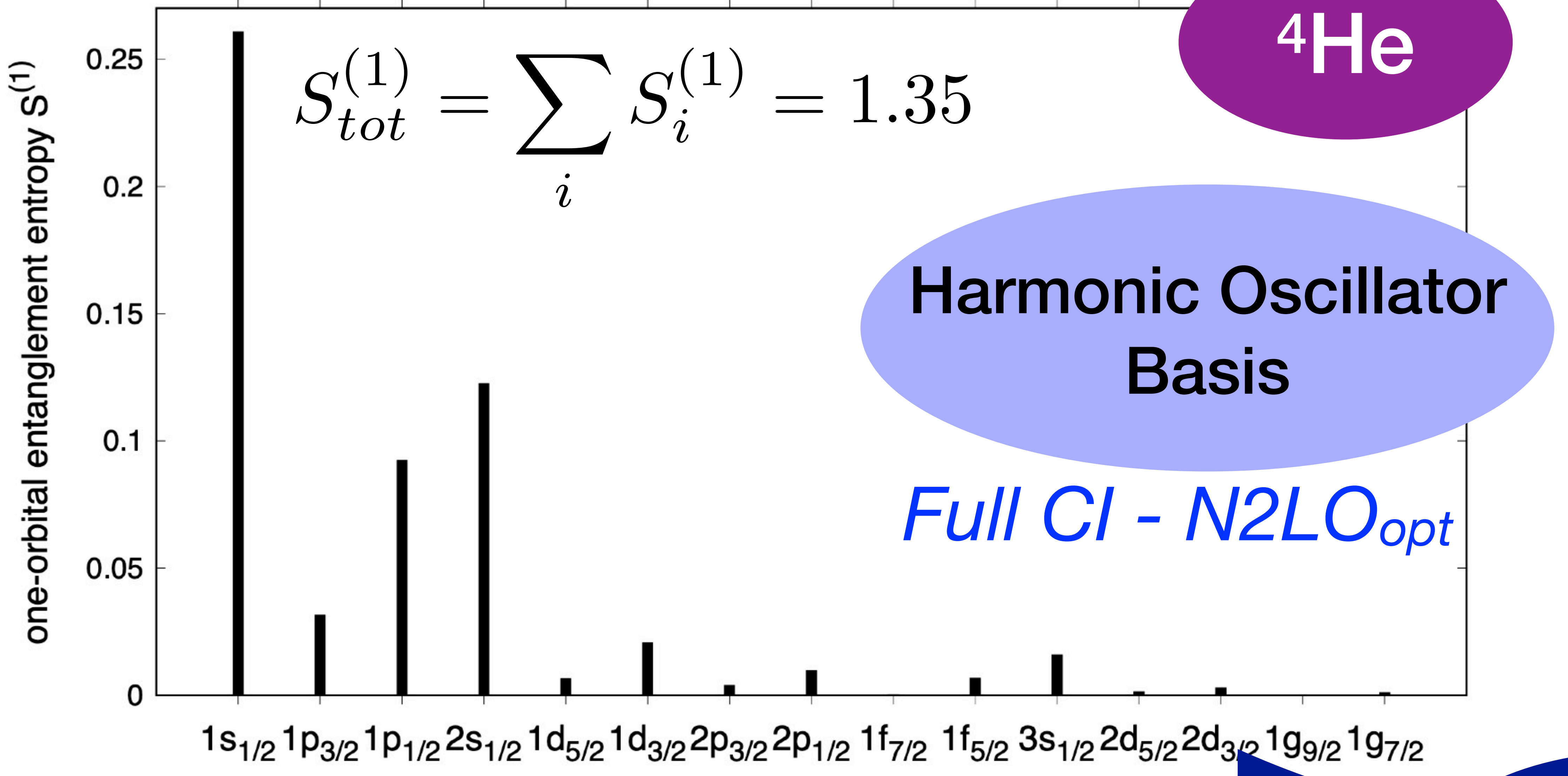
arXiv:2409.12064

Entanglement Rearrangement In Nuclei

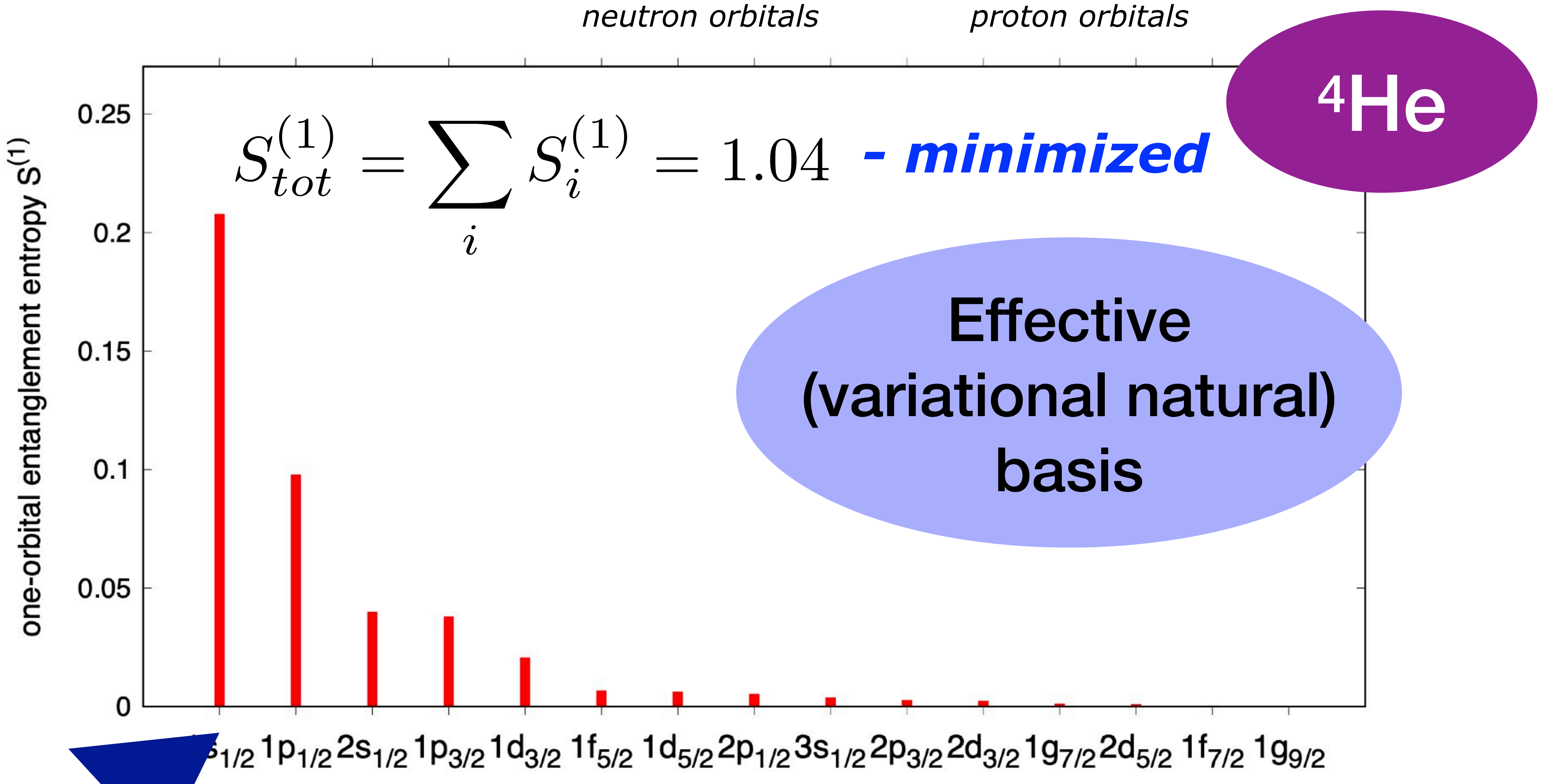
Single-orbital Von Neumann entropy:

$$S^{(1)}_{(i)} = -\mathrm{Tr}\left[\rho^{(i)} \ln \rho^{(i)}\right]$$





$$\hat{U} = e^{i\hat{T}}$$



$$\hat{T} = \sum_{ij} T_{ij} a^{\dagger}_i a_j$$

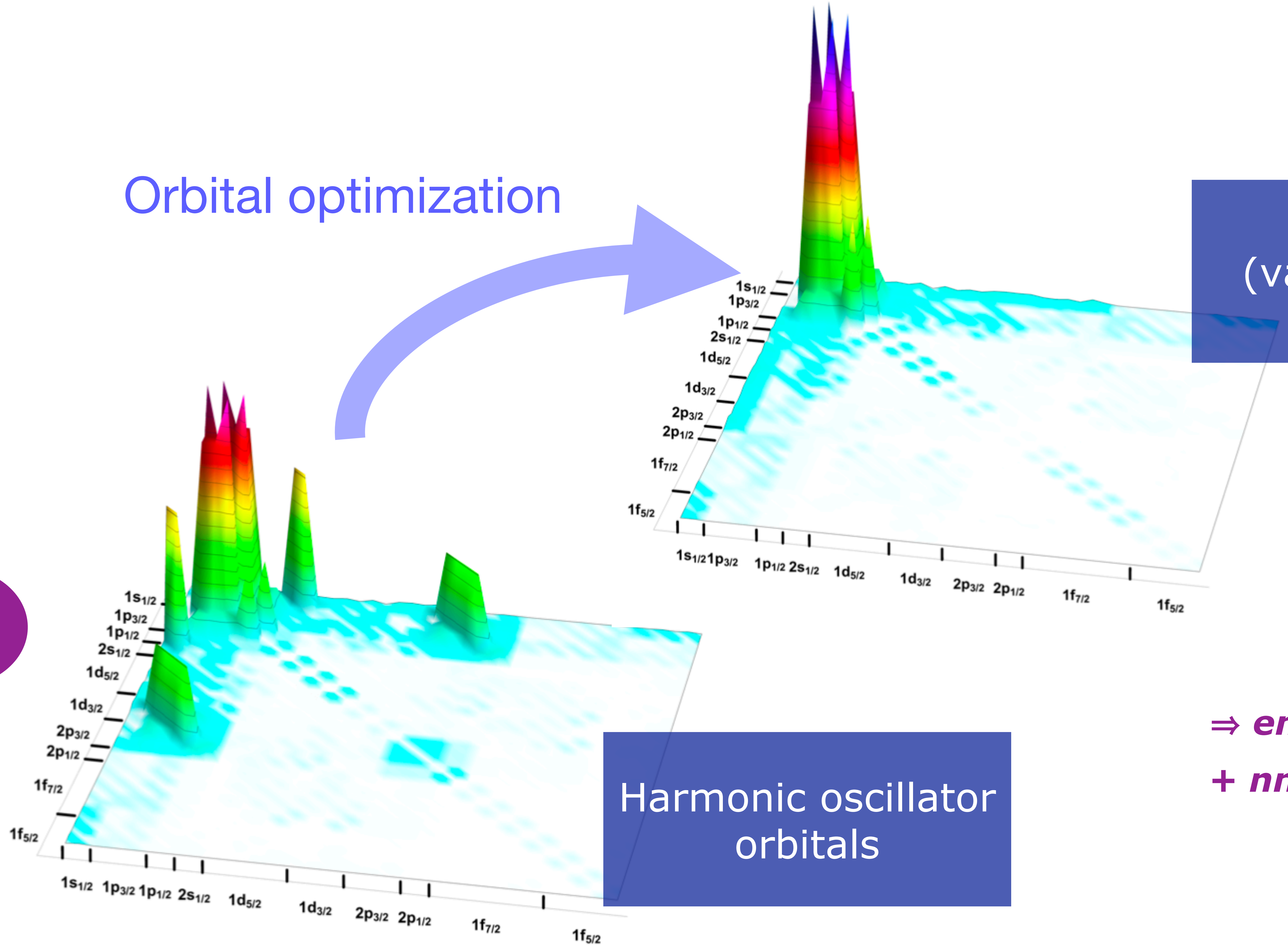
CR, Savage, Pillet, PRC 103, 034325 (2021)

Entanglement Rearrangement In Nuclei

Orbital optimization

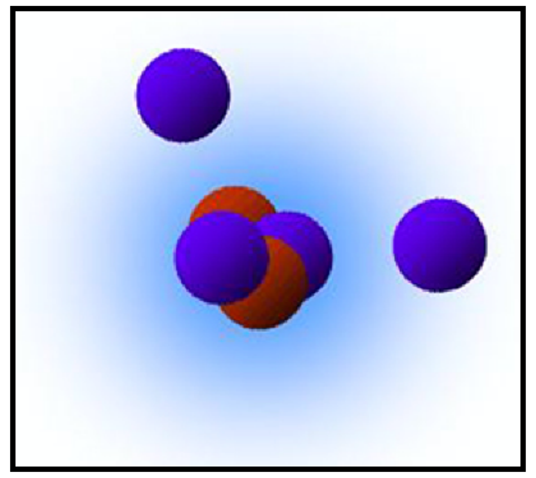
Neutron-neutron mutual information

^{6}He



Effective (variational natural) orbitals

Harmonic oscillator orbitals

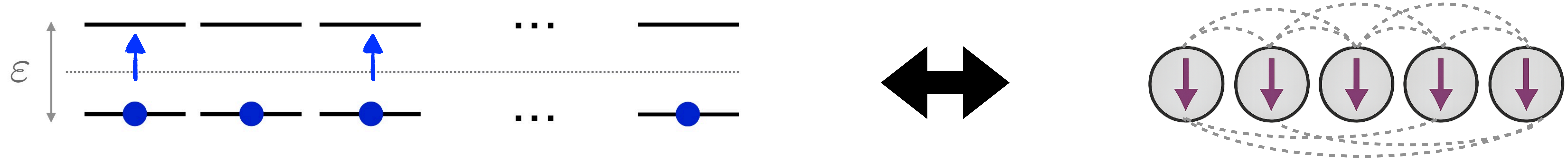


⇒ emergence of ^{4}He-core

+ nn-valence structure (?)

CR, Savage, Pillet, PRC 103, 034325 (2021)

Singling out Deformation: the Lipkin-Meshkov-Glick model



$$\hat{H} = \varepsilon \hat{J}_z - V_x(\hat{J}_x^2 + \chi \hat{J}_y^2)$$

$$\hat{J}_\alpha = \frac{1}{2}\sum_{i=1}^{N} \hat{\sigma}_\alpha^{(i)} \;, \quad \alpha = x, y, z \qquad \chi \in [-1, 0)$$

✳ Phase Transition: doubly degenerate ground state of mixed parity (N→∞)

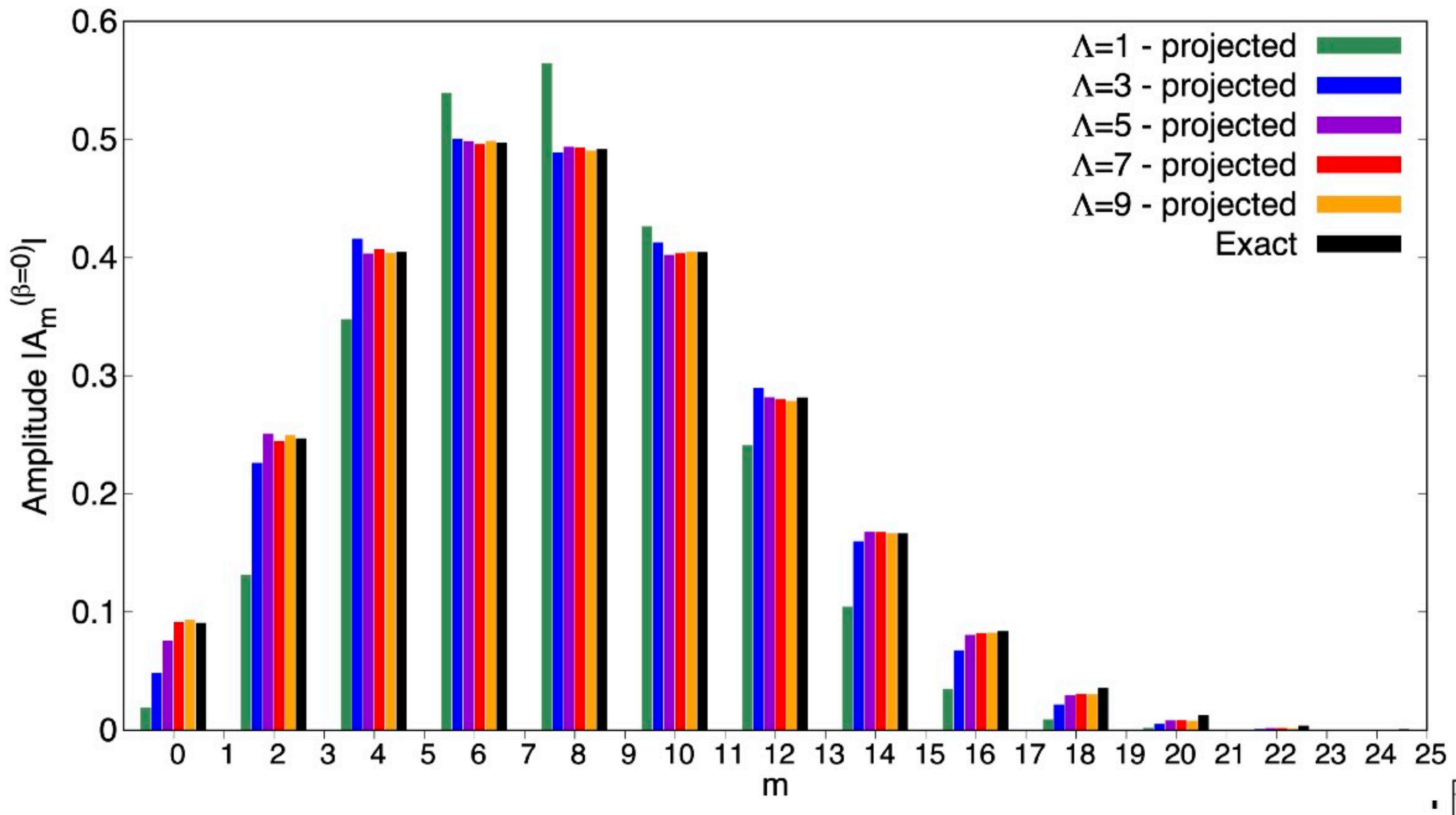
✳ Relevance for nuclear physics, condensed matter, trapped-ion quantum computing, quantum sensing/metrology…

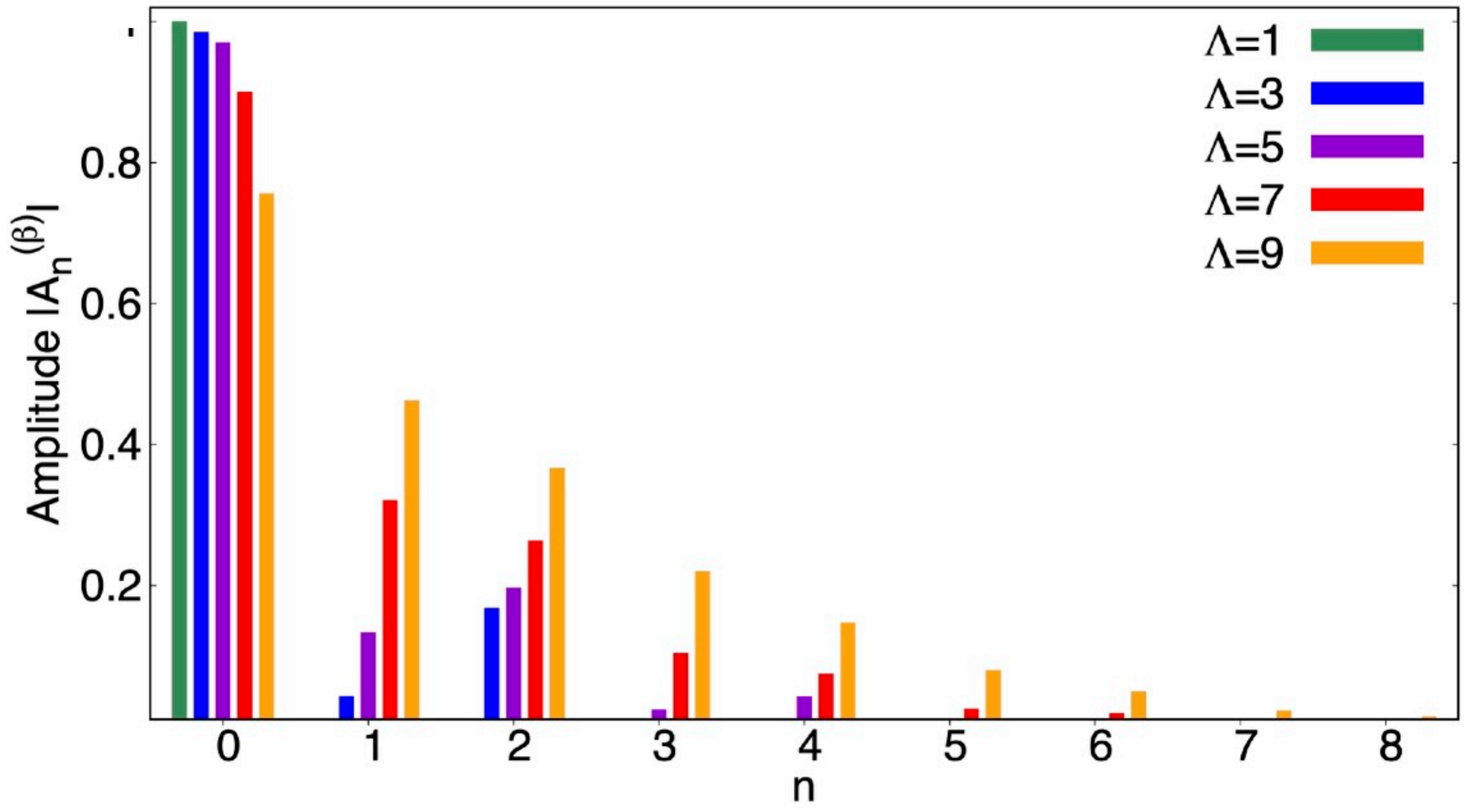
Lipkin, Meshkov, Glick, Nucl. Phys. 62, 188 (1965)

Information and Entanglement Rearrangement in Truncated Model Spaces

Basis optimization/Effective Hamiltonian techniques $\hat{H} \rightarrow \hat{U}(\vec{\beta})\hat{H}\hat{U}(\vec{\beta})^\dagger$

★ In the LMG model: $\hat{U}(\beta) = \prod_i \exp(-i\sigma_y\beta)$

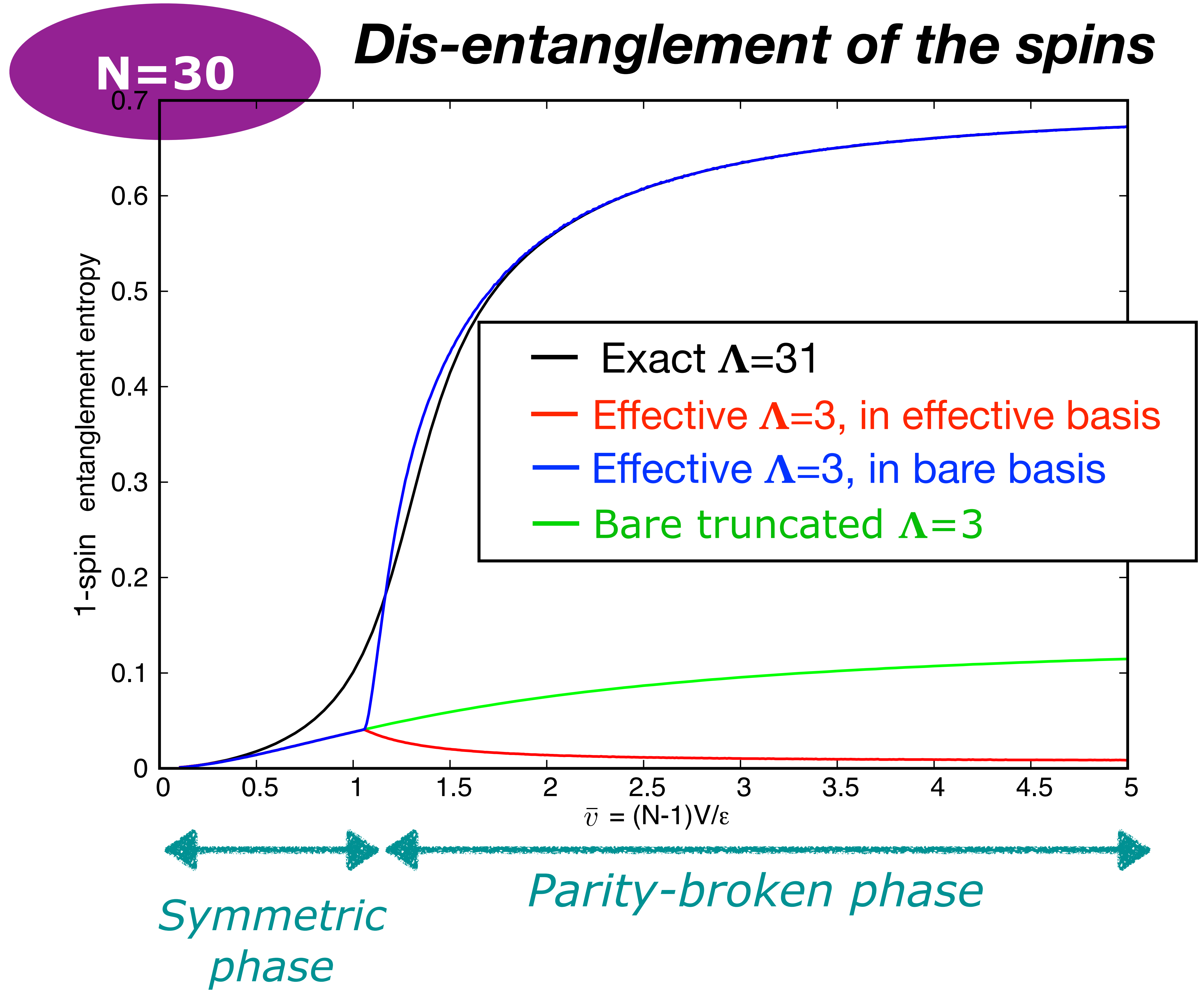




Localization of the many-body wave function in the Hilbert space

Λ= cut off on many-body configurations

Dis-entanglement of the spins



Hengstenberg, CR, Savage EPJA 59, 231 (2023)

CR, Savage PRC 108, 024313 (2023)

Beyond stabilizer Ground State with Magic Injection

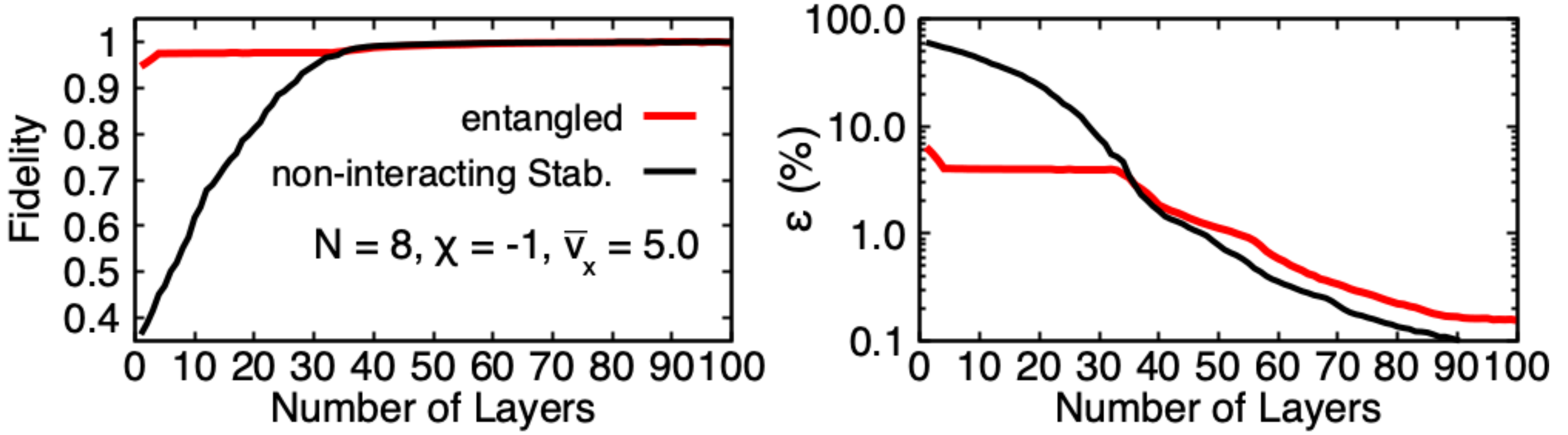
* Grimsley+ Nature Comm. 10, 3007 (2019)

★ *ADAPT-VQE **

$$|\Phi(\theta_1, ..., \theta_L)\rangle = \prod_{l=1}^{L} \mathrm{e}^{i\theta_l \hat{T}_l} |\phi\rangle \qquad \frac{\partial E}{\partial \theta_l}_{|\theta_l=0} = -i\langle \Phi_{l-1}| \left[\hat{T}_l, \hat{H}\right] |\Phi_{l-1}\rangle$$

For the LMG model: $\hat{T}_{ij}^{\pm} = X_i Y_j \pm Y_i X_j$

CR arXiv:2505.02923



- Long plateaux due to very small energy gradients for many iterations
- Destroys the coherence, and rebuilds it

Symmetry-guided mapping of the Agassi model onto qudit systems

★ Time evolution — circuits for simulations using qu5its

- **Hamiltonian mapping to qu5its:**

$$\hat{H} = \sum_{j=1}^{\Omega/2} \sum_{j' \neq j=1}^{\Omega/2} H^{(2)}_{(jj')}$$

Acts on 2 qu5its j, j'

$$\begin{aligned} H^{(2)} &\equiv \sum_a \hat{H}^{(2,a)} \\ &= \left[\varepsilon \hat{j}_z - (V+g)\hat{\mathcal{X}}_{13} - g\hat{N}_{\text{pairs}}\right] \otimes \hat{I}_5 \\ &\quad + \hat{I}_5 \otimes \left[\varepsilon \hat{j}_z - (V+g)\hat{\mathcal{X}}_{13} - g\hat{N}_{\text{pairs}}\right] \\ &\quad - V \sum_{r,s \in \{(12),(23)\}} \left(\hat{\mathcal{X}}_r \otimes \hat{\mathcal{X}}_s - \hat{\mathcal{Y}}_r \otimes \hat{\mathcal{Y}}_s\right) \\ &\quad - \frac{g}{2} \sum_{\substack{r,s \in \{(01),(03),\\ -(14),-(34)\}}} \left(\hat{\mathcal{X}}_r \otimes \hat{\mathcal{X}}_s + \hat{\mathcal{Y}}_r \otimes \hat{\mathcal{Y}}_s\right) \end{aligned}$$

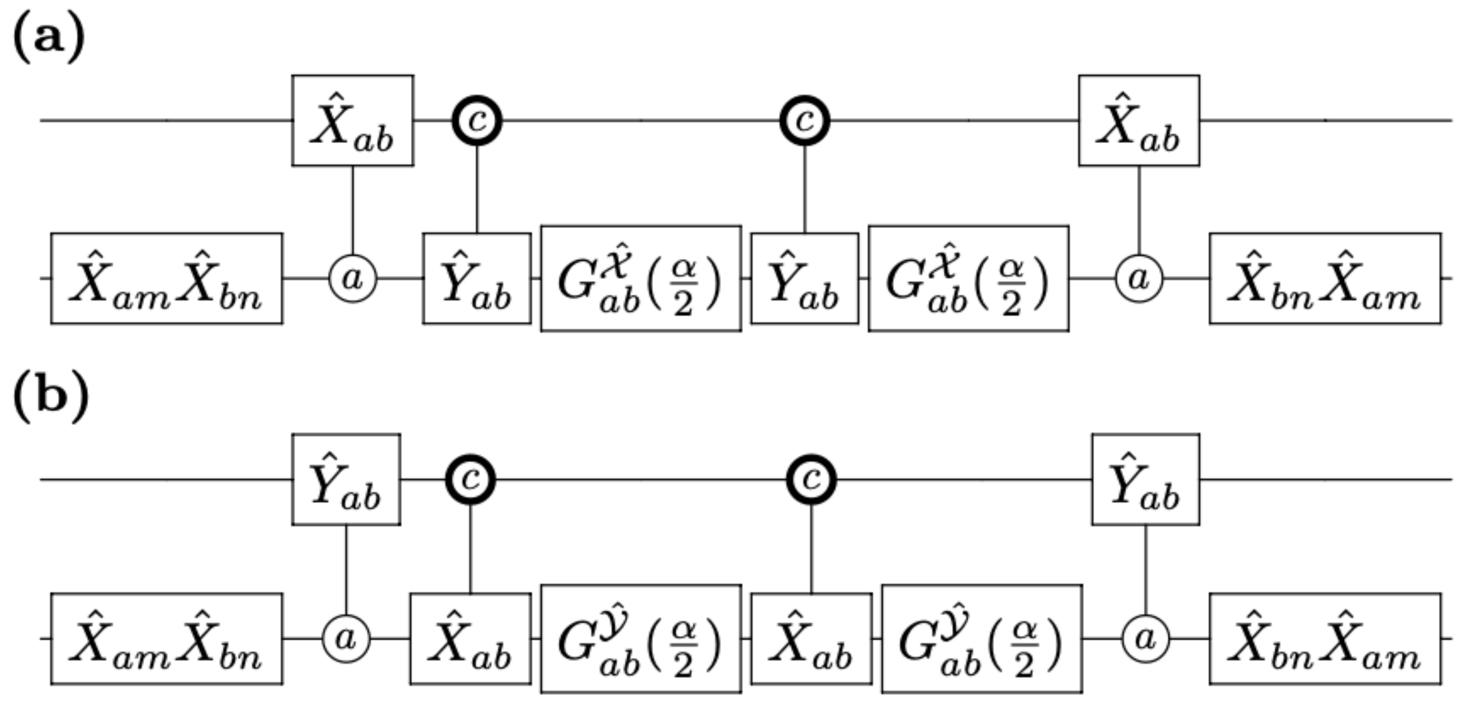
generators of Givens rotations

$$G^{\mathcal{X}\mathcal{X}}_{abmn}(\alpha) \qquad G^{\mathcal{Y}\mathcal{Y}}_{abmn}(\alpha)$$

- **Trotter decomposition at leading order:**

$$\hat{U}(t) = e^{-i\hat{H}t} \simeq \left(e^{-i\hat{H}\Delta t}\right)^{n_{Trot}}$$

$$e^{-i\hat{H}\Delta t} = e^{-i\sum_{jj'} \hat{H}^{(2)}_{jj'}\Delta t} \simeq \prod_{jj'} \prod_a e^{-i\hat{H}^{(2,a)}_{jj'}\Delta t}$$



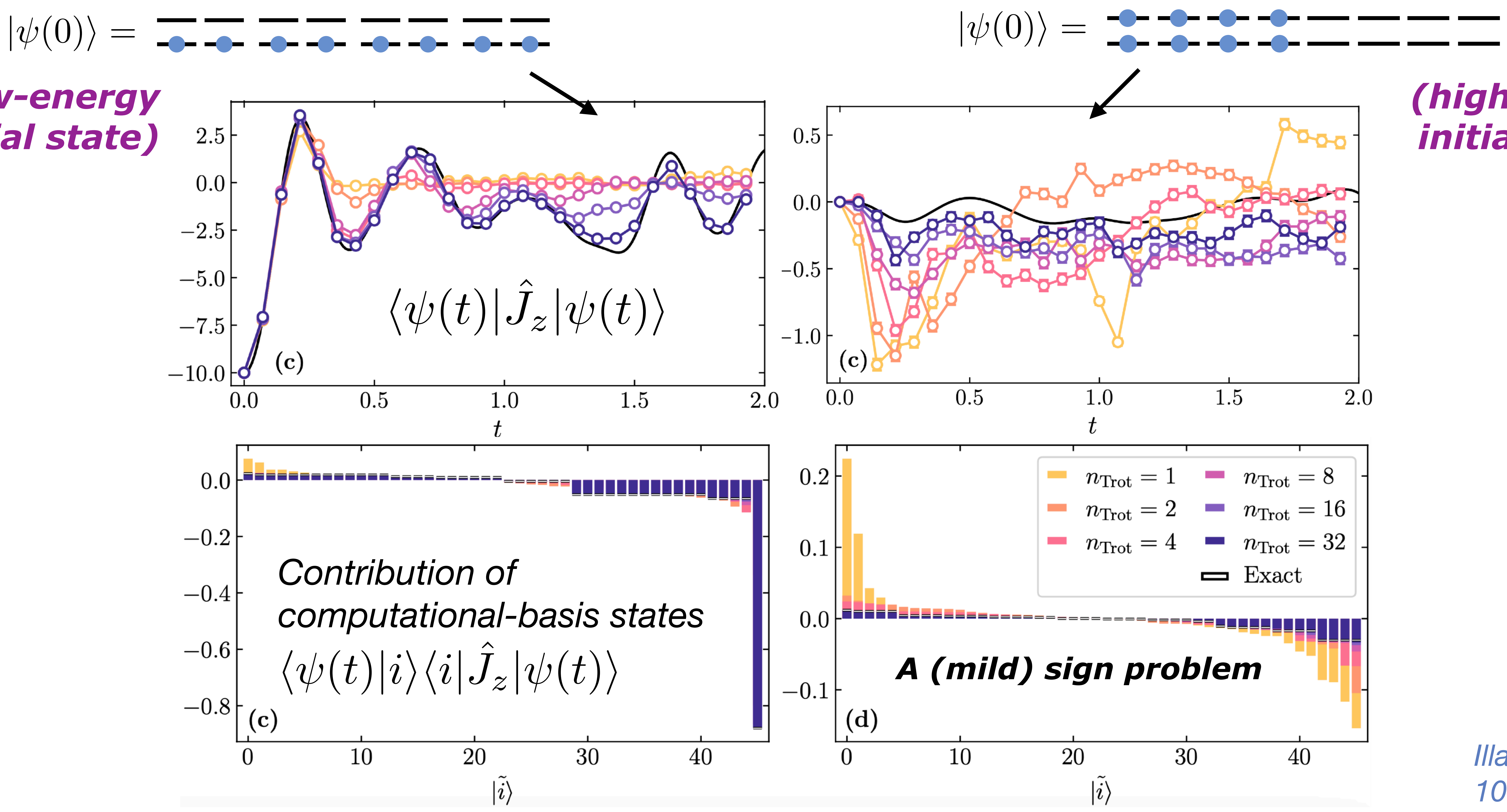
Circuits for $G^{\mathcal{X}\mathcal{X}}_{abmn}(\alpha)$ and $G^{\mathcal{Y}\mathcal{Y}}_{abmn}(\alpha)$

Dynamics simulations of the Agassi model: low- vs high-energy initial states

★ ***Trotterized time evolution on a qudit-system simulator using Google's `cirq` software:***

(low-energy initial state)

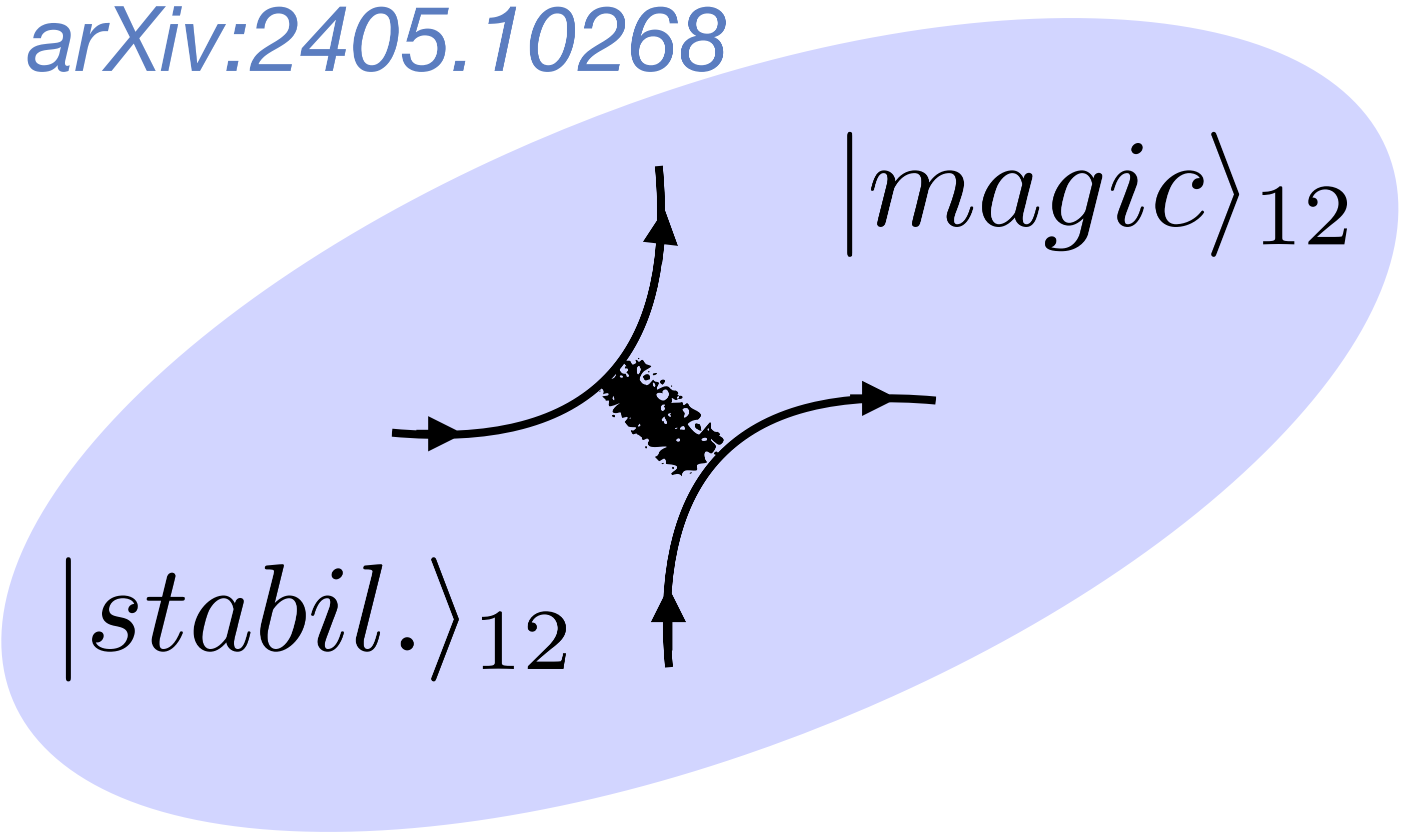
(high-energy initial state)



Illa, CR, Savage PRC 108, 064306 (2023)

The Magic Power of Nuclear and Hyper-Nuclear Forces

CR & M. J. Savage arXiv:2405.10268



Magic power of the S-matrix:

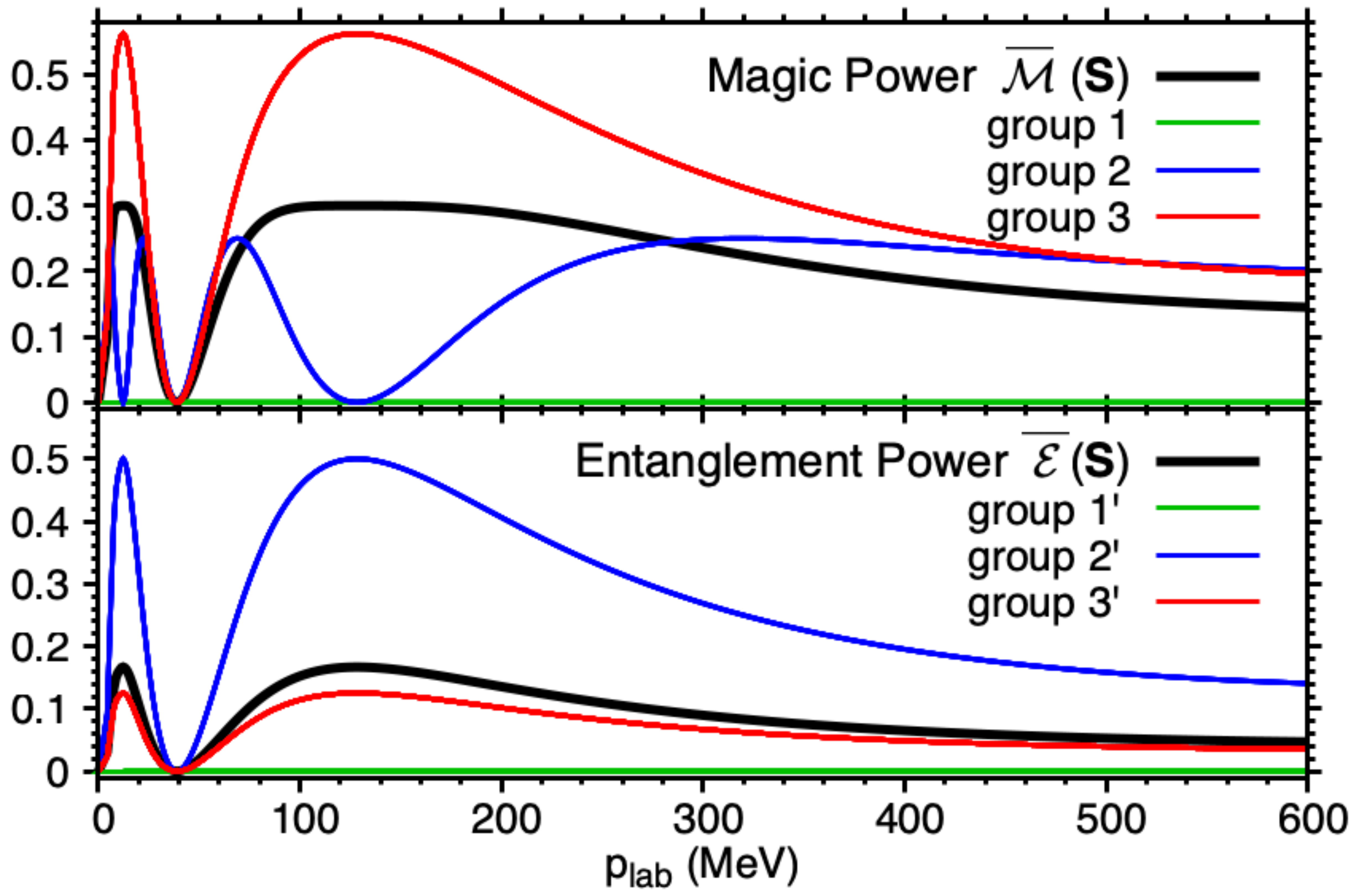
$$\overline{\mathcal{M}}(\hat{\mathbf{S}}) \equiv \frac{1}{\mathcal{N}_{ss}} \sum_{i=1}^{\mathcal{N}_{ss}} \mathcal{M}\left(\hat{\mathbf{S}}\,|\Psi_i\rangle\right)$$

Average fluctuations in magic induced by the S-matrix

Entanglement power of the S-matrix

$$\overline{\mathcal{E}}(\hat{\mathbf{S}}) \equiv \frac{1}{\mathcal{N}_{ss}^{TP}} \sum_{i=1}^{\mathcal{N}_{ss}^{TP}} \mathcal{E}\left(\rho_i^{(1)}(\hat{\mathbf{S}})\right)$$

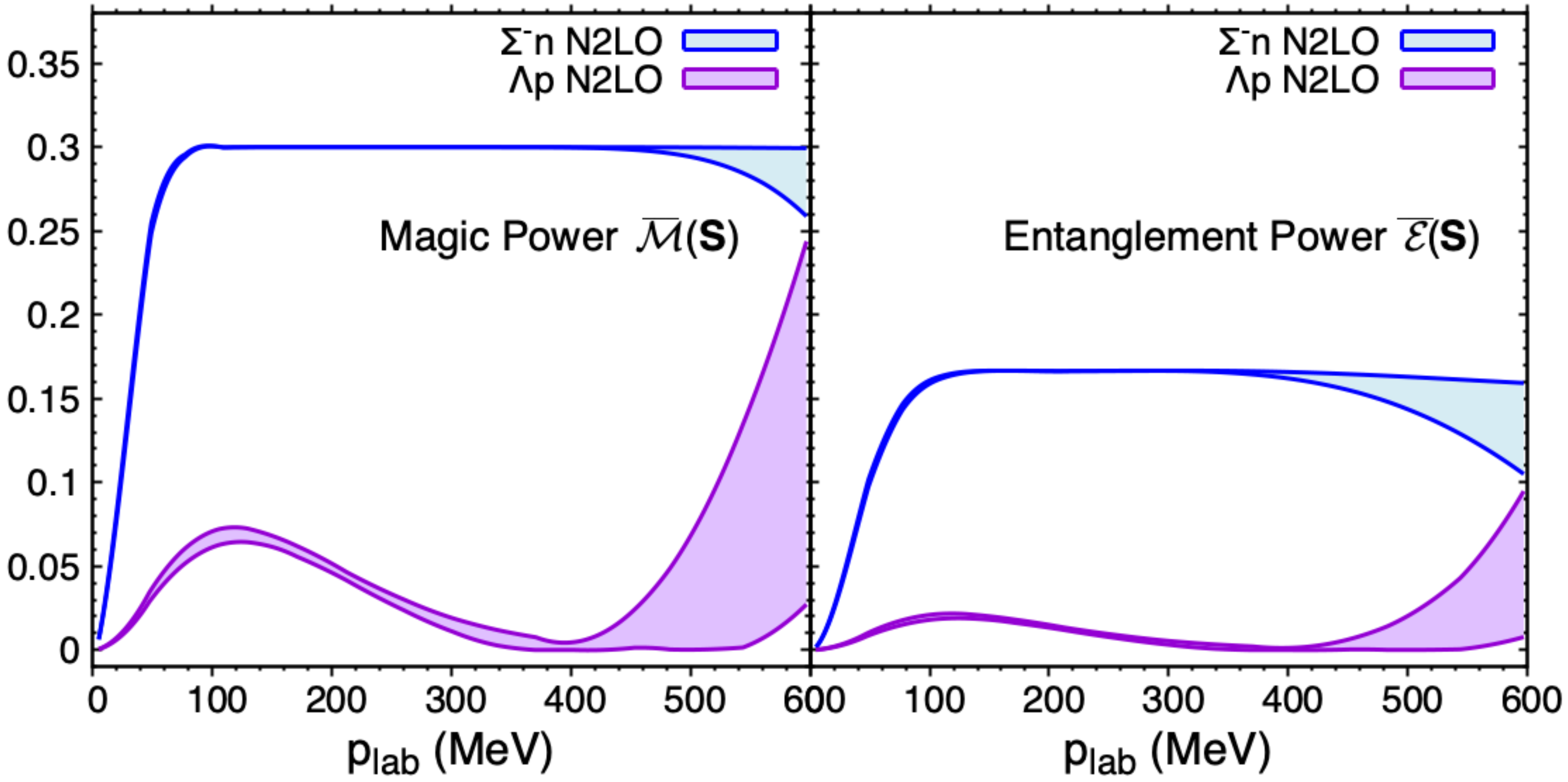
$$\overline{\mathcal{M}}(\hat{\mathbf{S}}) = \frac{3}{20}\Big(3+\cos(4\,\Delta\delta)\Big)\sin^2(2\,\Delta\delta) \qquad \overline{\mathcal{E}}(\hat{\mathbf{S}}) = \frac{1}{6}\sin^2(2\,\Delta\delta)$$

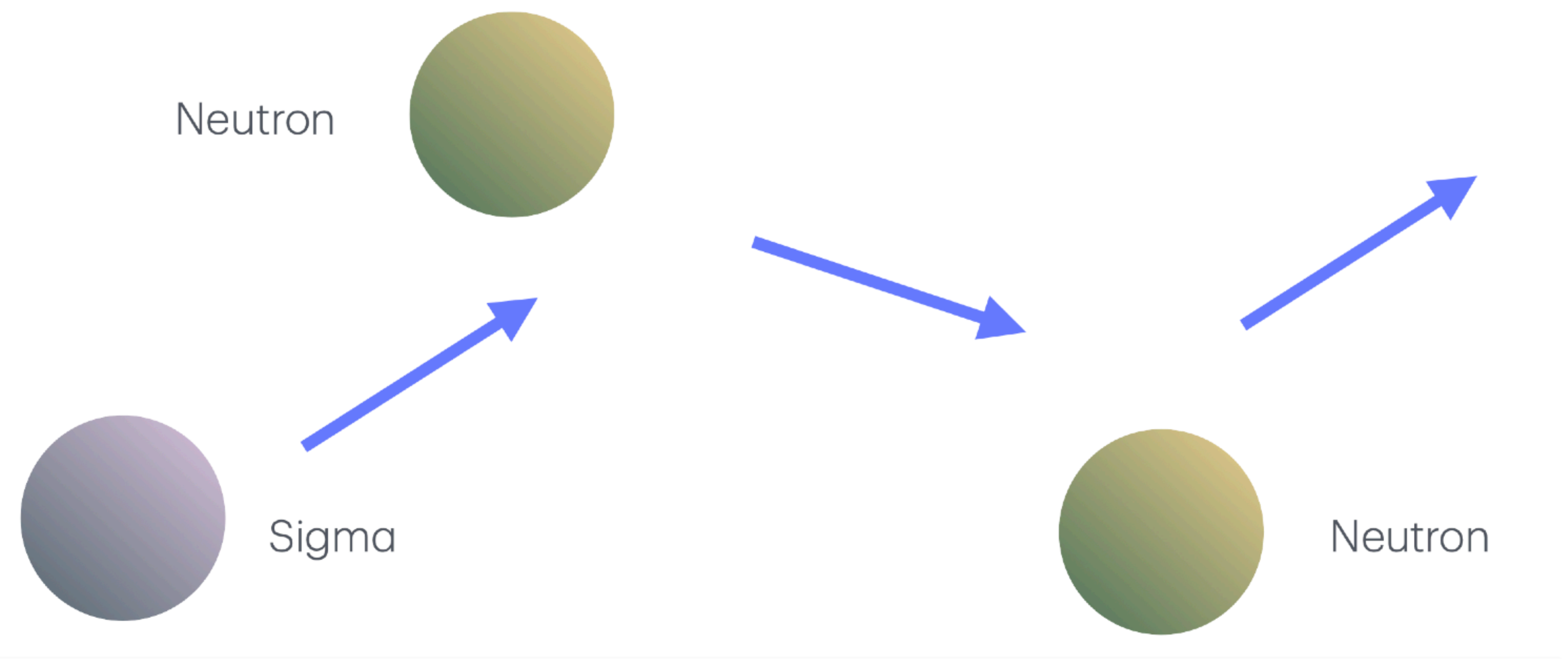


Same results as in Beane+ PRL 122, 102001 (2019) with continuous integration over spin orientations of initial tensor-product states

The Magic Power of Nuclear and Hyper-Nuclear Forces

CR & M. J. Savage arXiv:2405.10268





Magic between neutrons is induced by successive Σ^-n scatterings (assumed no decoherence)

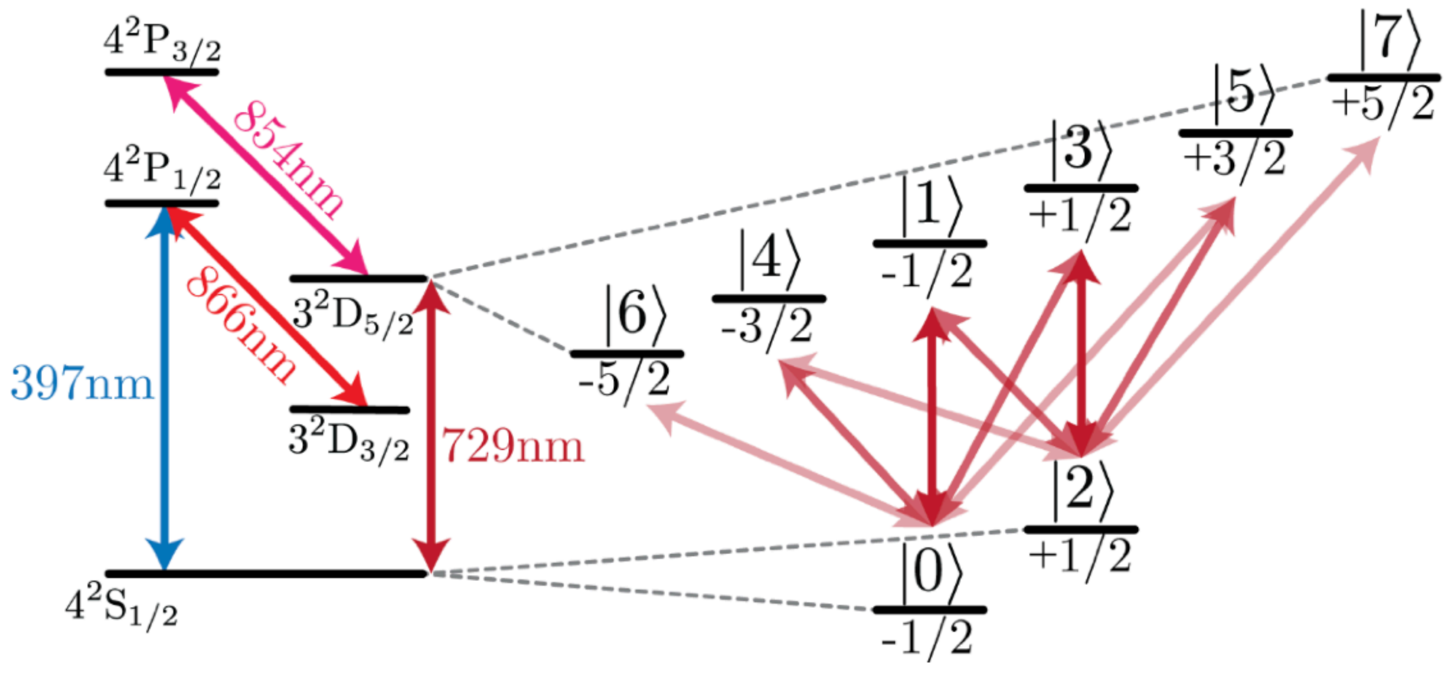
➡ Σ-baryon is identified as a potential candidate catalyst for enhanced spreading of magic and entanglement in dense matter

Entanglement power also in Beane+ PRL 122, 102001 (2019); Liu+ PLB 856, 138899 (2024)

Quantum Simulations with Qudits

Quantum Computers with qudits (d>2-level quantum systems) allow for naturally mapping symmetries of the systems of interest

Developed with trapped ions, superconducting devices and more — see e.g. Wang+, Front. Phys. 8, 589504 (2020)



Trapped ion system Innsbruck
https://www.quantumoptics.at/en/research/qudits.html

Can qudits reduce computational (gate) complexity?

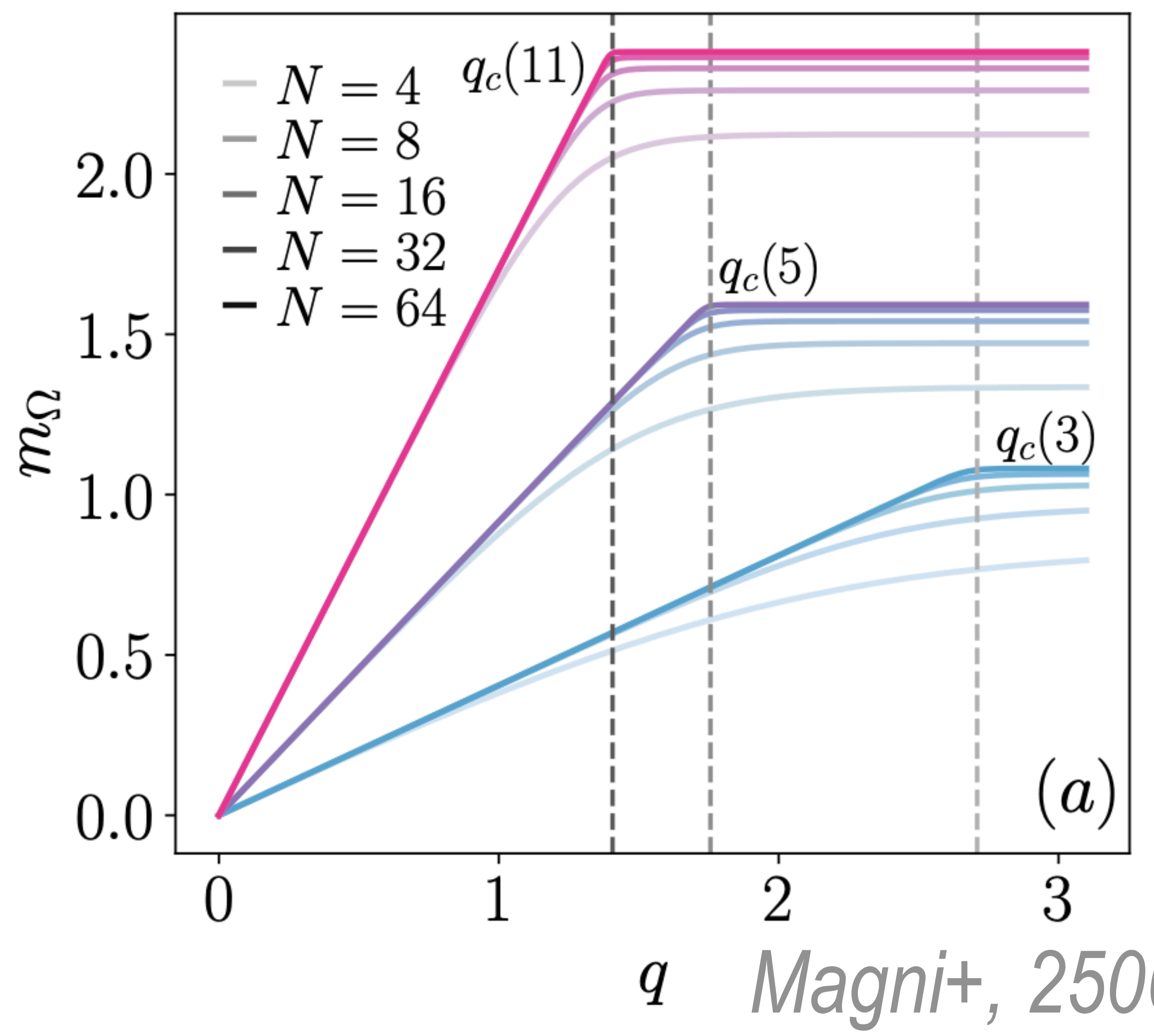
*e.g. Meth+ Nature Physics 21, 570–576 (2025), Calajó+ PRX Quantum **5**, 040309 (2024), Illa, CR, Savage, PRC 108, 064306 (2023), & PRD 110, 014507 (2024), Kürkçüoglu+ arXiv:2410.16414, Fromm+ EPJ Quant. Technol. **12**, 92 (2025), Turro+ PRD 111, 043038 (2025)…*

→ *Also see talk by Even Chiari*

Are qudits more efficient at generating/spreading many-body complexity?

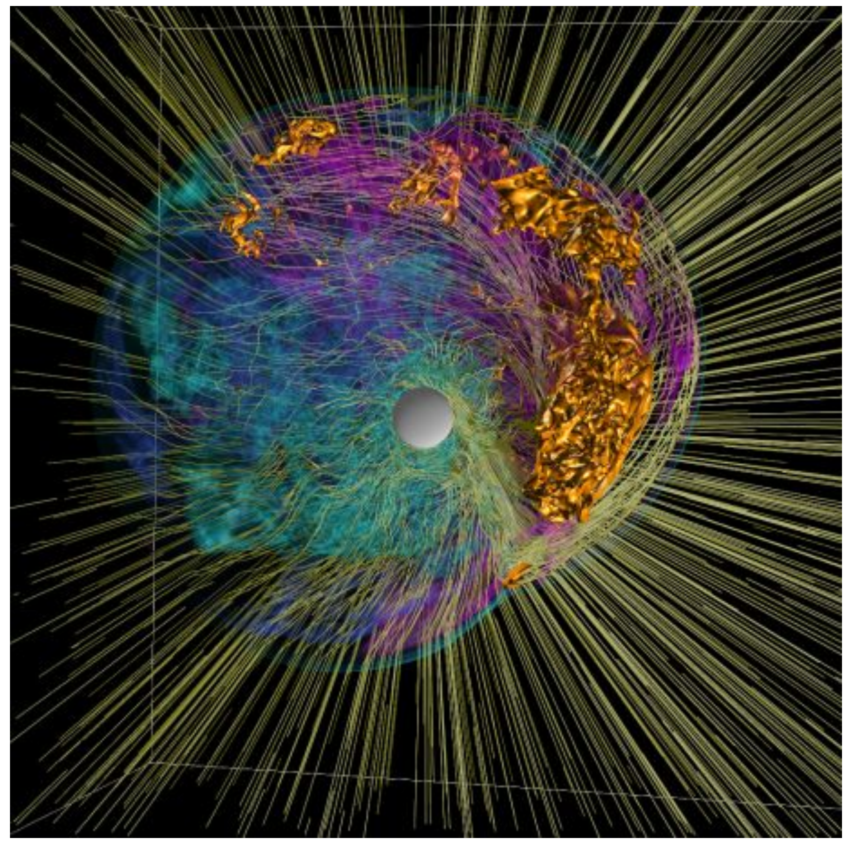
Studies of magic in random circuits, e.g.:

Turkeshi, Tirrito, Sierant, Nature Comm. 16, 2575 (2025)
Magni, Turkeshi arXiv:2506.02127



Magni+, 2506.02127

Neutrino Dynamics with Qutrits



Neutrinos from core-collapse supernovae

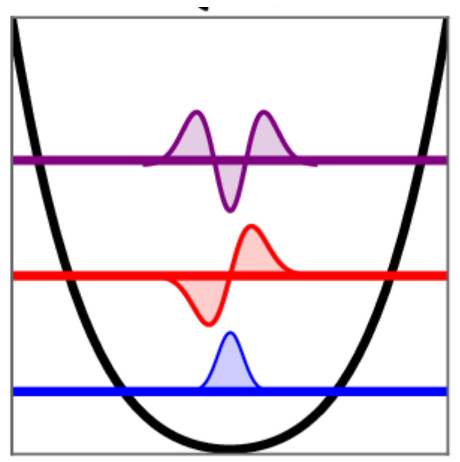
Wikimedia commons

Neutrinos = weakly-interacting, strongly-correlated particles

3 favours of neutrinos $|\nu\rangle_e, |\nu\rangle_\mu, |\nu\rangle_\tau$

related to mass eigenstates: $|\nu\rangle_F = U_{PMNS}|\nu\rangle_M$

Mapping onto qutrits:



|2> $\equiv |\nu_2\rangle_M$

|1> $\equiv |\nu_1\rangle_M$

|0> $\equiv |\nu_0\rangle_M$

Gate complexity
Qubits vs qudit mappings:

Turro+ PRD 111, 043038 (2025)

Qudit	Circuit	All-to-all 2-q gate count	All-to-all 2-q gate depth	Linear chain 2-q gate count	Linear chain 2-q gate depth
Qutrit	Fig. 1	4	4	4	4
Qubit	A (Fig. 3)	24	13	42	31
	B (Fig. 4)	18	12	30	25

TABLE II. The two-qudit entangling gate count and depth for the two-neutrino quantum circuits proposed, involving two qutrits or four qubits.

See also works by Alessandro Roggero & Denis Lacroix, see talk by Mariane Mangin Brinet

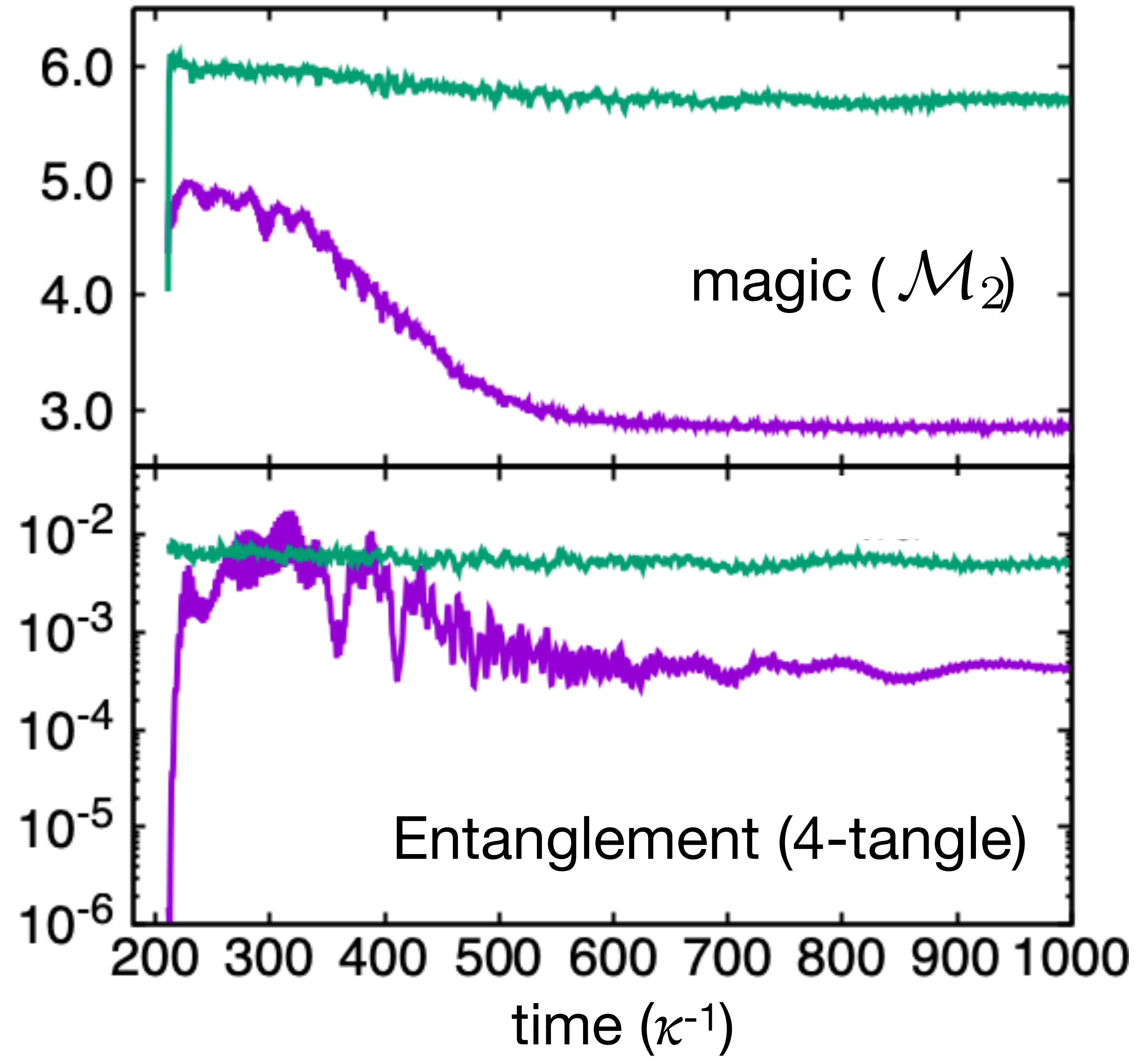
Magic and Entanglement in Neutrino Dynamics

Chernyshev, CR, Savage PRR 7 023228 (2025)

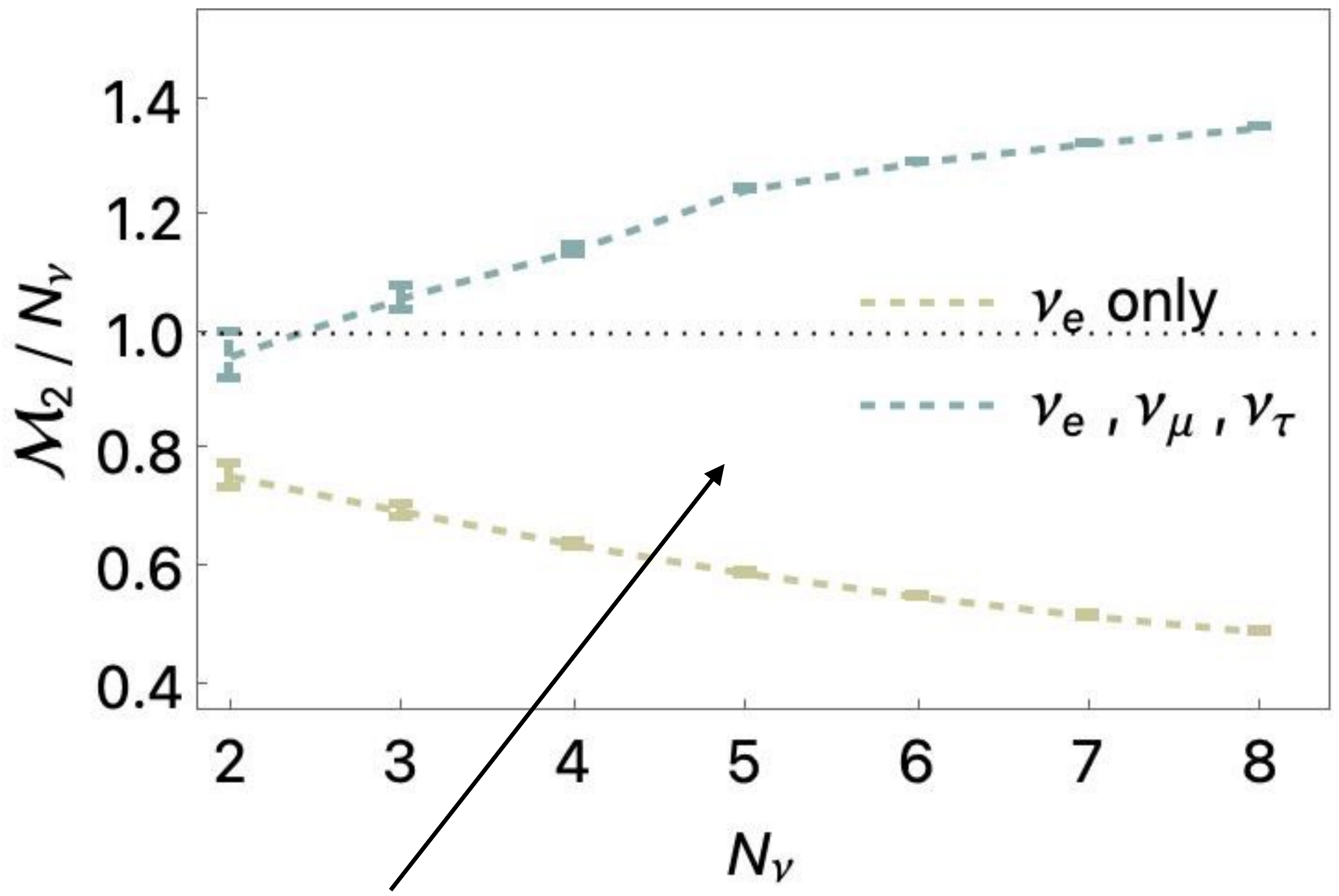
1-flavour vs *3-flavour*

$|\nu_e\nu_e\nu_e\nu_e\nu_e\rangle$ $|\nu_e\nu_e\nu_\mu\nu_\mu\nu_\tau\rangle$

initial state:



Asymptotic values:



Max value for a tensor-product state

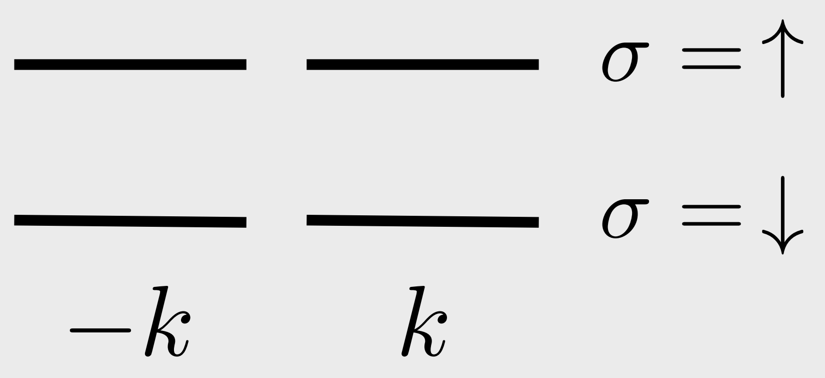
☞ *Flavour mixing enhances complexity during the evolution*

Symmetry-guided mapping of the Agassi model onto qudit systems

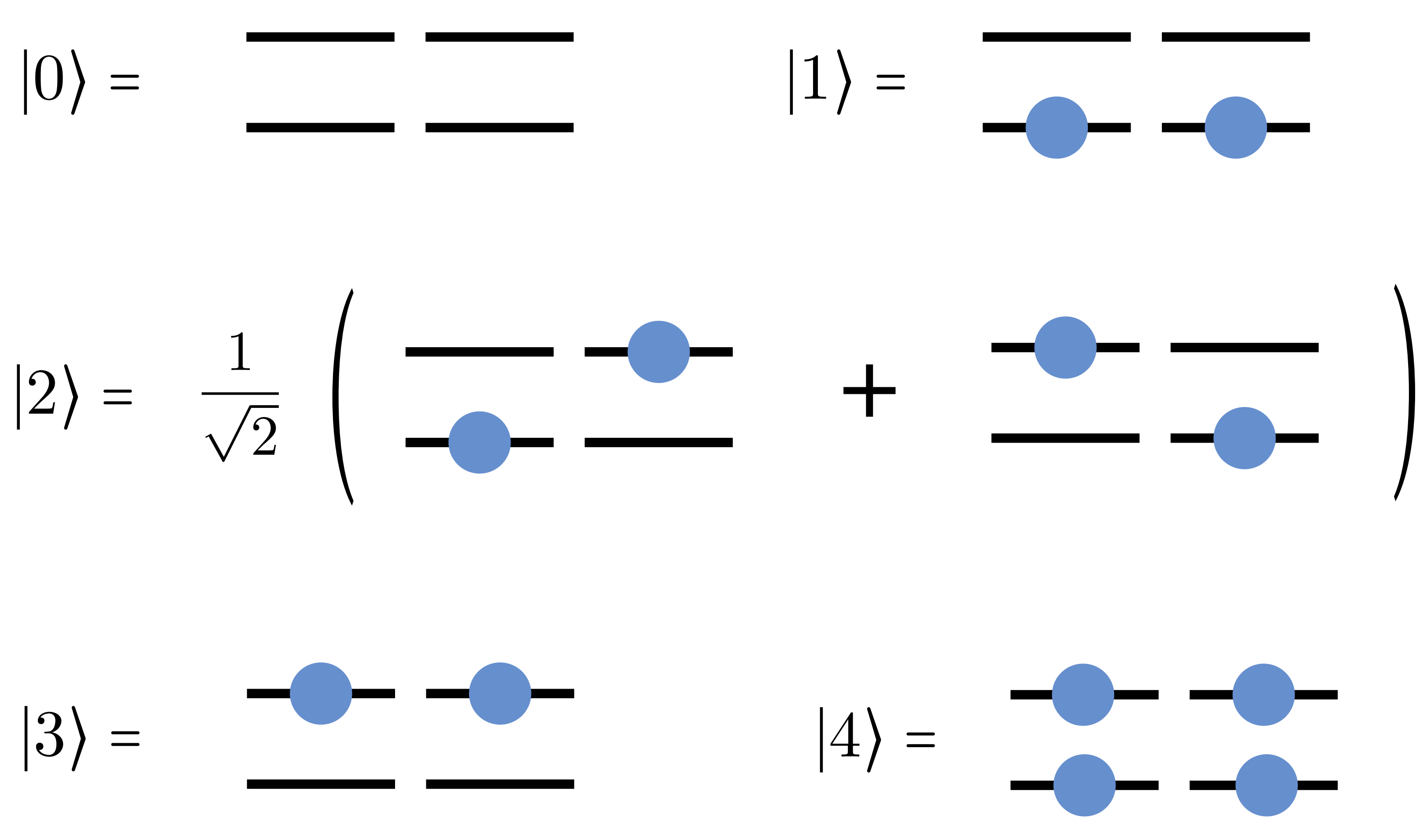
Make use of the SO(5) symmetry:

$J_z, J_\pm, B_{\uparrow,\downarrow}, B^\dagger_{\uparrow,\downarrow}$ = generators of SO(5)

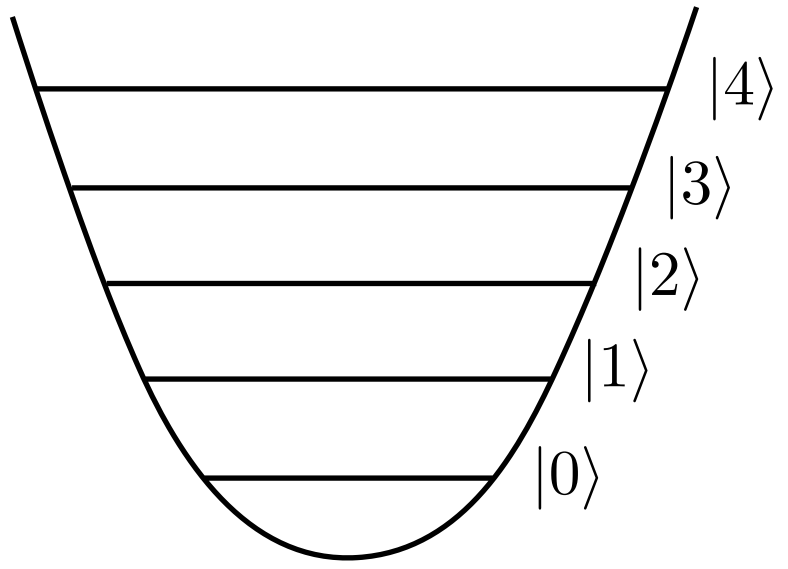
Degrees of freedom = pairs of modes



⇒ 5 states:



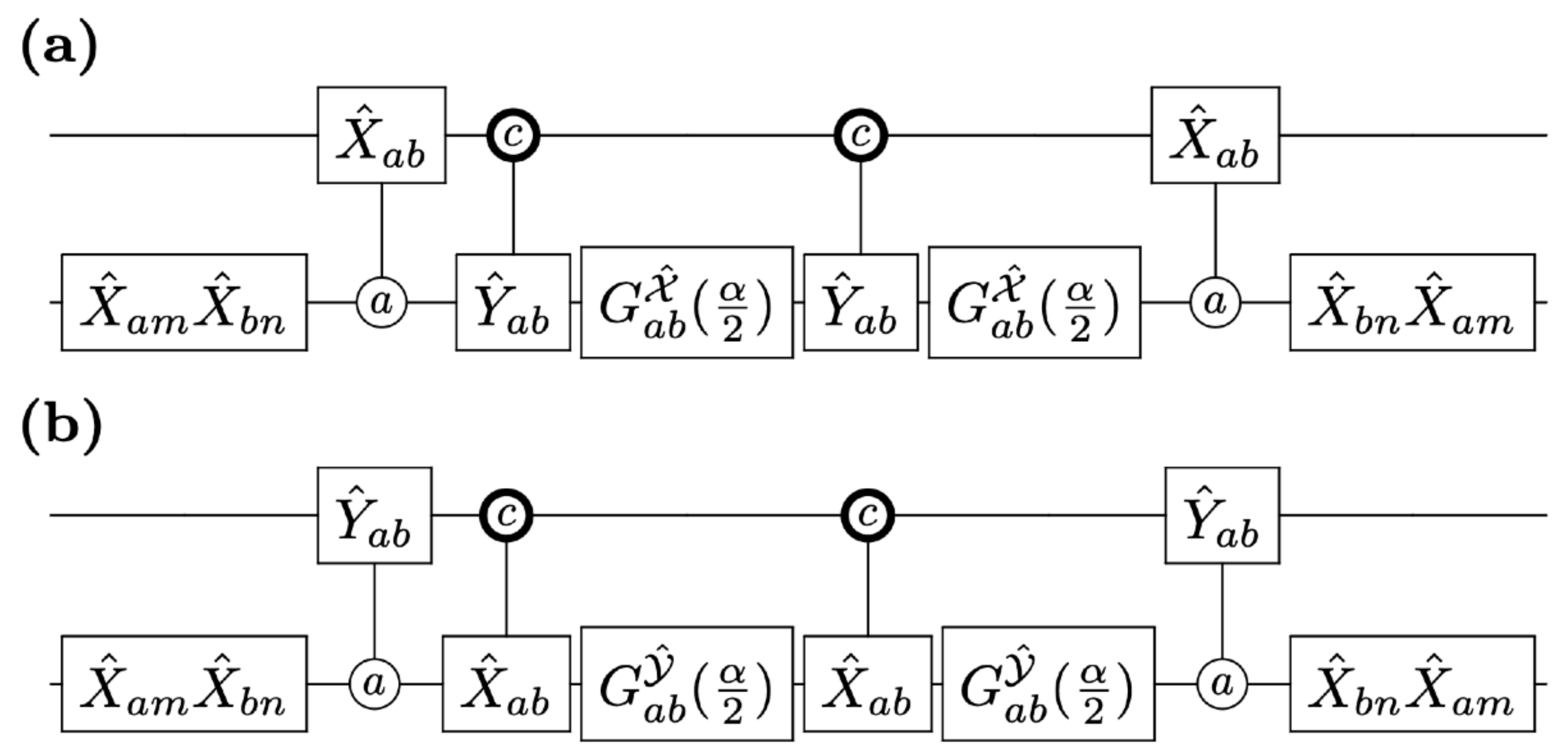
Naturally maps onto "qu5its"
[qudits with d=5]



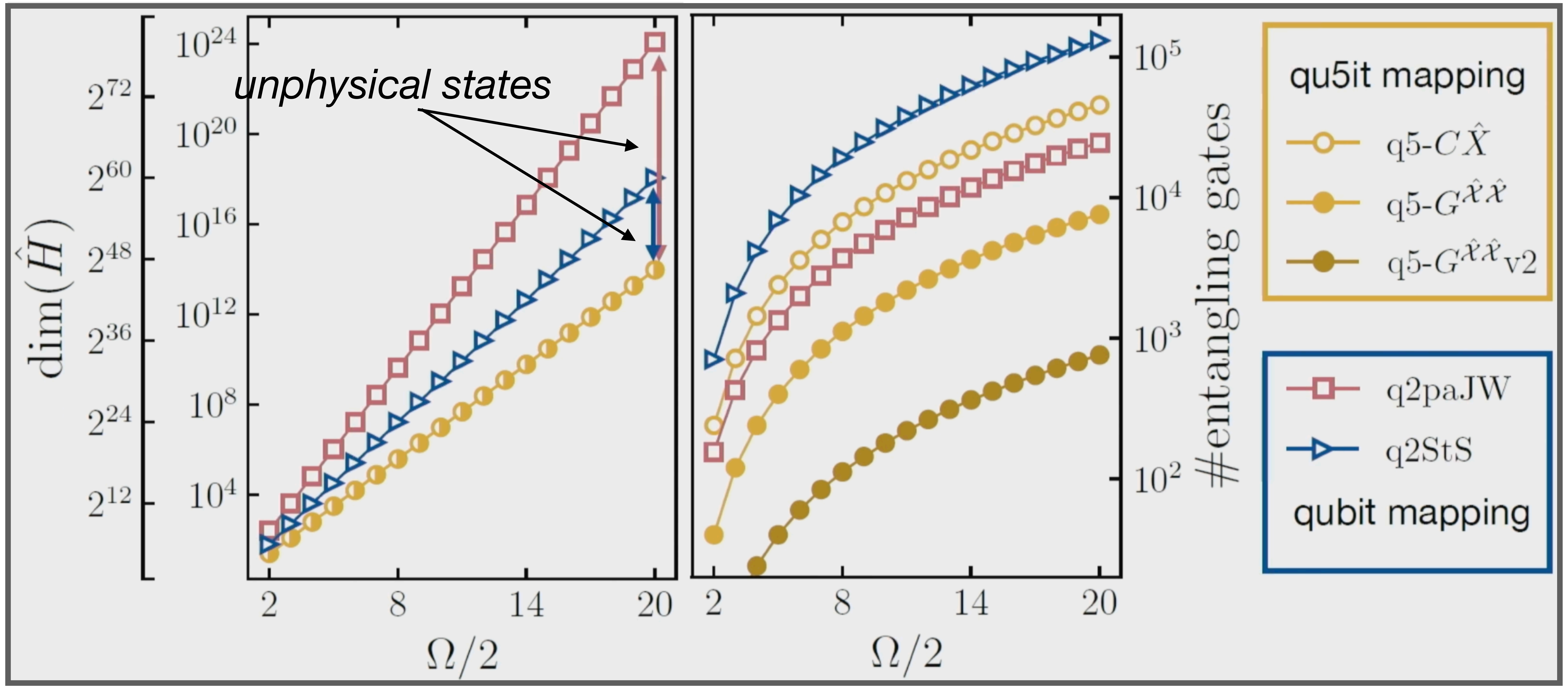
Illa, CR, Savage PRC 108, 064306 (2023)

Symmetry-guided Quantum Simulations with Qudits

Developed a qudit-system classical simulator for unitary dynamics using Google's `cirq`



Errors & Resource Requirements for Simulating Real-Time Dynamics (LO Trotter):



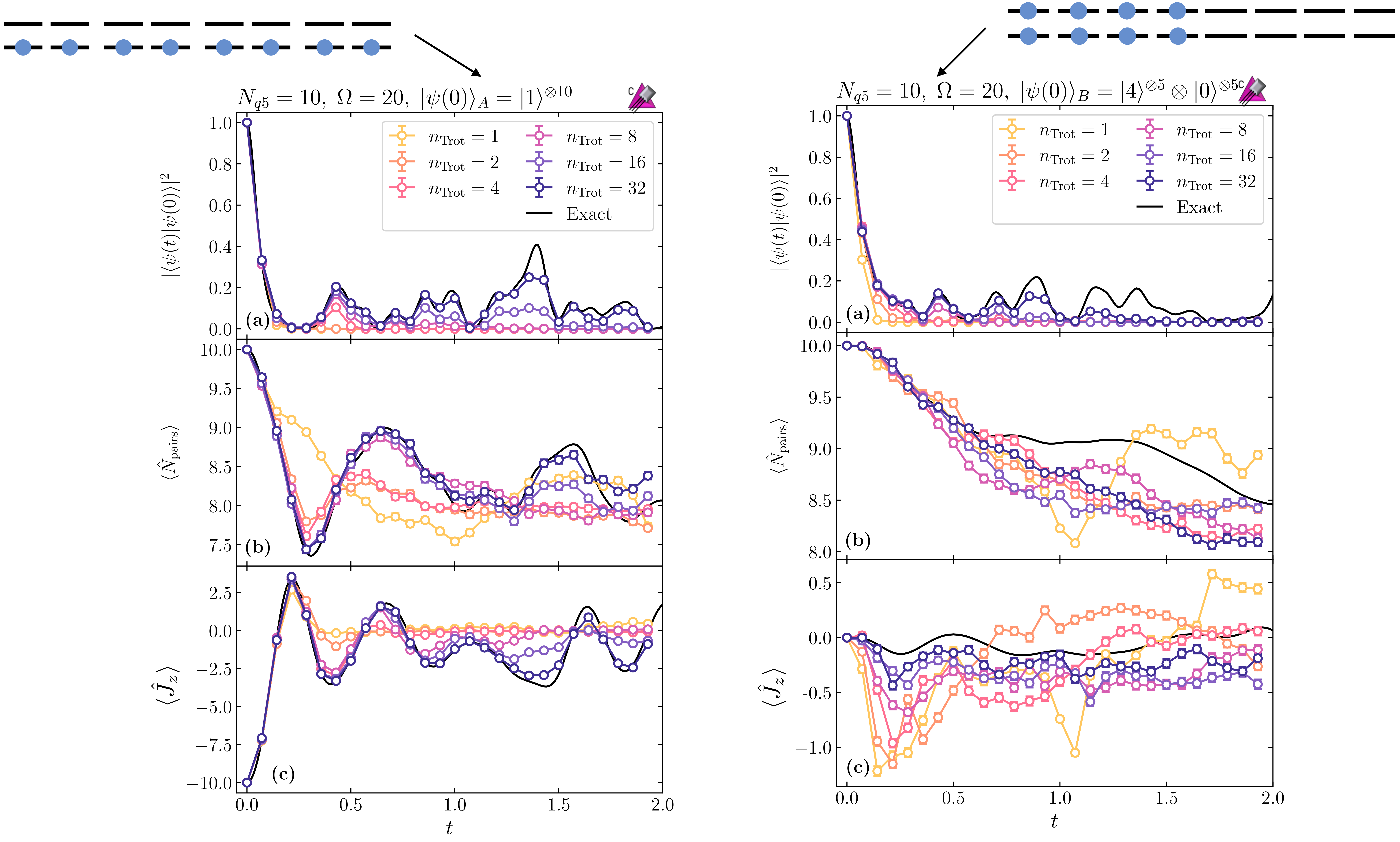
"Physics-aware" JW mapping to qubits

qu5it-state to qubit-states mapping

Illa, CR, Savage PRC 108, 064306 (2023)

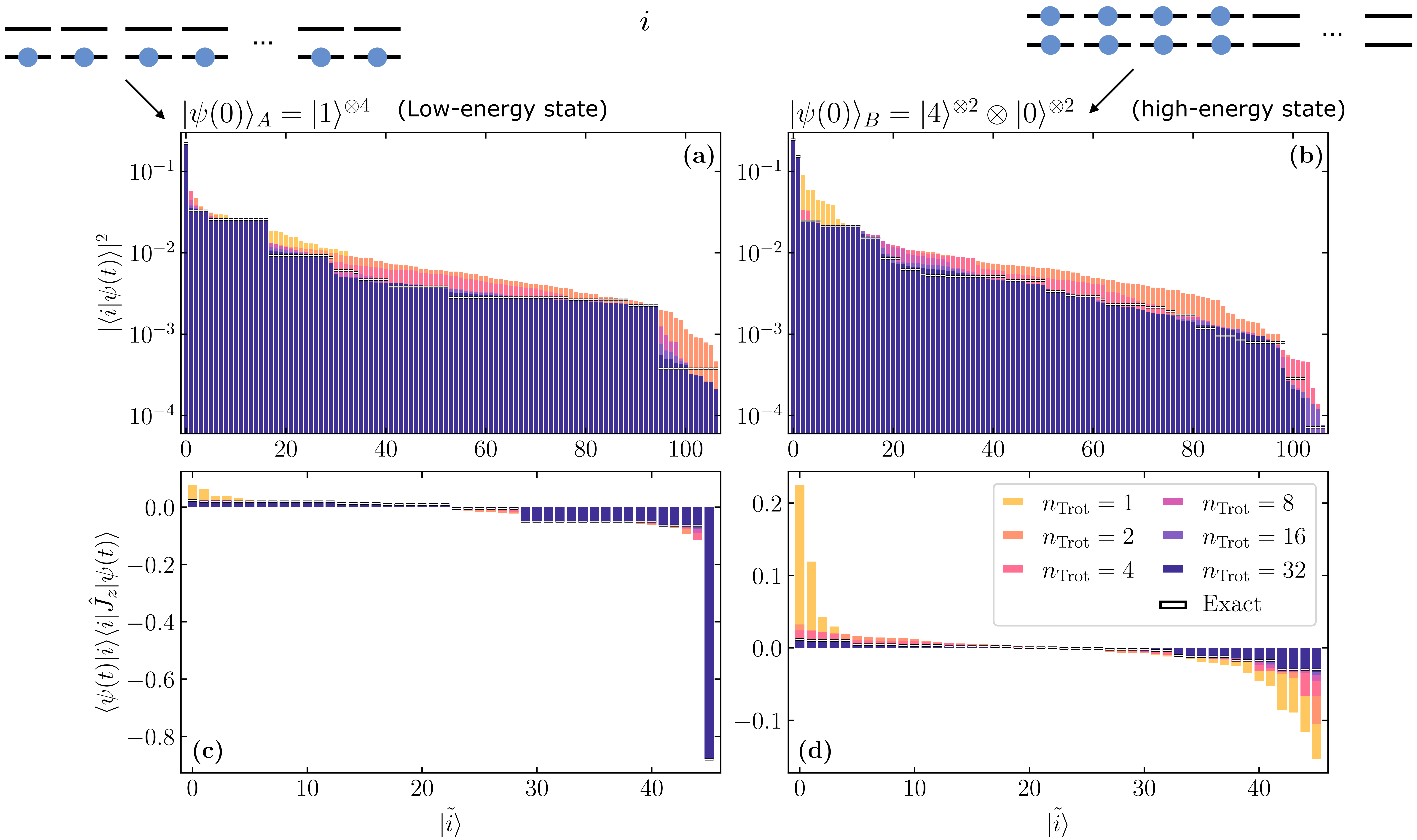
Symmetry-guided mapping of the Agassi model onto qudit systems

★ ***Developed a qudit-system simulator using Google's `cirq` software:***



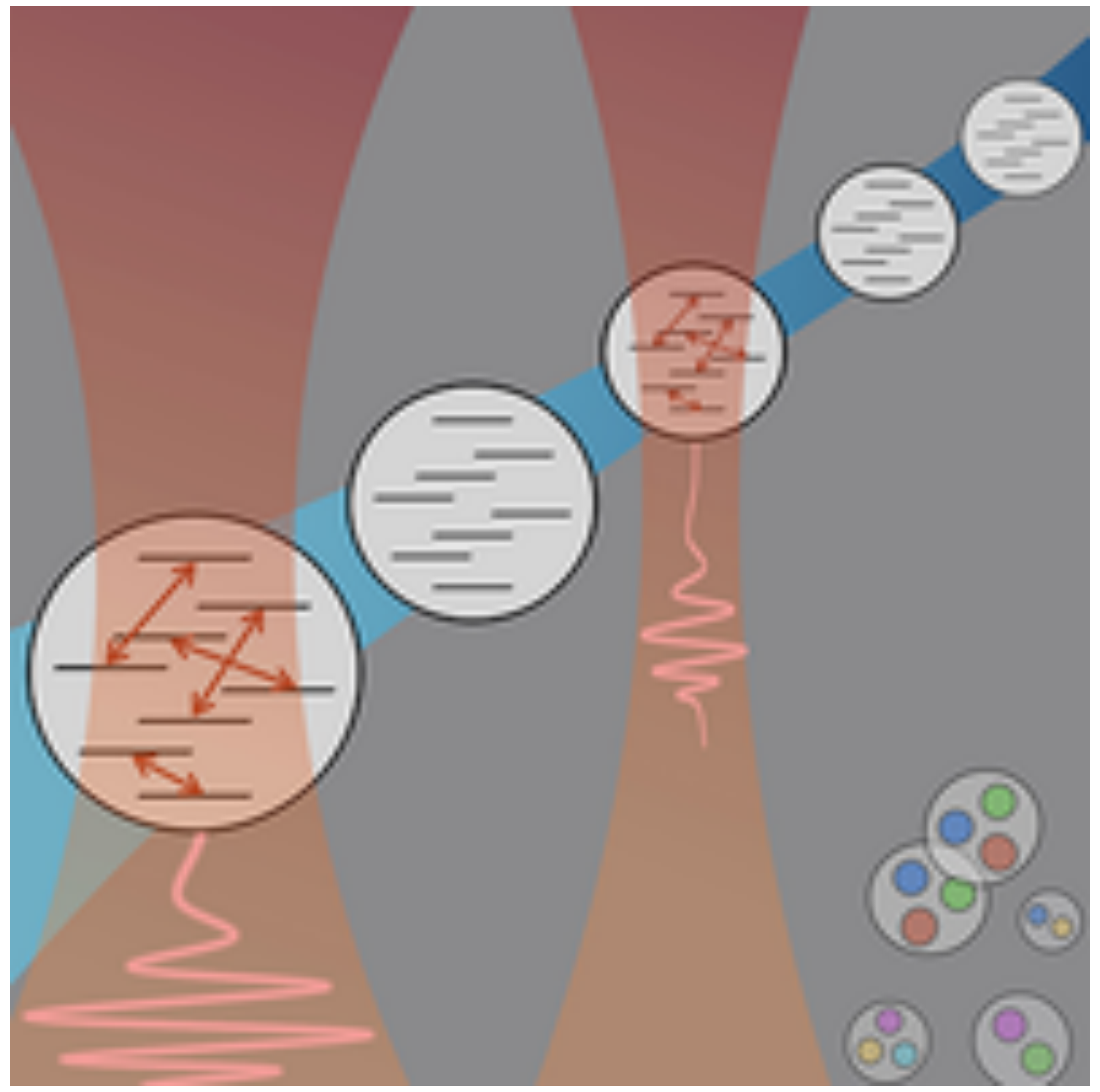
A new sign problem for quantum simulations

★ **A new sign problem:** $|\psi(0)\rangle = \sum_i c_i(0)|i\rangle$ ← Computational-basis states

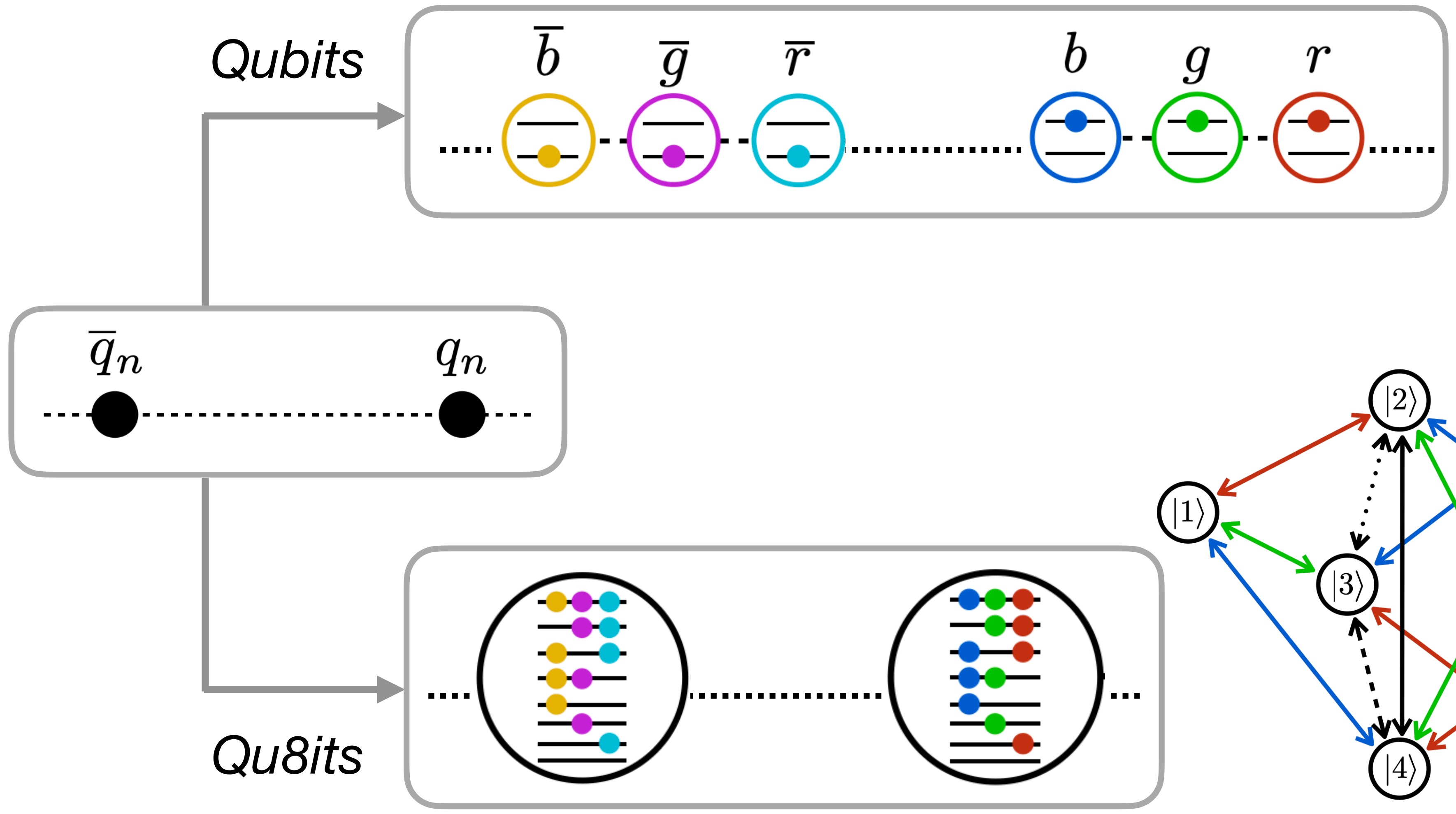


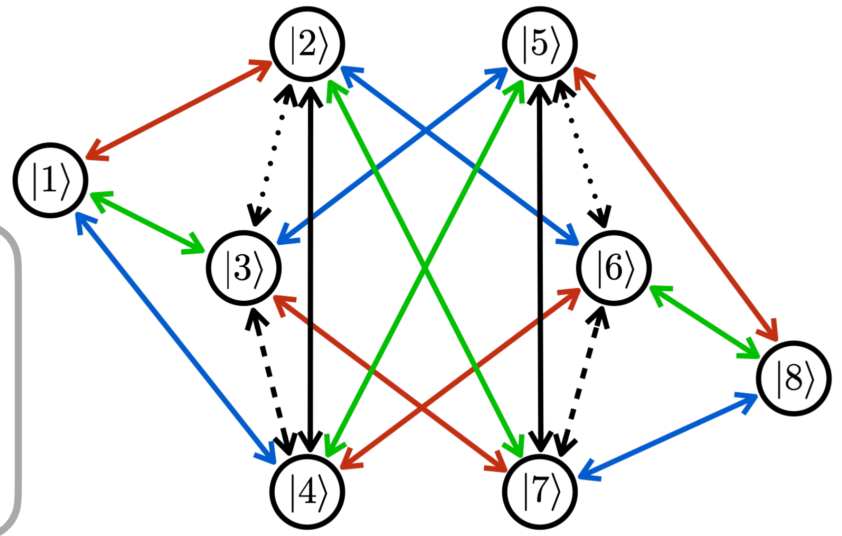
Qudits for Quantum Simulations of 1+1D SU(3) Lattice QCD

Illa, CR, Savage, PRD 110, 014507 (2024)
Editor's suggestion



$$H=\sum_f\left[\frac{1}{2}\sum_{n=0}^{2L-2}\left(\phi_n^{(f)\dagger}\phi_{n+1}^{(f)}\ +\ \text{h.c.}\right)\ +\ m_f\sum_{n=0}^{2L-1}(-1)^n\phi_n^{(f)\dagger}\phi_n^{(f)}\right]\ +\ \frac{g^2}{2}\sum_{n=0}^{2L-2}\sum_{a=1}^{8}\left(\sum_{m\le n}Q_m^{(a)}\right)^2$$





Resource for time evolution (single Trotter step):

Qudits	Number of qudits	U_{kin} ent. gates	U_{el} ent. gates
Qubit ($d=2$)	$6N_fL$	$6N_f(8L-3)-4$	$N_f(2L-1)[23N_f(2L-1)-17]$
Qu8it ($d=8$)	$2N_fL$	$6N_f(2L-1)$	$4N_f(2L-1)[N_f(2L-1)-1]$
Reduction in resources ($L\to\infty$)	3	4	5.75