Pitch-angle Diffusion and Bohm-type approximations in Diffusive Shock Acceleration

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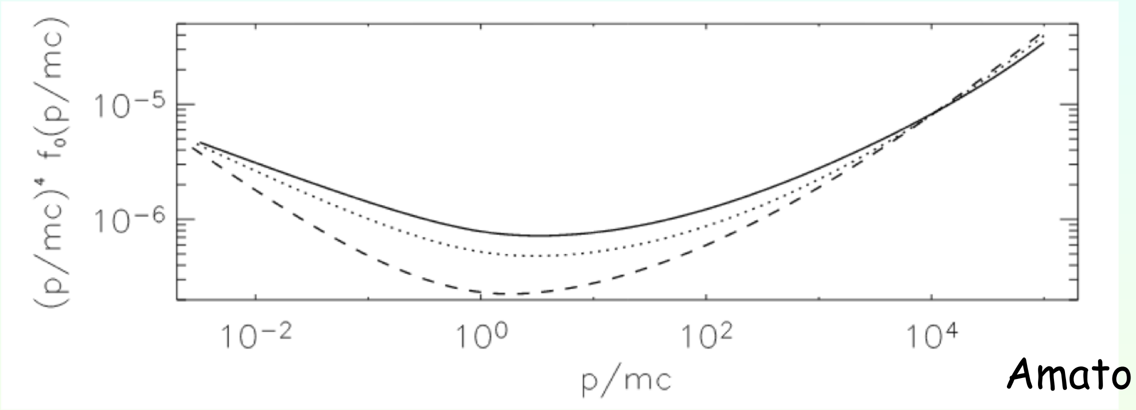
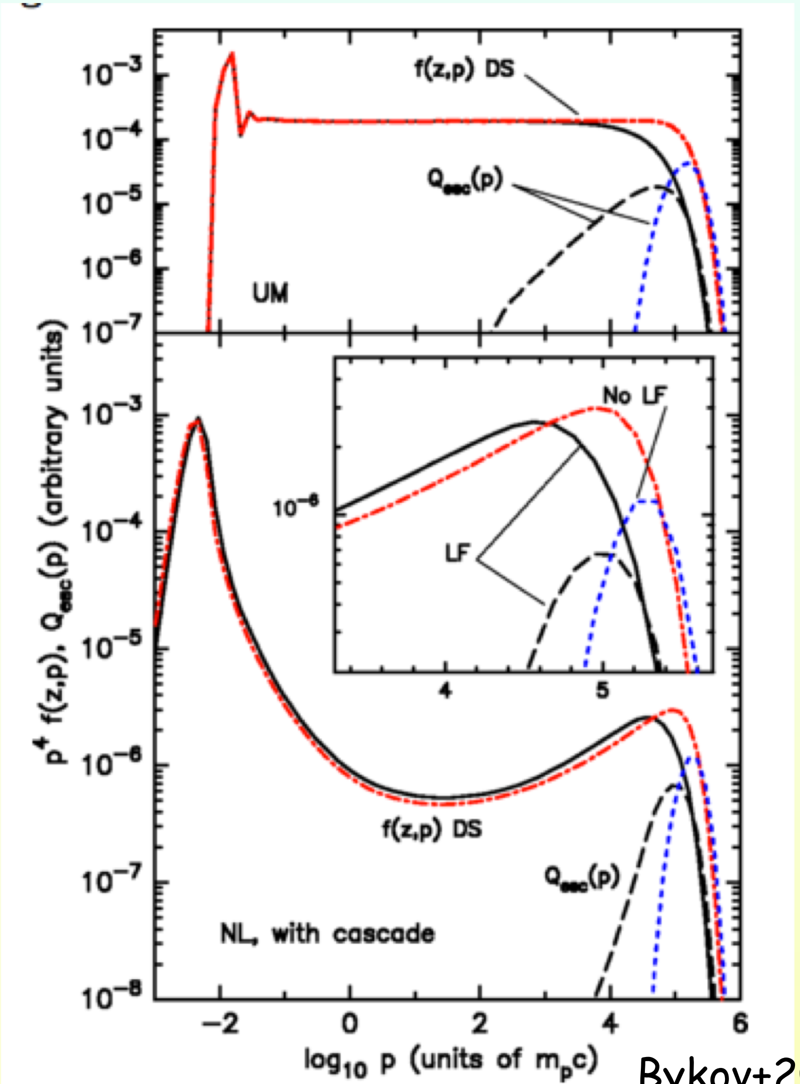
How to study acceleration of particles ?

1. (semi) - Analytic



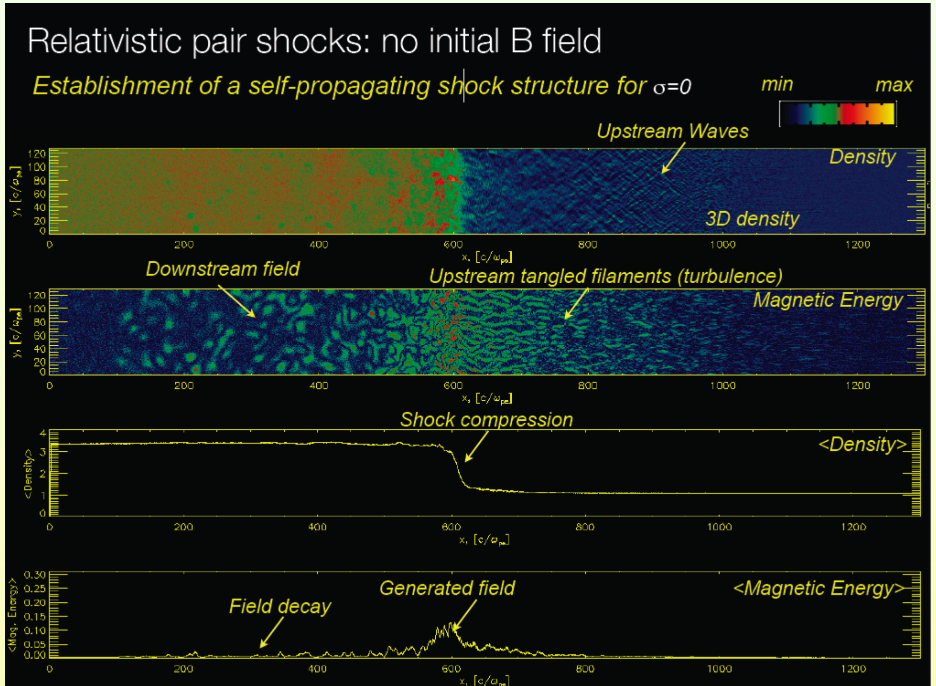
Amato & Blasi 2005

2. Monte-Carlo codes



Bykov+2017

3. Particle-in-cell (PIC)



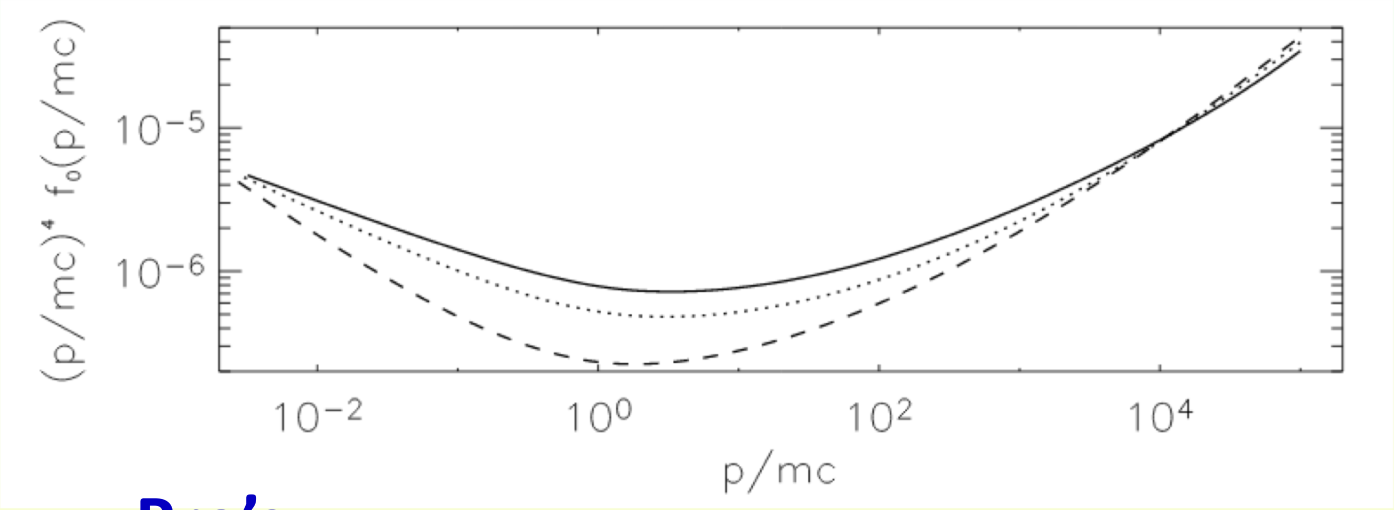
Spitkovsky 2008

How to study acceleration of particles ?

1. (semi) - Analytic

-Analytical expression for particle distribution function

-Solve transport equation



Kirk & Heavens 1989,

Malok 1997,

Amato & Blasi 2005

Caprioli+2010

Lemoine 2019, …

Pro's

-Fast

-Easily(?) tractable

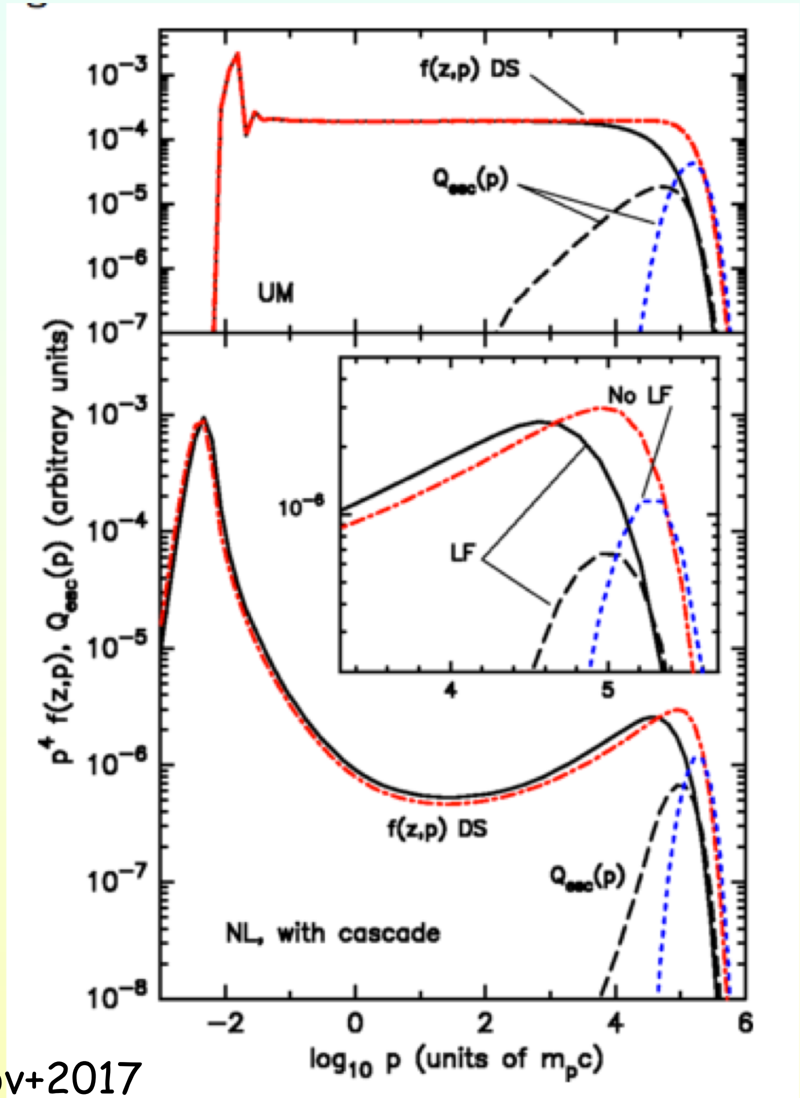
Co's

-(very) limited parameter space

-Heuristic diffusion model

How to study acceleration of particles ?

2. Monte-Carlo codes



Bykov+2017

-Trajectories of individual particles are tracked over avg. background field

Ellison 1990, Achterberg+2001, Vladimirov+ 2006, Sommerlin & Baring 2011, Bykov+ 2017,...

Pro's

- probes large parameter space region
- Fast
- Covers a large spatial region

Co's

Simplified assumptions:

- magnetic field structure
- Interactions field-particles

How to study acceleration of particles ?

3. Particle-in-cell (PIC)

-Simultaneous solutions of particle trajectories and EM fields self-consistently

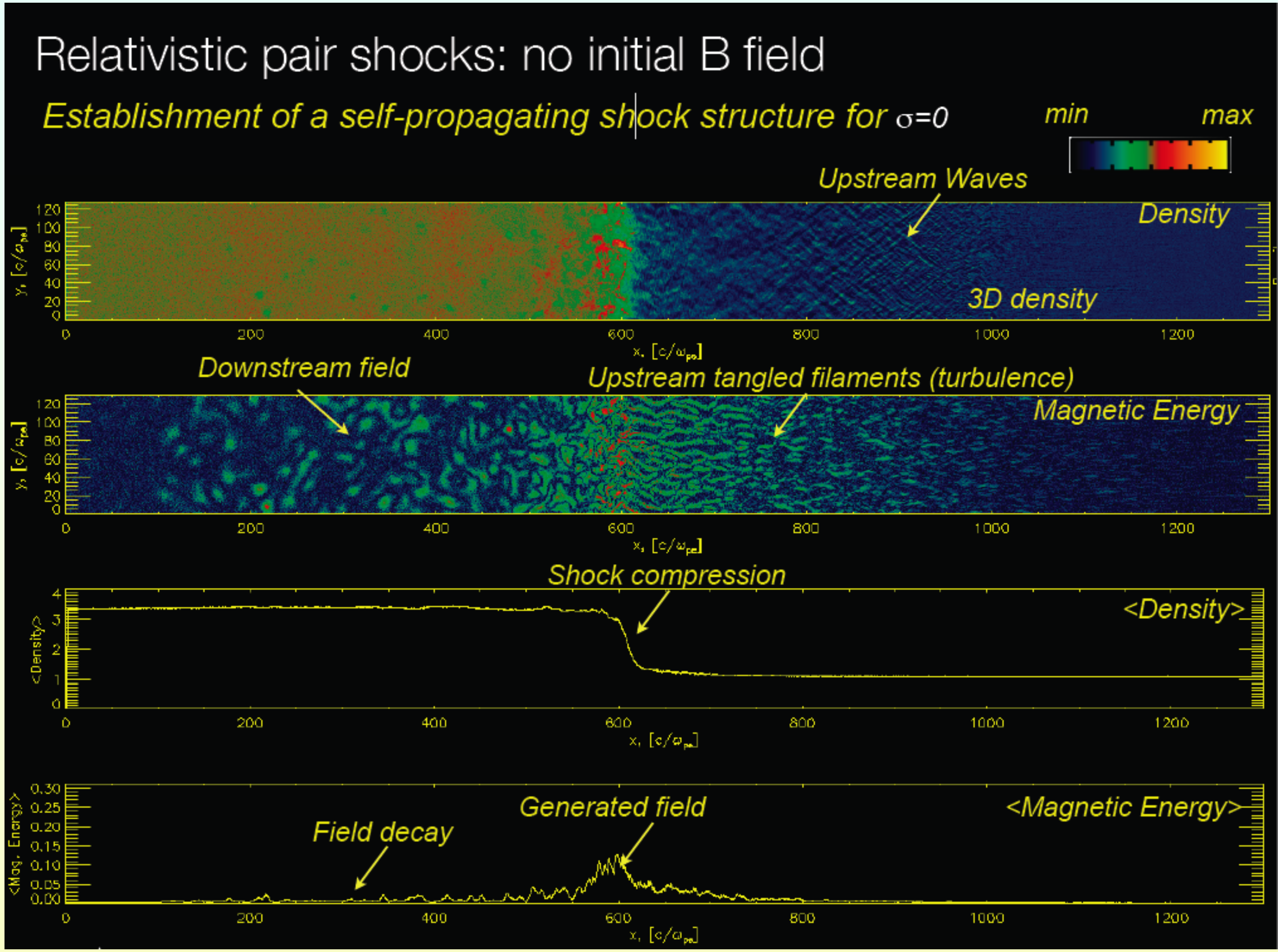
Silva+03, Frederiksen+04, Spitkovsky 2008, Bai+2014
Guo+ 2014, Sironi & Cerutti, 2017, Umeda+2019…

Pro's

- Exact
- Self consistent

Co's

- Computationally expensive: Can simulate ~10^{-8} of accelerated region



"Tristan" code   Spitkovsky 2008

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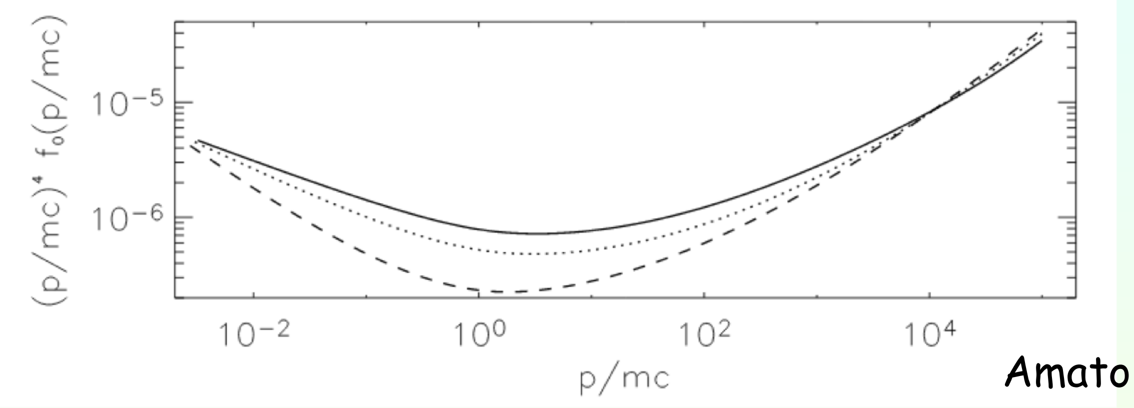

Amato & Blasi 2005

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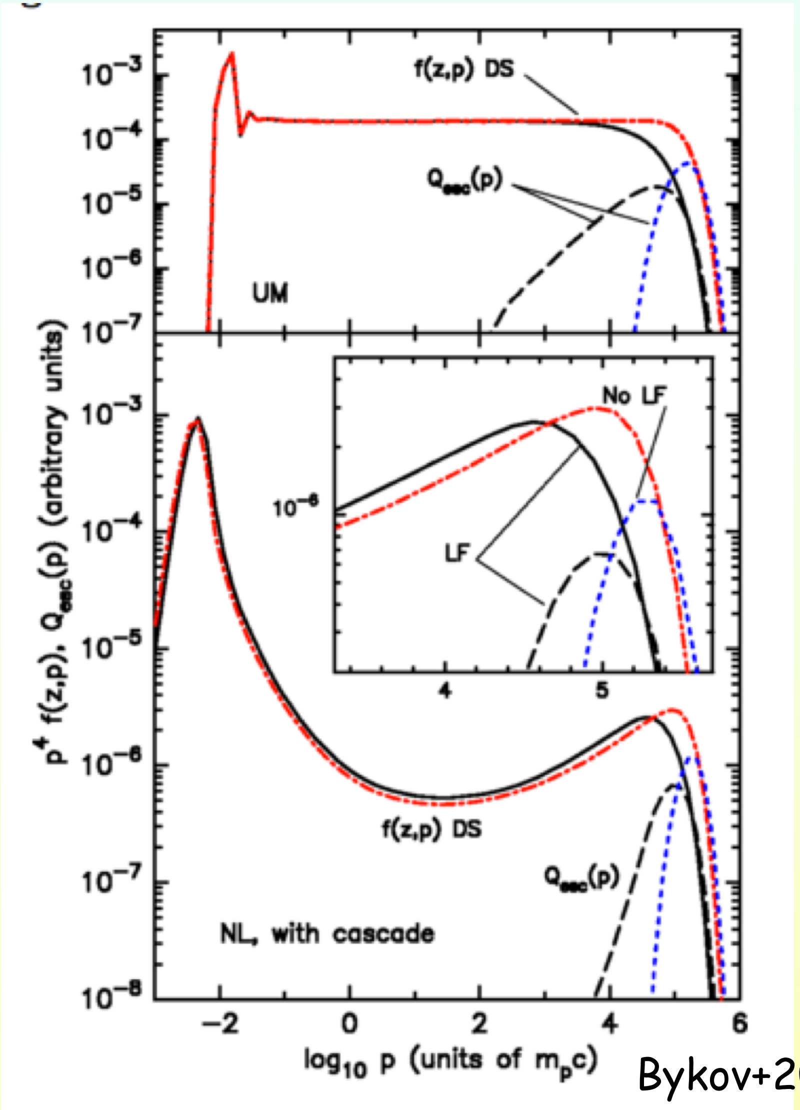

Bykov+2017

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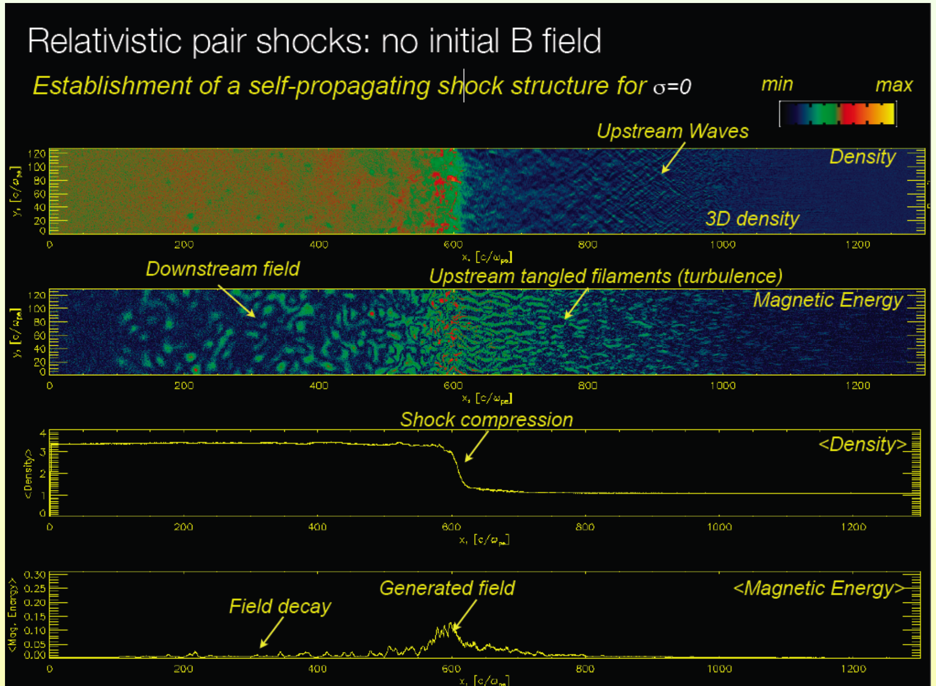

Spitkovsky 2008

How to study acceleration of particles ?

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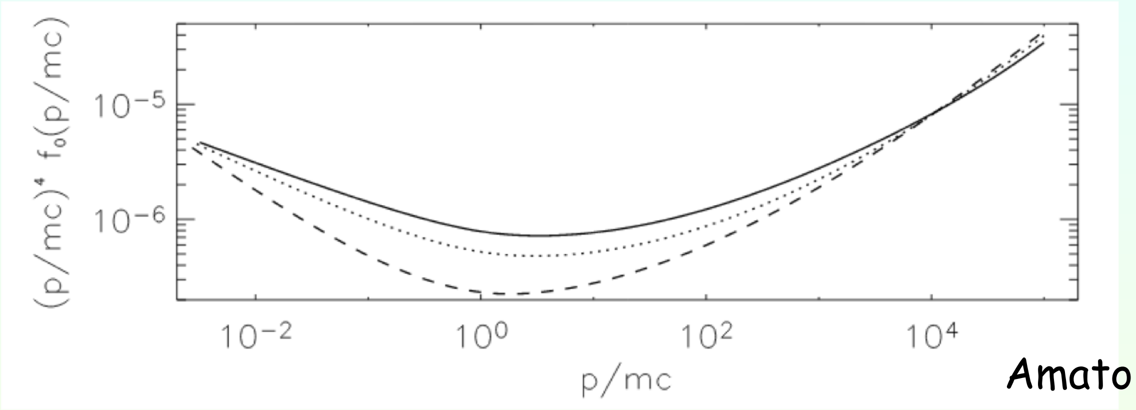

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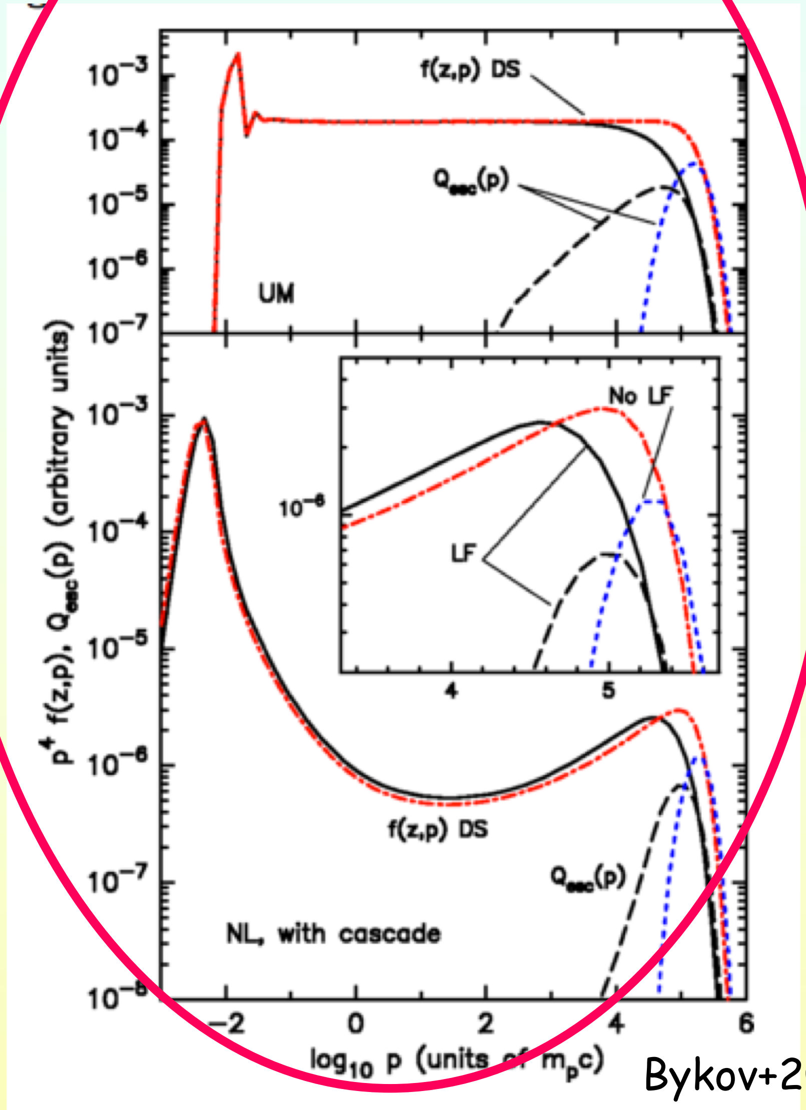

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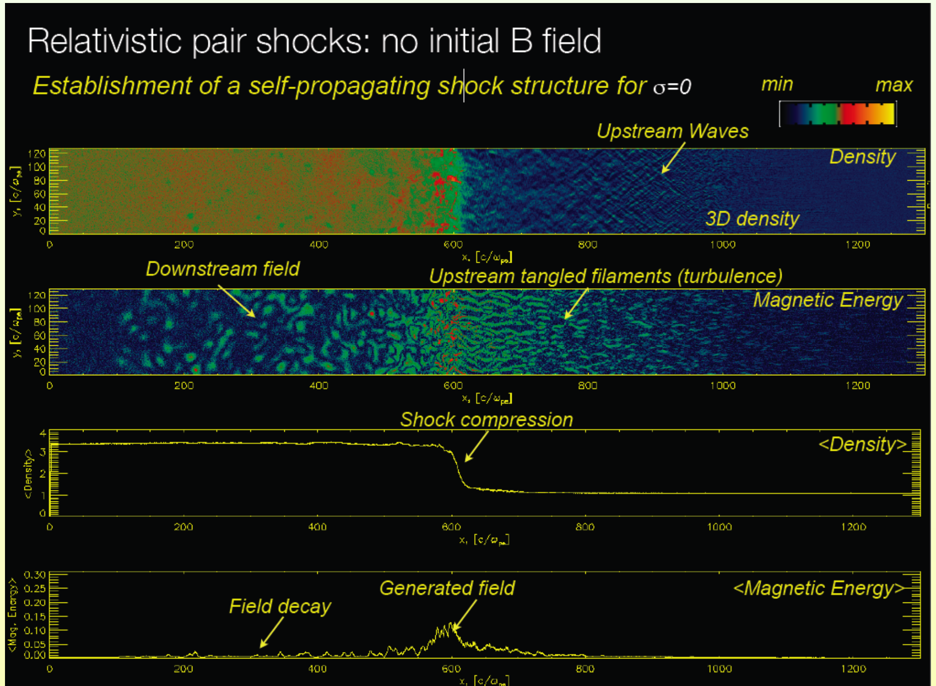
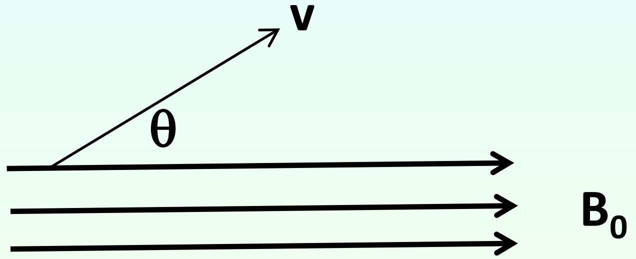


Spitkovsky 2008

Quantifying particle-field interactions in MC codes

Particle "scatter" when interacting with B-field (Pitch angle scattering - "random walk")



QLT – quasi-linear theory

-Deviation from helical orbit is treated perturbatively – averaging wave contribution over many gyrotimes.

- Resonance condition – $k \sim 1/r_g$

-True in the limit $\delta B/B_0 << 1$

Bohm diffusion

- A particle undergoes discrete, isotropic scattering

- No resonance

- "Mean free path": $\lambda_{mfp} \sim r_g^{\alpha}$

$\alpha \approx 1$

Improving the prescription for MC simulation:
Pitch-angle diffusion and Bohm-type approximaion

Modeling B-field:

-Guiding field B_0

- Turbulentic B-field $\delta B/B_0$ spectral shape:

$$\delta\vec{B} = \sum_{waves} A_k e^{i(k\cdot x+\phi_k)}\vec{n}$$

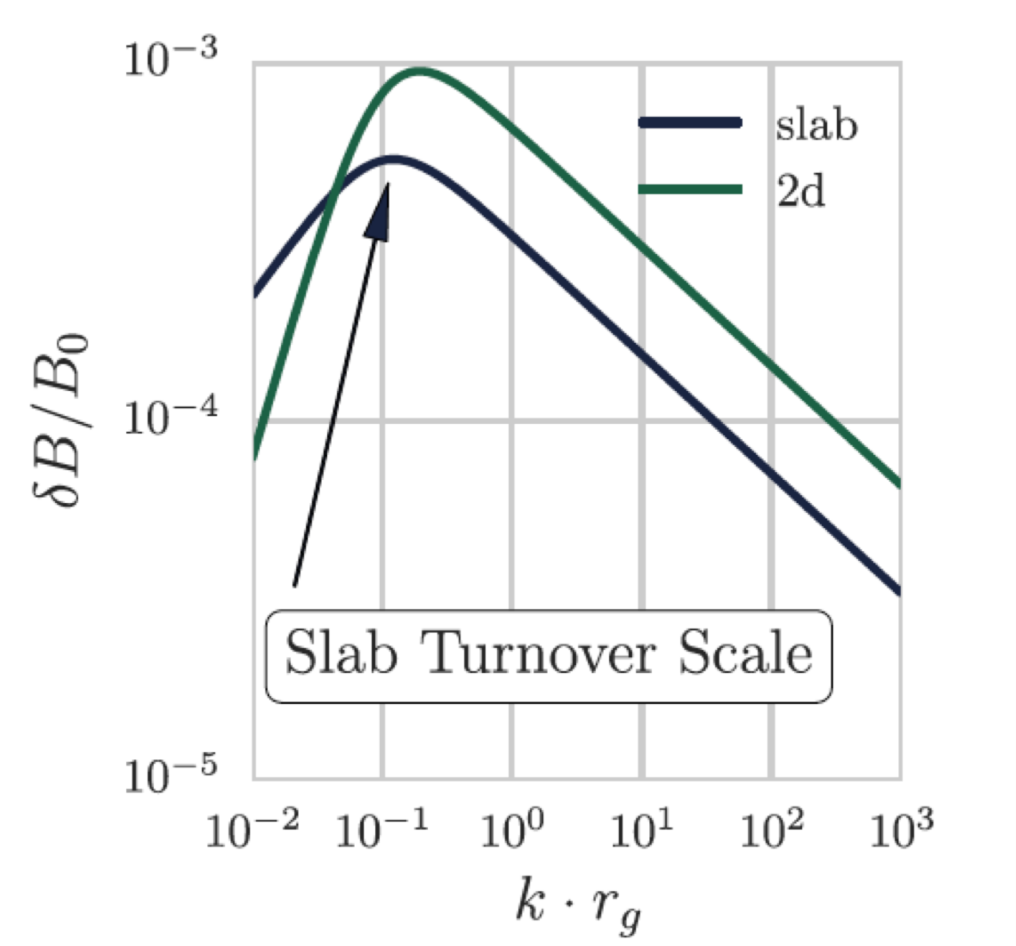
$\vec{k} = \left(0,0,k_{\parallel}\right)$ Slab waves

$\vec{k} = k_{\perp}\left(\cos\theta,\sin\theta,0\right)$ 2d waves

$$A_k^2 \propto \Delta k \frac{k^q l^{q+1}}{\left[1+(kl)^2\right]^{\frac{s+q}{2}}}$$

l – turnover scale

– *s, q*: various turbulence spectra (Kolmogorov, Goldreich-Sridhar, etc.)



Riordan & Pe'er, 2019

Improving the prescription for MC simulation:
Pitch-angle diffusion and Bohm-type approximaion

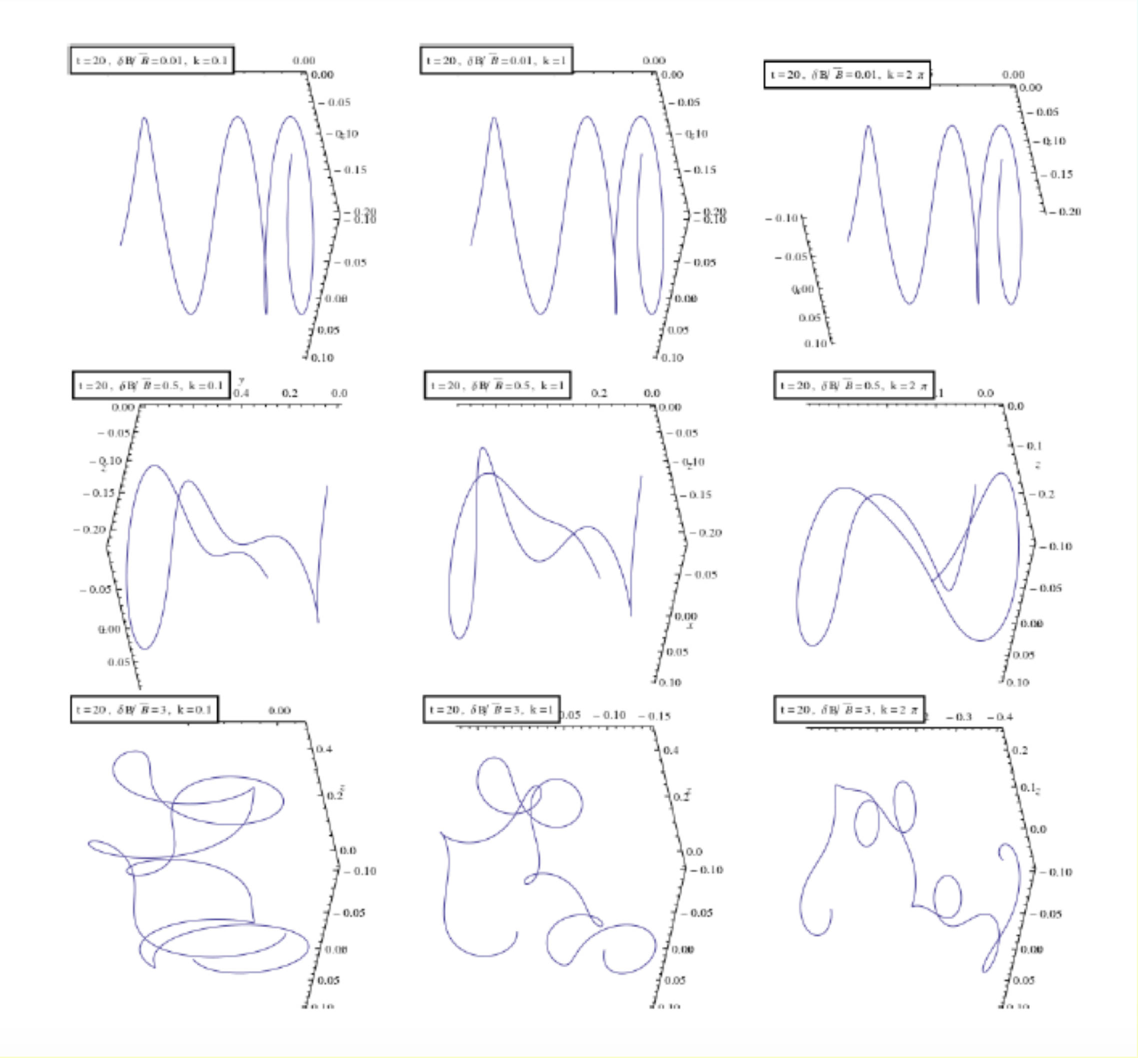
Modeling B-field:

$$\delta\vec{B} = \sum_{waves} A_k e^{i(k\cdot x+\phi_k)}\vec{n}$$

$n_w = 4096$, $10^{-4}<k<10^{6}$

- Inject 256 particles / random seed (turbulence realization) * 8 seeds

- follow trajectories



Measure

-Diffusion time

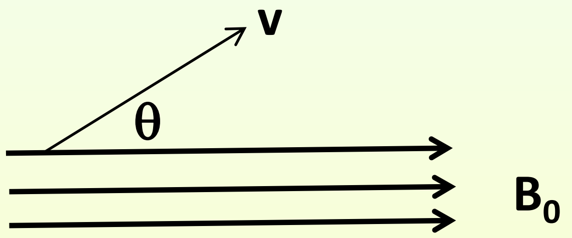
-Scattering time

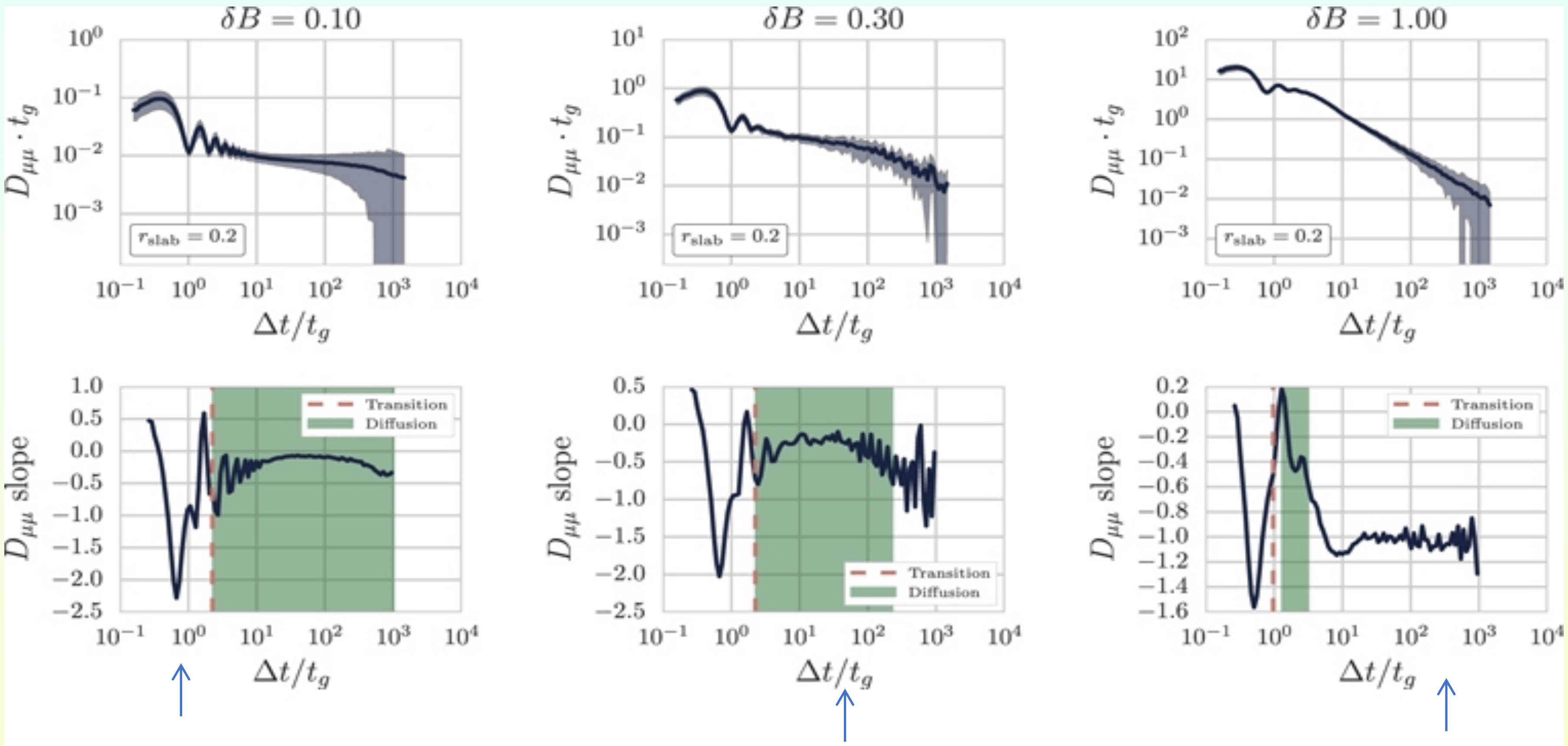
Riordan & Pe'er, 2019

Result 1: particle pitch-angle diffusion

$$D_{\mu\mu} = \frac{\left\langle (\Delta\mu)^2 \right\rangle}{\Delta t}$$

$$\mu = \cos\theta = \frac{v_z}{v}$$





Ballistic regime

Diffusive regime

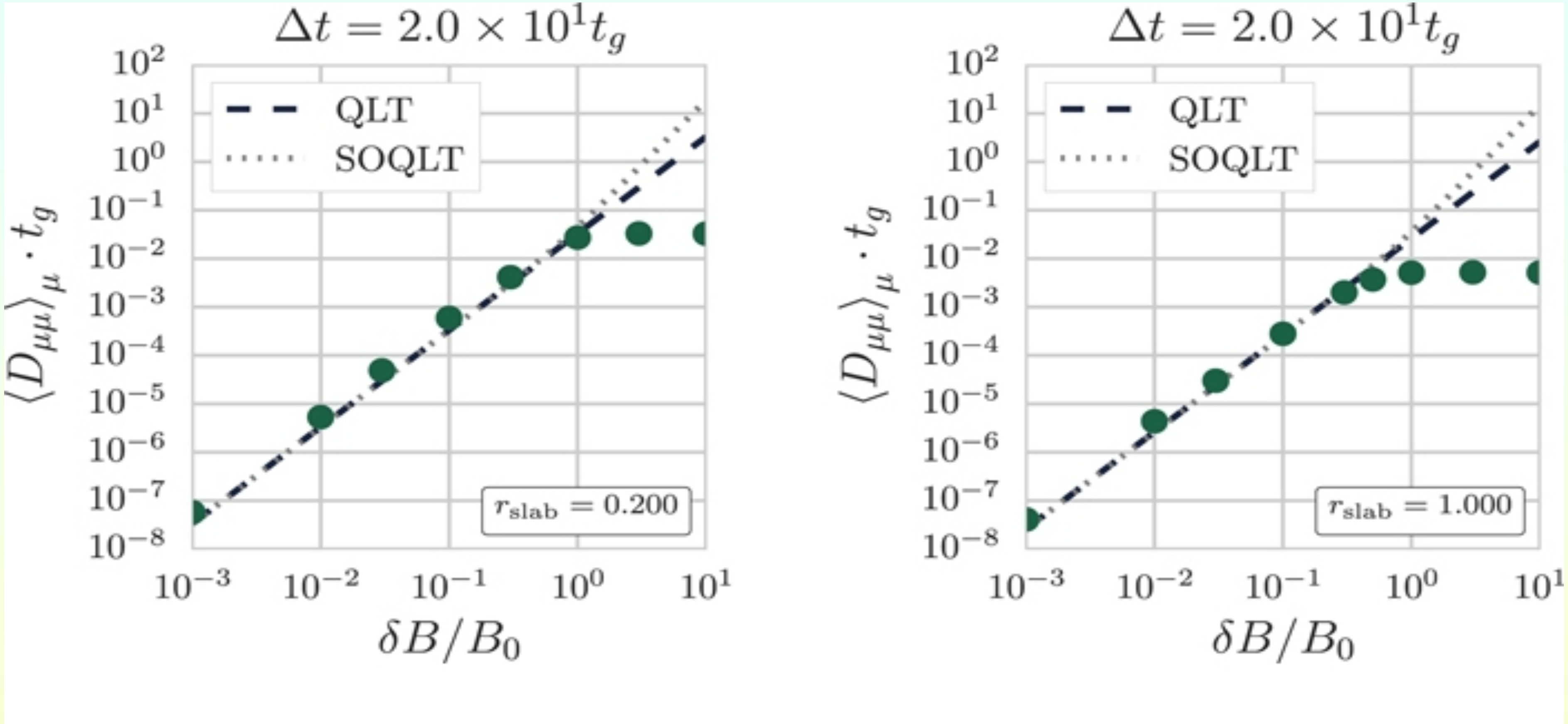
Sub-diffusive tail

t_g = gyrotime=$2\pi\gamma m/qB$

For a given δB/B, diffusion model is valid during a limited time

Result 2: Limited validity of analytical (QLT) models

Diffusion coefficient



$$D_{\mu\mu}(QLT) \propto \left(\frac{\delta B}{B} \right)^2$$

Valid for δB/B <~0.1

Riordan & Pe'er, 2019

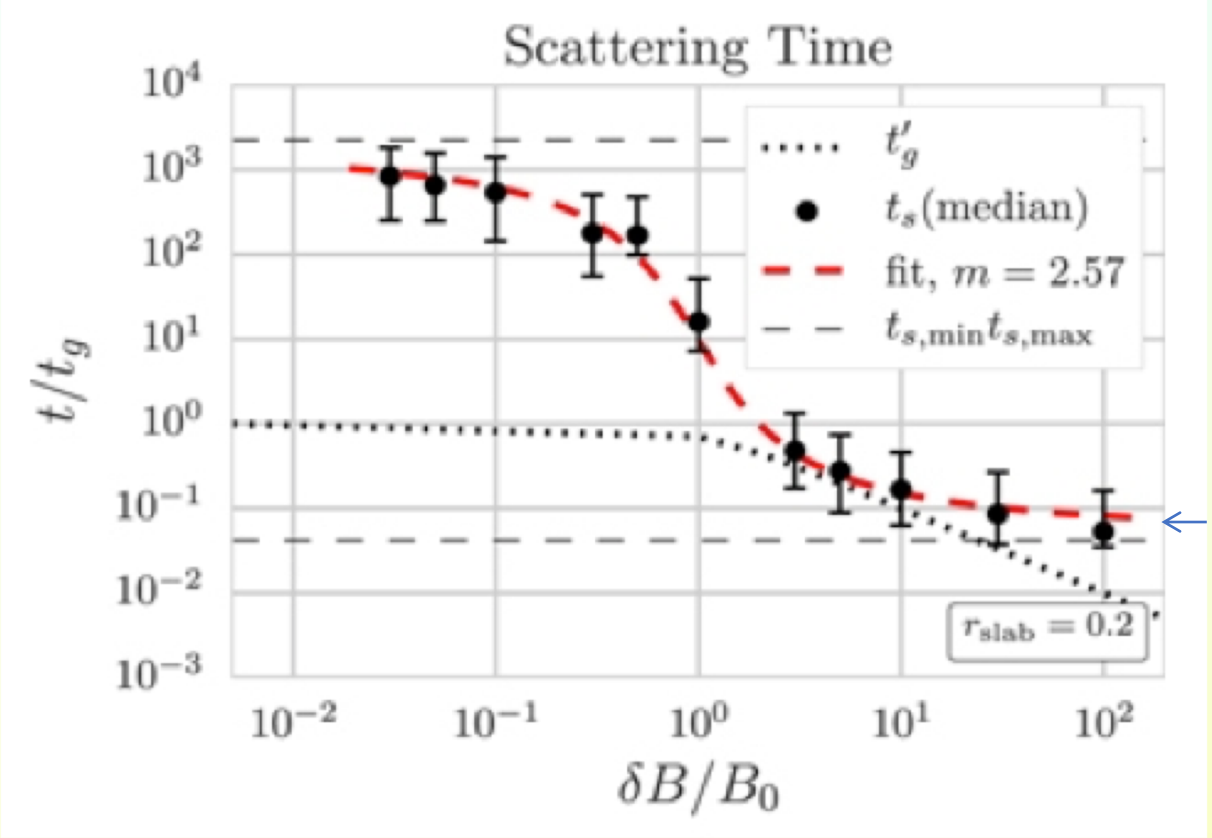
Result 3: limitations of the Bohm diffusion model

A leading model in MC simulations.

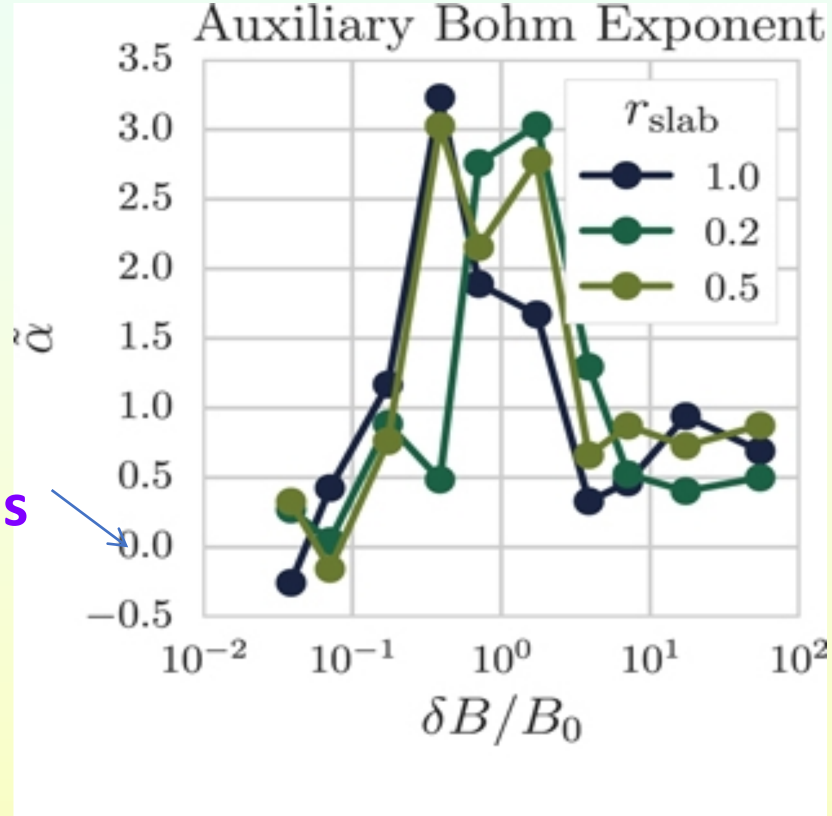
Particle moves in straight lines, and scatters

$$\lambda_{mfp} \sim r_g^{\alpha}$$

$\alpha \approx 1$ ←→ $t_s \sim t_g$



Analytical Expressions



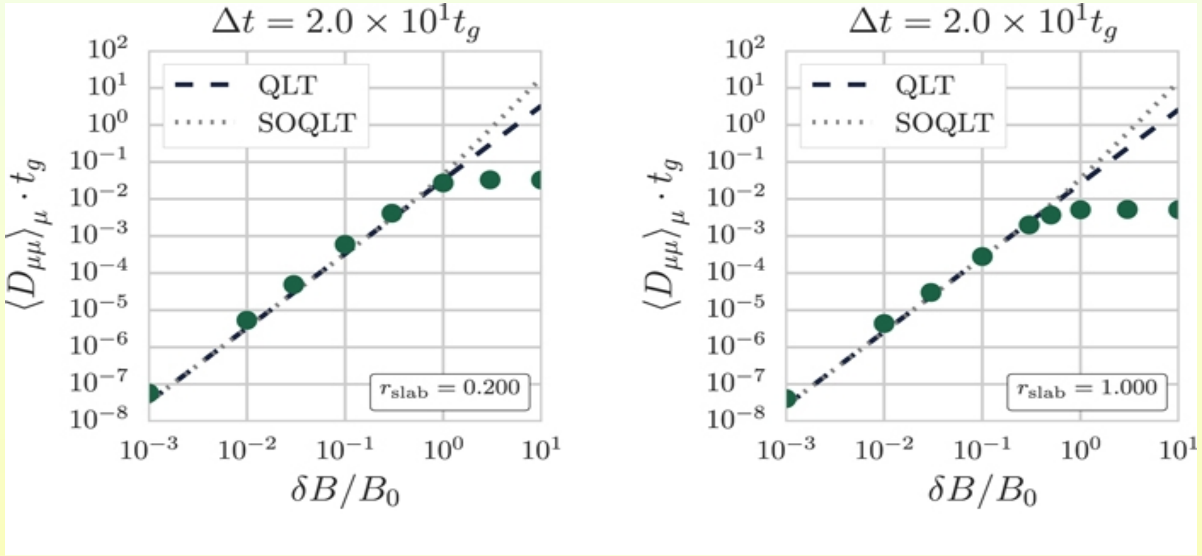
Bohm model is valid only for δB/B~1-10

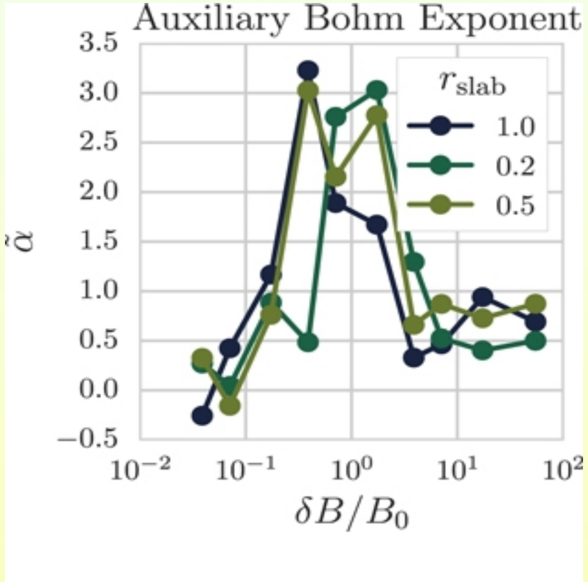
Generalized Bohm exponent

Riordan & Pe'er, 2019

Summary

- MC simulations are likely the best method to study acceleration
- QLT model is limited to δB/B <~0.1
- Bohm model has a limited validity
- Present analytical expression for Bohm exponent





Riordan & Pe'er, 2019, Ap.J., 873, 13